Data-Driven Model Order Reduction of Parameterized Dissipative Linear Time-Invariant Systems

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Abstract. We introduce a framework for data-driven model order reduction of parameterized LTI systems with guaranteed uniform dissipativity. The strategy casts the problem into a multivariate rational fitting scheme that formally preserves the bounded realness of the model response. The formulation relies on the solution of a semi-definite program arising from a rational parameterization based on Bernstein polynomials. The models can be employed in system-level simulations both in the frequency and time domain.

1 Introduction

Parameterized Reduced Order Models (pROMs) of dissipative systems are valuable tools for enabling fast simulation and optimization of complex electrical components depending on a number of free design parameters. These models reproduce the input-output behavior of the underlying structure and its dependency on the parameters, making use of a minimal set of explanatory variables. Use of pROMs drastically reduces simulation time requirements at the system level, especially for what concerns transient analyses.

In order to be fully exploitable within large system-level simulations, pROMs of physically passive structures must be compliant with the dissipativity property of the reference system for all the admissible parameters values. Even if accurate, pROMs that do not exhibit this property may be the root cause of spurious numerical instabilities, that compromise the validity of the results.

By restricting the focus on Linear Time-Invariant (LTI) systems, this contribution presents a novel data-driven framework for generating pROMs that are dissipative by construction. Differently from recent approaches based on port-Hamiltonian realizations [1,7], the proposed approach represents the model as a rational transfer function with parameterized coefficients. Based on this structure, the model identification stage involves the solution of a sequence of constrained convex rational fitting problems. By representing the parameterized coefficients of the model transfer function as Bernstein polynomials expansions, we show that the involved infinite-dimensional frequency domain conditions for dissipativity can be formulated as Linear Matrix Inequalities (LMI) of finite dimension, which are enforced in polynomial time making use of robust convex optimization solvers.

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2 Background and Notation

In the following, s will denote the Laplace variable, $\mathscr{S}_n$ the set of symmetric matrices of size n. The symbol $\otimes$ will stand for the Kronecker product, while the superscripts $\top$, $\star$ and $*$ will denote transposition, hermitian transposition and complex conjugation, respectively. The functions $b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\mathbf{x}), \mathbf{x} \in [0,1]^d \subset \mathbb{R}^d$ are multivariate Bernstein polynomials whose degree in each scalar variable is collected in the d-dimensional multi-index $\bar{\boldsymbol{\ell}} = (\ell_1, \ldots, \ell_d)$. Accordingly, $\boldsymbol{\ell}$ is an identifier for each component of this basis and $\mathscr{I}_{\bar{\boldsymbol{\ell}}}$ is the set of admissible multi-indices spanning the basis.

2.1 Problem Statement

Our goal is to generate a pROM of a P-port dissipative LTI system depending on a set of external (normalized) parameters $\boldsymbol{\vartheta} \in \Theta = [0,1]^d \subset \mathbb{R}^d$. We assume that the equations describing the reference system are not known in closed form, but that samples of its parameterized input-output frequency response $\breve{H}(s, \boldsymbol{\vartheta}) \in \mathbb{C}^{\mathrm{P}\times\mathrm{P}}$ are made available by real or virtual high-fidelity measurements

$$\breve{H}_{k,m} = \breve{H}(j\omega_k, \boldsymbol{\vartheta}_m), \quad 1 \le k \le \bar{k},\ 1 \le m \le \bar{m}, \tag{1}$$

retrieved for fixed frequency-parameter configurations. The problem is thus to synthesize a small order transfer function $\mathsf{H}(s, \boldsymbol{\vartheta})$ fitting the available samples

$$\mathsf{H}(j\omega_k, \boldsymbol{\vartheta}_m) \approx \tilde{H}_{k,m}, \quad k = 1, \ldots, \bar{k}, \quad m = 1, \ldots, \bar{m} \tag{2}$$

and at the same time preserving the dissipativity property of the underlying system. For parameterized LTI P-port systems, dissipativity can be characterized in terms of the associated transfer function. Given the following conditions

