

Non-linear Finite Element Restorative Analysis of Low Shear Resistant RC Beams Strengthened with Bio-Sisal and GFRP

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Abstract. Restoration is strengthening of structural components to its original strength characteristics or with an even better strength characteristic. Damaging events such as earthquakes incepts various degradation processes in structures, restoration aids in making the structures overcome these damaging effects and enhances its characteristics thereby leading to structural life enhancement. Nonlinear finite element method (NLFEM) is a mathematical tool by means of which various strengthening or restorative methodologies can be evaluated through virtual platform and thus it results in economic savings and aids in decision making before actually being implemented with accuracy and optimization in the field works. In this paper the restorative analysis by means of NLFEM is carried out for evaluating the restorative potential of sisal fibre made fibre reinforced polymer (FRP) and a comparative analysis with that of GFRP has been done, in enhancing the shear resistance of RC beams (designed for shear-oriented failure) with these respective bonded FRPs. Sisal fibres have huge environmental positive contributing factors over artificially fabricated glass fibres and its composites, as glass is mainly obtained from silica, which is again obtained from non-renewable resources. Concrete crushing parameter alongside concrete cracking parameters, small deflection, static analysis with appropriate boundary conditions, and suitable sub-steps were all considered in the NLFEM analysis of bio-sisal FRP affixed RC beam and also the GFRP affixed RC beam. FRP composite and RC beam affixation was virtually designed buy using contact elements. The results obtained demonstrated huge potential in shear resistance enhancements in RC beams using both sisal FRP and GFRP affixation process. The paper ended with a recommendation to practicing engineering for utilizing natural sustainable bio based FRPs over artificial ones for restorative purpose.

Keywords: Analytical verification · finite element analysis · shear · FRP · Sisal · Beams

1 Introduction

Restoration is the science of parametrical enhancements of various building components with strength, stiffness and lateral stability, so that it can again be brought back to life with improved functionalities. The process of restoration can be done with fibre

reinforced polymer composites for strength parametrical enhancements using different fibres of artificial nature such as carbon based fibre reinforced polymer composite, glass fibre based reinforced polymer composite, and aramid fibre based reinforced polymer composites, successfully as past research has divulged $[1-10]$ $[1-10]$. There are various advantages of using the fibre reinforced polymer (FRP) restorative methodology over other methodologies such as reinforced concrete affixation on top of or surrounding the various structural components, steel plate affixation, ferrocement affixation, supplemental damper affixation, wing-wall affixation etc. FRPs are completely light weight than reinforced concrete layer affixation over a particular structural component or addition of wing wall component. FRPs have higher flexibility in the affixation process over all other processes. FRPs possess very high ultimate tensile strength and tensile modulus as compared to other restorative methodologies. Affixation process with FRPs are very simple over other methodologies. Non-linear finite element method (NLFEM) is a mathematical tool by means of which various restorative methodologies can be evaluated through virtual platform and thus it results in economic savings and aids in decision making before actually being implemented with accuracy and optimization in the field works. In reality, experimental procedures are intensively labour dependent and bear heavy economic liabilities, hence before practically implementing any new methodology with newer materials, numerical simulation aids us in proper justification of the methodology to be implemented. In this paper the restorative analysis by means of NLFEM is carried out for evaluating the restorative potential of sustainable and renewable based material which is bio-sisal obtained from local resources and using completely organic fibres which are connected with rural economy and a comparative study with the restorative potential of glass fibre reinforced polymer composite (GFRP) which bear non-renewable primary pre-cursors and are a processed industrial product, have been studied.

Effectiveness and accuracy of various linear and non-linear analysis with static pre and post deflection models, has been strongly established by various past researchers, for simulating the effect and restorative potential of various forms of FRP attached to reinforced concrete surfaces [\[11](#page-9-2)[–32\]](#page-11-0). Predominantly non-linear analysis of RC beams designated with shear failure characteristics are of important aspect-oriented study, the reason being, shear, associated with damage which is completely brittle in nature. And can result in sudden, non-deflecting, highly catastrophic failures without giving out prior signs or warnings to the building occupants $[1, 3, 6, 9, 10]$ $[1, 3, 6, 9, 10]$ $[1, 3, 6, 9, 10]$ $[1, 3, 6, 9, 10]$ $[1, 3, 6, 9, 10]$ $[1, 3, 6, 9, 10]$ $[1, 3, 6, 9, 10]$ $[1, 3, 6, 9, 10]$ $[1, 3, 6, 9, 10]$. The study incorporates reinforced concrete RC beams designated with shear failure characteristics and understanding the restorative potential of the same, when affixed with bio-sisal FRP vis-à-vis GFRP respectively. Understanding the crushing and cracking features and the maximum load at shear before the RC beams goes into the plastic deformation phase, of these respective models, with the aid of the NLFEM of analysis forms the core of the paper.