1. $\mathsf{G}(s, \boldsymbol{\vartheta})$ regular for $\Re\{s\} > 0 \quad \forall \boldsymbol{\vartheta} \in \Theta$
2. $\mathsf{G}^*(s, \boldsymbol{\vartheta}) = \mathsf{G}(s^*, \boldsymbol{\vartheta}) \quad \forall s \in \mathbb{C}, \forall \boldsymbol{\vartheta} \in \Theta$
3. a. $\mathbb{I}_{\mathrm{P}} - \mathsf{G}^\star(s, \boldsymbol{\vartheta})\mathsf{G}(s, \boldsymbol{\vartheta}) \succeq 0$ for $\Re\{s\} > 0, \forall \boldsymbol{\vartheta} \in \Theta$ Scattering
   b. $\mathsf{G}^\star(s, \boldsymbol{\vartheta}) + \mathsf{G}(s, \boldsymbol{\vartheta}) \succeq 0$ for $\Re\{s\} > 0, \forall \boldsymbol{\vartheta} \in \Theta$ Immittance

a parameterized transfer function $\mathsf{G}(s, \boldsymbol{\vartheta})$ in immittance representation is Positive Real (PR) if it satisfies conditions 1, 2, 3b, while a transfer function in scattering representation is Bounded Real (BR) if it satisfies 1, 2, 3a. The poles of PR or BR transfer functions are always stable, as required by condition 1. Immittance transfer functions are also classified as Strictly Positive Real (SPR) if they satisfy condition 1 also for $\Re\{s\} = 0$ and

$$\mathsf{G}^\star(j\omega, \boldsymbol{\vartheta}) + \mathsf{G}(j\omega, \boldsymbol{\vartheta}) \succ 0, \forall \omega \in \{\mathbb{R} \cup \infty\}, \forall \boldsymbol{\vartheta} \in \Theta \tag{3}$$

in place of 3b. A SPR transfer function exhibits no poles nor zeros on the imaginary axis [8].

Models with (S)PR or BR transfer functions are dissipative. Therefore our problem is to obtain the model transfer function $\mathsf{H}(s, \boldsymbol{\vartheta})$ in such a way that it fulfills (2) and that is PR or BR, depending on the model representation.

2.2 Model Structure

Our approach performs model generation based on model structure [5]

$$\mathsf{H}(s,\boldsymbol{\vartheta}) = \frac{\mathsf{N}(s,\boldsymbol{\vartheta})}{\mathsf{D}(s,\boldsymbol{\vartheta})} = \frac{\sum_{i=0}^{n}\sum_{\boldsymbol{\ell}\in\mathscr{I}_{\overline{\boldsymbol{\ell}}}} R_{i,\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta})\,\varphi_i(s)}{\sum_{i=0}^{n}\sum_{\boldsymbol{\ell}\in\mathscr{I}_{\overline{\boldsymbol{\ell}}}} r_{i,\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta})\,\varphi_i(s)}. \tag{4}$$

In the above, the rational dependence on the variable s is induced by the basis functions $\varphi_i(s)$, constructed from a set of fixed poles $\{q_1,\ldots,q_n\}$ with $\Re\{q_i\}<0\,\forall i>0$ as

$$\begin{cases} \varphi_i(s) = (s-q_i)^{-1}, & q_i\in\mathbb{R} \\ \varphi_i(s) = [(s-q_i)^{-1}+(s-q_i^*)^{-1}] & q_i\in\mathbb{C} \\ \varphi_{i+1}(s) = j[(s-q_i)^{-1}-(s-q_i^*)^{-1}] & q_{i+1}=q_i^*\in\mathbb{C}, \end{cases} \tag{5}$$