The organization of the article (research as presented) basically deals with the following sub-steps: Firstly the design of the reinforced concrete (RC) beams for shear failure criteria (so that bending failure takes place after the onset of shear dominated cracks and failure), then preparation of the RC beam reinforcement detailing, followed by the generation of the finite element model in the Ansys virtual platform considering the reinforcement detailing and the RC beam properties. Then, considering all the

evaluated values of the various components such as Concrete, reinforcement steel, FRP composites of Bio-sisal and Glass and the steel plates for loading and support, and assigning respective properties in the Ansys virtual environment. Thereafter crack modelling parameters and failure criteria parameters and properties being set in the Ansys virtual environment, also ensuring proper inter-elemental connectivity for monolithic effect, thereafter the loadal application and the support system modeling in the Ansys virtual environment, then the setting of the convergence criteria in the Ansys virtual environment for solution obtainment. And finally obtaining the Ansys based crack pattern and detailed analysis-based output and presentation of the Ansys generated results and thereby giving a comparative statement of the maximum ultimate shear load being carried by bio-sisal FRP wrapped RC beams and glass FRP wrapped RC beams.

2 Models for Non-linear NLFEM

IS- 456:2000 [\[33\]](#page-11-1) has been referred to for the design of the reinforced concrete beam designated for shear failure. The reinforced concrete beam design was such that shear failure was predominant and the beam board higher flexural resistance in-order to generate shear cracks before the onset of bending cracks. Model ShearRC1 is that finite element model where no FRP was bonded on the face of the beam. Model Bio-Sisal2, is that finite element model where sisal FRP composite was bonded over the three faces of the RC beam, and finally, Model GFRP3 is that finite element model where GFRP composite was bonded over the three faces of the RC beam.

2.1 Design for Predominant Shear Failure in the NLFEM of Analysis

Shear brittle failure is characterized by shear cracks which appear to be almost $45⁰$ inclined cracks near the support regions of the beams. These come without any prior deflections so that sufficient warning signs can be vetted out. As these cracks are sudden and brittle in nature without any display of post bending deformations, hence study of the same becomes extremely important to avoid and prevent any catastrophic damage to the structures. The design in the simulative environment i.e. NLFEM of analysis using the Ansys 12.0 software was done considering the provision of additional flexural reinforcements i.e. longitudinal tensile reinforcements considering double bending effect, thereby providing in higher tensile reinforcement of 159.435 mm². It was observed that considering only single bending effect, the tensile reinforcement requirement was of 106.29 mm^2 , whereas by considering the double bending effect, the total area of tensile reinforcement almost increased by 50%. In this way, determinant shear cracks can be obtained in the simulative environment. In total 4 numbers of 8 mm dia bars was provided in the pure bending zone, as the tensile reinforcement. Considering single shear effect, such that shear resistance of the RC beam is lesser than the bending resistance, 2 legged 8 mm dia steel stirrup bars at a clear spacing.

Fig. 1. Reinforcement detailing of Model ShearRC1, Model Bio-Sisal2 and Model GFRP of 130 mm C/C was provided throughout the beam. For all models i.e. Model ShearRC1, Model Bio-Sisal2 and Model GFRP3, the same reinforcement design was carried out and followed while modeling in the simulative environment. Figure 1 Shows the detailing of the steel reinforcement of all the beams.