and $\varphi_0(s)=1$. Bases $b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta})$ are multivariate Bernstein polynomials that parameterize the model with respect to $\boldsymbol{\vartheta}$. Finally, $r_{i,\boldsymbol{\ell}}\in\mathbb{R}$ and $R_{i,\boldsymbol{\ell}}\in\mathbb{R}^{\mathsf{P}\times\mathsf{P}}$ are the unknown model coefficients. We remark that N and D are rational transfer functions sharing the same set of common poles, but exhibiting different parameterized residues. Since the common poles simplify in (4), each element of $\mathsf{H}(s,\boldsymbol{\vartheta})$ is actually a ratio of n-degree polynomials of s, with parameterized coefficients. The poles of $\mathsf{H}(s,\boldsymbol{\vartheta})$ are the parameterized zeros of $\mathsf{D}(s,\boldsymbol{\vartheta})$.

We will make use of the following state space realizations associated to $\mathsf{D}(s,\boldsymbol{\vartheta})$

$$\mathsf{D}(s,\boldsymbol{\vartheta}) \leftrightarrow \Sigma_{\mathsf{D}} = \left(\begin{array}{c|c} A_1 & B_1 \\ \hline C_1(\boldsymbol{\vartheta}) & D_1(\boldsymbol{\vartheta}) \end{array}\right), \tag{6}$$

$$\mathbb{I}_{\mathsf{P}}\mathsf{D}(s,\boldsymbol{\vartheta}) \leftrightarrow \left(\begin{array}{c|c} \mathbb{I}_{\mathsf{P}}\otimes A_1 & \mathbb{I}_{\mathsf{P}}\otimes B_1 \\ \hline \mathbb{I}_{\mathsf{P}}\otimes C_1(\boldsymbol{\vartheta}) & \mathbb{I}_{\mathsf{P}}\otimes D_1(\boldsymbol{\vartheta}) \end{array}\right) = \left(\begin{array}{c|c} A & B \\ \hline C_\otimes(\boldsymbol{\vartheta}) & D_\otimes(\boldsymbol{\vartheta}) \end{array}\right), \tag{7}$$

where the constant matrices A_1, B_1 are

$$A_1 = \text{blkdiag}\{A_{1,i}\}\in\mathbb{R}^{n\times n}, \quad B_1 = [\ldots,B_{1,i},\ldots]^\top\in\mathbb{R}^n, \tag{8}$$

$$A_{1,i} = \begin{cases} q_i, & q_i\in\mathbb{R} \\ \begin{bmatrix} \sigma_i & \omega_i \\ -\omega_i & \sigma_i \end{bmatrix}, & q_i=\sigma_i\pm j\omega_i\in\mathbb{C} \end{cases}, \quad B_{1,i} = \begin{cases} 1, & q_i\in\mathbb{R} \\ \begin{bmatrix} 2 & 0 \end{bmatrix}, & q_i=\sigma_i\pm j\omega_i\in\mathbb{C} \end{cases} \tag{9}$$

Here, (A_1,B_1) is controllable and A_1 is Hurwitz. The parameterized output matrices are Bernstein polynomial expansions defined as

$$C_1(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{\ell}\in\mathscr{I}_{\overline{\boldsymbol{\ell}}}} C_1^{\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}), \quad C_1^{\boldsymbol{\ell}} = [r_{1,\boldsymbol{\ell}},\ldots,r_{n,\boldsymbol{\ell}}]\in\mathbb{R}^{1\times n}, \tag{10}$$

$$D_1(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{\ell}\in\mathscr{I}_{\overline{\boldsymbol{\ell}}}} D_1^{\boldsymbol{\ell}}\, b_{\boldsymbol{\ell}}^{\overline{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}), \quad D_1^{\boldsymbol{\ell}} = r_{0,\boldsymbol{\ell}}\in\mathbb{R}. \tag{11}$$