3 Non-linear Restorative Analysis Using Non-linear Finite Element Method

3.1 Methodology of the Modeling and Meshing in the Simulative Environment

Finite element method of analysis is an elemental level analysis which basically starts with the discretization technique. Using the elemental discretization procedure, various composite elements in the model to be built up is broken down into elements connected at nodes, having individual characteristics or properties. In order to discretize concrete the SOLID-65 element has been very widely accepted with good accuracy of results. LINK-8 element has been widely accepted for simulating the steel reinforcement in discreet analysis using the FEM platform. SOLID-45 element has been widely used by past researchers for simulating the effect of steel loading plates and steel supports, and these have provided very good results without causing any deformities and have provided monolithic effect without any in-compatibility within the various elements in the structural model. FRPs of glass fibre reinforced polymer and bio-degradable sisal fibre reinforced polymer composites have been discretized using the SHELL-99 element. SHELL-99 element has been successfully used for discretizing reinforced polymer composites in various engineering simulative problems by various past researchers [\[11](#page-9-2)[–25\]](#page-10-0). This elemental discretization of various concrete constituents such as hardened cement,

steel reinforcement, loading and support plating system utilizing these stated elements have been widely studied and results with good ac curacy has been established by past works by various researchers [\[11–](#page-9-2)[25\]](#page-10-0). After assigning the elemental properties, material properties were assigned to each of these elements for the generation of the RC beam model. The concrete element which is basically done elemental discretization using the Solid65 element in Ansys uses Material Model Number 1 for its property classification. Concrete elemental discretization basically uses both characteristics of linear and multi-linear isotropic models. The Willam and Warnke model [\[26\]](#page-10-1) aids in defining the multilinear characteristics of the failure criterion of reinforced concrete in conjunction with the Von Misses failure criterion. The modulus of elasticity of the concrete (Ec) is defined in the Ansys platform by the parameter EX, and PRXY is the parameter which defines the Poisson's ratio (v) of reinforced concrete. The modulus of elasticity of concrete can be evaluated as per the equation as defined by (Clause. 6.2.3.1 of IS 456: 2000), $Ec = 5000\sqrt{fck}$, with the characteristic compressive strength value of *fck* being considered as 20 MPa. Poisson's ratio of reinforced concrete is considered as 0.2 (Kachlakev et al., [\[16\]](#page-10-2) for the said analysis. SOLID-65 was considered for multilinear isotropic

values and the following stress-strain graph as shown in Fig. [2](#page-5-0) was considered to model the bending characteristics of concrete. For modeling the crack-crush characteristics following characteristics for concrete have been used, shear transfer between elemental interfaces plays a vital role in the crack modeling of the non-linear finite element analysis of reinforced concrete structures, the coefficient of transfer of shear between elemental interfaces is considered in the range of 0.0 to 1.0. Basically coefficient of 0 describes smooth elemental crack in the SOLID65 concrete model and coefficient of 1 describes rough elemental crack in the SOLID65 concrete model. Kachlakev et al. [\[16\]](#page-10-2) has carried out massive research for determining the shear transfer coefficients for crack modelling of reinforced concrete using finite element method. Their study ranged from determining coefficients of shear transfer from smooth to rough cracks and also cracks falling in the intermediate sections, i.e. semi rough cracks. In non-linear finite element analysis convergence of the solution and obtainment of results is the primary criterion. It has been reported that non-convergence of solution (related to crack modeling of concrete) takes place when the shear transfer coefficient for cracks which are considered "open" falls below 0.2. Considering the said the coefficient of shear-transfer for open crack is considered 0.3 here, for the carried out analysis.

The reinforcement in the concrete is described by the parameter, Material Model Number 2. This parameter describes the elemental properties of the reinforcement defined by the Link8 element. Link8 element has been considered for elemental discretization of the longitudinal reinforcements at the beam bottom and top (reinforcement bars) and also the stirrup reinforcements, used for taking care of the shear forces. The said material, as defined in Material Model Number 2, is considered as bilinear-isotropic and has been derived from the failure criterion of "Von Mises failure criteria". This bilinear model has a requisite of two parameters being defined, one is the yield stress designated by *fy,* which is considered here as 415 MPa, and the other is the hardening modulus of the steel reinforcement, which is also required to be defined. For reinforcement steel modeling, the parameter of the tangent modulus (mainly considered in the plastic zone of the stress-strain curve) is considered to be zero. The modulus of elasticity for the link8

Fig. 2. Stress-strain curve for concrete as presented by by Ibrahim and Mahmood (2009), Wolanski [\[27\]](#page-10-3) and Kachlakev et al. [\[16\]](#page-10-2).

model is considered to be 2,00,000 MPa, which is designated as EX. The Poisson's ratio, which is designated as PRXY is considered as 0.3. Figure [3](#page-5-1) shows the stress-strain curve for steel reinforcement, i.e. modeled using the Link8 element. Same characteristics have been used for describing the real constant properties of Solid45 element used for elemental discretization of the loading and the supports.