Defining $A = \mathbb{I}_{\mathrm{P}} \otimes A_1$ and $B = \mathbb{I}_{\mathrm{P}} \otimes B_1$, $\mathsf{N}(s, \boldsymbol{\vartheta})$ admits the realization

$$\mathsf{N}(s,\boldsymbol{\vartheta}) \leftrightarrow \Sigma_{\mathsf{N}} = \left(\begin{array}{c|c} A & B \\ \hline C_2(\boldsymbol{\vartheta}) & D_2(\boldsymbol{\vartheta}) \end{array}\right) \tag{12}$$

$$C_2(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{\ell}\in\mathscr{I}_{\bar{\boldsymbol{\ell}}}} C_2^{\boldsymbol{\ell}} b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}) \qquad C_2^{\boldsymbol{\ell}} \in \mathbb{R}^{\mathrm{P}\times n\mathrm{P}}, \tag{13}$$

$$D_2(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{\ell}\in\mathscr{I}_{\bar{\boldsymbol{\ell}}}} D_2^{\boldsymbol{\ell}} b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}) \qquad D_2^{\boldsymbol{\ell}} = R_{0,\boldsymbol{\ell}} \in \mathbb{R}^{\mathrm{P}\times\mathrm{P}}. \tag{14}$$

For any $\boldsymbol{\ell}$, $C_2^{\boldsymbol{\ell}}$ collects the entries of $R_{i,\boldsymbol{\ell}}$, $i > 0$ with suitable ordering. Being ν a placeholder for either 2 or $\otimes$, we define for brevity the matrices

$$X_\nu(\boldsymbol{\vartheta}) = \begin{bmatrix} C_\nu^\top(\boldsymbol{\vartheta}) \\ D_\nu^\top(\boldsymbol{\vartheta}) \end{bmatrix} \begin{bmatrix} C_\nu(\boldsymbol{\vartheta}) \; D_\nu(\boldsymbol{\vartheta}) \end{bmatrix} = \sum_{\boldsymbol{m}\in\mathscr{I}_{\bar{\boldsymbol{m}}}} X_\nu^{\boldsymbol{m}} b_{\boldsymbol{m}}^{\bar{\boldsymbol{m}}}(\boldsymbol{\vartheta}), \tag{15}$$

with $\bar{\boldsymbol{m}} = 2\bar{\boldsymbol{\ell}}$. Similarly, we define the following augmented-degree representation for the output matrices of Σ_{N}

$$Y(\boldsymbol{\vartheta}) = \begin{bmatrix} C_2^\top(\boldsymbol{\vartheta}) \\ D_2^\top(\boldsymbol{\vartheta}) \end{bmatrix} = \sum_{\boldsymbol{\ell}\in\mathscr{I}_{\bar{\boldsymbol{\ell}}}} \begin{bmatrix} C_2^{\boldsymbol{\ell}\top} \\ D_2^{\boldsymbol{\ell}\top} \end{bmatrix} b_{\boldsymbol{\ell}}^{\bar{\boldsymbol{\ell}}}(\boldsymbol{\vartheta}) = \sum_{\boldsymbol{m}\in\mathscr{I}_{\bar{\boldsymbol{m}}}} Y^{\boldsymbol{m}} b_{\boldsymbol{m}}^{\bar{\boldsymbol{m}}}(\boldsymbol{\vartheta}), \tag{16}$$

which is always possible thanks to the degree elevation property of the Bernstein polynomials. In the above, each $Y^{\boldsymbol{m}}$ is obtained as a linear combination of the matrices $\begin{bmatrix} C_2^{\boldsymbol{\ell}} \; D_2^{\boldsymbol{\ell}} \end{bmatrix}^\top$, with predefined coefficients. See [2] for further details.

3 Model Dissipativity Conditions

Conditions 1, 2, 3a, 3b depend continuously on the Laplace variable and on the parameter vector $\boldsymbol{\vartheta}$. Verifying numerically the dissipativity of a pROM based on these conditions is not feasible, as it would require checking an infinite number of constraints, one for each fixed frequency-parameter configuration. The following theorem represents our main result, providing sufficient conditions to assess the dissipativity of the pROM in terms of a finite number of semidefinite constraints on the model coefficients. Without loss of generality, we provide the statement for models in scattering representation. Similar results can be derived for the immitance case.

Theorem 1. *Let* $\Omega(P,Q,R) = \begin{bmatrix} P^\top R + RP & RQ \\ Q^\top R & 0 \end{bmatrix}$. *Then,*

a) $\mathsf{H}(s,\boldsymbol{\vartheta})$ *in* (4) *is uniformly asymptotically stable over* Θ *if*

$$\exists L^{\boldsymbol{\ell}} \in \mathscr{S}_n : K^{\boldsymbol{\ell}} = \Omega(A_1, B_1, L^{\boldsymbol{\ell}}) - \begin{bmatrix} 0 & C_1^{\boldsymbol{\ell}\top} \\ C_1^{\boldsymbol{\ell}} & 2D_1^{\boldsymbol{\ell}} \end{bmatrix} \prec 0 \quad \forall \boldsymbol{\ell} \in \mathscr{I}_{\bar{\boldsymbol{\ell}}} \tag{17}$$

b) $\mathsf{H}(s,\boldsymbol{\vartheta})$ *is uniformly Bounded Real over* Θ *if, additionally,*

$$\exists P^{\boldsymbol{m}} \in \mathscr{S}_{nP} : J^{\boldsymbol{m}} = \begin{bmatrix} \Omega(A,B,P^{\boldsymbol{m}}) - X_\otimes^{\boldsymbol{m}} & Y^{\boldsymbol{m}} \\ Y^{\boldsymbol{m}\top} & -\mathbb{I}_P \end{bmatrix} \preceq 0, \quad \forall \boldsymbol{m} \in \mathscr{I}_{\bar{\boldsymbol{m}}}. \tag{18}$$

We provide here a sketch of the proof for Theorem 1, more detailed derivations are available in [3]. The uniform model stability condition (a) stems from enforcing the denominator $\mathsf{D}(s, \boldsymbol{\vartheta})$ to be SPR. In fact, the zeros of a SPR function are guaranteed stable, and since the zeros of $\mathsf{D}(s, \boldsymbol{\vartheta})$ are the poles of $\mathsf{H}(s, \boldsymbol{\vartheta})$, uniform stability follows. The SPR conditions on $\mathsf{D}(s, \boldsymbol{\vartheta})$ are written in algebraic form using a (parameterized form of) the Kalman-Yakubovich-Popov (KYP) Lemma, which is then discretized in the parameter space by a Bernstein polynomial expansion. A straightforward derivation leads to the sufficient condition for uniform stability expressed by (17). A similar process is used to derive the uniform dissipativity condition in (b). The KYP lemma is again used to eliminate dependence on frequency in condition 3a, and a Bernstein polynomial expansion provides a discretized form of the corresponding parameterized LMI condition for BR-ness. The result is the sufficient condition in (18). The key enabling factors on which this proof is built are the model structure (4) with the associated parameterized state-space realizations of Sect. 2.2, and the properties of the Bernstein polynomials which allow proving positivity (negativity) of a parameter-dependent matrix by constraining the sign of its Bernstein coefficients.

4 Model Generation

In this section, we present our approach to generate dissipative pROMs in scattering representation via semidefinite programming, exploiting the theoretical results of Theorem 1. As in standard approaches based on model structure (4), we meet condition (2) iteratively, enforcing a sequence of linearized approximations

$$\frac{\mathsf{N}^{\mu}(j\omega_k, \boldsymbol{\vartheta}_m) - \mathsf{D}^{\mu}(j\omega_k, \boldsymbol{\vartheta}_m)\breve{H}_{k,m}}{\mathsf{D}^{\mu-1}(j\omega_k, \boldsymbol{\vartheta}_m)} \approx 0, \quad k = 1, \ldots, \bar{k}, \quad m = 1, \ldots, \bar{m}, \tag{19}$$

where $\mu = 1, 2, \ldots$ is an index for the iteration. At iteration μ, $\mathsf{D}^{\mu-1}$ is numerically available[1] so that relation (19) can be recast in matrix form as

$$\begin{bmatrix} \Psi_x^{\mu} & \Psi_y^{\mu} \end{bmatrix} \begin{bmatrix} x^{\mu} \\ y^{\mu} \end{bmatrix} \approx 0 \tag{20}$$

where vectors x^{μ}, y^{μ} collect the current numerator and denominator coefficients $R_{i,\ell}$ and $r_{i,\ell}$, respectively, and Ψ_x^{μ} and Ψ_y^{μ} are known matrices. The approximation (20) is then enforced in a least-squares sense. The iteration stops whenever $\mathsf{D}^{\mu}(j\omega, \boldsymbol{\vartheta}) \simeq \mathsf{D}^{\mu-1}(j\omega, \boldsymbol{\vartheta})$, so that (19) becomes equivalent to (2).

Since only the denominator variables $y^{\mu-1}$ are required to set up problem (20), the iteration admits a fast implementation based on the elimination of the variables x^{μ}, that are computed only once convergence is met. The elimination procedure is based on computing the QR factorization of the matrix $\begin{bmatrix} \Psi_x^{\mu} & \Psi_y^{\mu} \end{bmatrix}$, as thoroughly discussed in [6]. After the variable elimination, (20) is replaced by the smaller denominator estimation problem

$$\Gamma_y^{\mu} y^{\mu} \approx 0, \tag{21}$$

[1] We set $\mathsf{D}^0(j\omega, \boldsymbol{\vartheta}) = 1$ at the first iteration to initialize the denominator estimate.

being Γ_y^{μ} a known matrix. We constrain the estimation with the stability conditions (17), by solving the following semi-definite program

$$\min_{y^{\mu}, L^{\boldsymbol{\ell}}} \|\Gamma^{\mu} y^{\mu}\|_2 \text{ subject to: } \quad \Omega(A_1, B_1, L^{\boldsymbol{\ell}}) - \begin{bmatrix} 0 & C_1^{\boldsymbol{\ell}\top} \\ C_1^{\boldsymbol{\ell}} & 2D_1^{\boldsymbol{\ell}} \end{bmatrix} \prec 0 \quad \forall \boldsymbol{\ell} \in \mathscr{I}_{\overline{\boldsymbol{\ell}}}, \tag{22}$$

so that the resulting y^{μ} guarantees a stable model by construction.

Problem (22) is solved repeatedly until convergence, that is practically met when the condition

$$\delta^{\mu} = \frac{\left\|y^{\mu} - y^{\mu-1}\right\|_2}{\|y^{\mu}\|_2} \leq \varepsilon \tag{23}$$

holds with a user-defined small threshold $\varepsilon > 0$. Supposing this condition is met at iteration $\bar{\mu}$, we complete the model generation by estimating the numerator unknowns $x^{\bar{\mu}}$. This can be done by substituting the available denominator coefficients $y^{\bar{\mu}}$ in (20) and enforcing the resulting condition

$$\Psi_x^{\bar{\mu}} x^{\bar{\mu}} \approx -\Psi_y^{\bar{\mu}} y^{\bar{\mu}}. \tag{24}$$

Since (17) holds by construction, we enforce (24) in such a way that the solution satisfies (18), so that $\mathsf{H}(s, \boldsymbol{\vartheta})$ is BR and the final model is dissipative. To this aim, we observe that the matrices $X_{\otimes}^{\boldsymbol{m}}$ in (18) are known, as they are defined upon the available denominator coefficients $y^{\bar{\mu}}$. Since the terms $Y^{\boldsymbol{m}}$ are obtained as linear combinations of the numerator unknowns $x^{\bar{\mu}}$ according to (16), we enforce (24) in a least-squares sense, by solving another semi-definite program

$$\min_{x^{\bar{\mu}}, P^{\boldsymbol{m}}} \left\|\Psi_x^{\bar{\mu}} x^{\bar{\mu}} + \Psi_y^{\bar{\mu}} y^{\bar{\mu}}\right\|_2 \quad \text{s.t.} \quad \begin{bmatrix} \Omega(A, B, P^{\boldsymbol{m}}) - X_{\otimes}^{\boldsymbol{m}} & Y^{\boldsymbol{m}} \\ Y^{\boldsymbol{m}\top} & -\mathbb{I}_{\mathrm{P}} \end{bmatrix} \preceq 0, \quad \forall \boldsymbol{m} \in \mathscr{I}_{\overline{\boldsymbol{m}}}, \tag{25}$$

which guarantees the bounded realness of $\mathsf{H}(s, \boldsymbol{\vartheta})$.

We remark that since Theorem 1 provides only sufficient conditions for the verification of model dissipativity, enforcing constraints (17), (18) may introduce some conservativity in the model generation process, by over-restricting the set of feasible model coefficients. A systematic approach based on the degree elevation property of the Bernstein polynomials can be used to arbitrarily reduce this conservativity, at the price of introducing additional instrumental variables. Full details about this procedure are available in [3].

5 A Test Case

The proposed strategy is applied to generate a dissiparive pROM of a high-speed interconnect link, designed as in [4]. We let the structure behavior depend on $d = 2$ geometrical parameters related to the vertical interconnect on a Printed Circuit Board, namely the pad radius $\vartheta_1 \in [100, 300]$ μm and the associated antipad radius $\vartheta_2 \in [400, 600]$ μm. The dataset (1) is retrieved by performing virtual measurements of the 2×2 scattering matrix of the structure, computed from a physics-based Maxwell equations solver in the bandwidth $[0, 5]$ GHz. Then we generate the pROM as described in Sect. 4, fixing the model order to $n = 25$. The remarkable accuracy of the resulting model is demonstrated in Fig. 1, through a comparison with a set of validation responses (not used for training) for different parameter configurations.

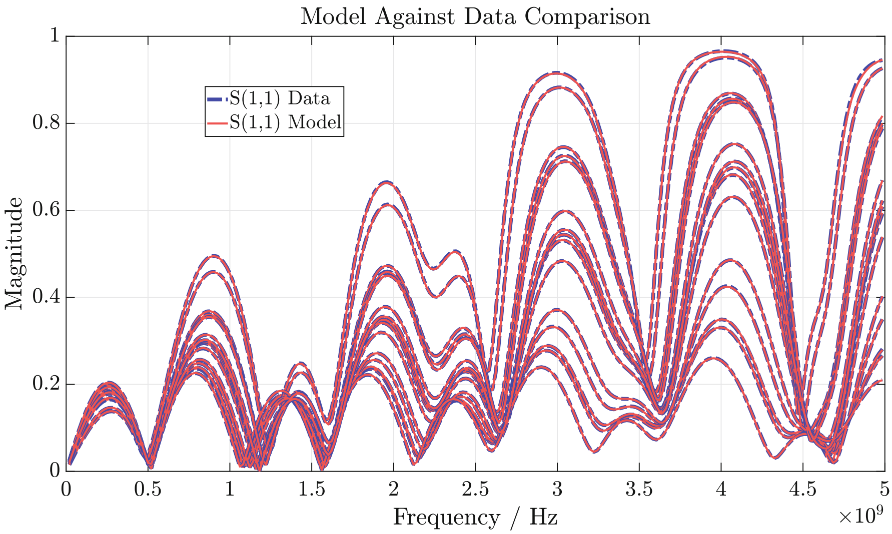

Fig. 1. Model-data comparison for different parameter configurations.

6 Conclusions

We presented a novel data-driven approach for generating pROMs with theoretical dissipativity certification. The method constrains the model training with finite dimensional linear matrix inequalities that are shown to guarantee the dissipativity of the model throughout the parameter space. Being based on convex programming, the approach is fully deterministic and returns accurate and compact parameterized models.

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