Fig. 3. Tension-compression (bending) curve for steel-reinforcement and loading as well as support roller system.

For assigning material properties to the FRP composites, SHELL-99 element was used for simulating Bio-sisal FRP composites, modulus of elasticity as designated by EX is considered as 234 MPa and the characteristic Poisson's ratio as designated by PRXY is considered as 0.32, with a material thickness of 3.98 mm for the bio-sisal FRP composites. SHELL-99 element was also used for elemental discretization of the GFRP composites, modulus of elasticity as designated by EX is considered as 678 MPa and the characteristic Poisson's ratio as designated by PRXY is considered as 0.21, with a material thickness of 1.4 mm for the GFRP composites.

Volumetric modeling of all individual elements was carried out, and all volumes were overlapped. Any orphaned nodes, which basically loose connection with the adjacent nodes, were carefully removed. All precautions were taken such that the nodes which are orphaned are removed, and all volumes are "glued" together so as to result in all the good properties to be satisfied by the displacement function, i.e. the convergence and the compatibility criteria are all fulfilled. Rectangular mesh of concrete element has been recommended by Wolanski, [\[27\]](#page-10-3) and Kachlakev et al., [\[16\]](#page-10-2) and the same has been carried out. All volumes of concrete, and steel rollers for loading and the steel support were meshed using the rectangular "mapped" meshing option. Even the composites of bio-sisal FRP and GFRP and their respective volumes were mapped mesh in the Model Bio-Sisal2 and Model GFRP3 for understanding the evaluating the restorative effect on reinforced concrete beams. Only half length of the RC beam alongside the composite face has been modeled due to symmetricity. NLFEM of analysis makes use of symmetrical configuration of loading and geometry and automatically projects the output for the complete beam. Symmetrical BC option was utilized for activating this feature.

Concentrating nodes, defined on a single plane of the bottom steel support (modeled at the support location) was given the constraint to model or simulate the support condition, which is "simply-supported" in this case of analysis. (degree of freedom to be constrained) restrainment was generated in the global UY and the global UZ directions, by applying 0 constraint. By this method of restrainment generation, the "simply supported" beam will be allowed to rotate at the support. The loadal force, P, is basically applied at top location where the steel plate for load application, has been modeled. The application of the vertical force is done by dividing the total force by the total number of nodes present along the plane of force application, so that all the nodes can balance the entire applied force equally and can result in equal force distribution at the center-line of nodes present on the steel plate for load application. Small deflection alongside static analysis was carried out with a total number of sub-steps being considered as 100. The convergence criteria was based on the structural load value, which was set at 200 KN (Figs. [4](#page-7-0) and [5\)](#page-7-1).

3.2 Finite Element Restorative Analysis and Conclusionary Studies

A lot of research has been undertaken by the past researchers in understanding the crack and crush characteristics of reinforced concrete [\[2,](#page-9-6) [6,](#page-9-4) [12](#page-9-7)[–14,](#page-10-4) [28](#page-10-5)[–32\]](#page-11-0). Red colour of vector plots in the form of red lines represents cracks in reinforced concrete with lesser width, whereas the blue-green colour of the vector plots in the form of blue-green lines represents critical failure criteria resulting in cracks of larger width which ultimately results in failure. The blue-green colour crack pattern basically results in the ultimate failure of the

Fig. 4. NLFEM of analysis and its constructive model generation of: (a) steel reinforcement inside the RC beam; (b) The beam with the FRP composite in a three sided wrapping configuration.

Fig. 5. NLFEM of analysis and its constructive model generation of: (a) symmetricity in the X longitudinal direction; (b) Loading generation at the top of the loading plate; (c) Support reaction at the support end.

specimens. So, in order to understand the NLFEM of analysis generated failure patterns, understanding the trajectory of blue-green vector plots becomes extremely important. Also the non-linear FEM of analysis stores and saves the cracking pattern generated by all the individual load step and finally presents a cumulative crack-crush vector plot at the ultimate or the failure load of the respective specimen. So, a composite load crack and crush characteristic vector plot of all the load sub steps is presented at the end of the iteration when the convergence criteria is met.

We can see well from the Fig. $6(a)$ $6(a)$ that the beam where no FRP was bonded on the face of the beam, i.e. Model ShearRC1, the failure pattern is that of diagonal shear cracking, indicating weakness in the shear resistance over the bending resistance. From Fig. [6\(](#page-8-0)b) i.e. Model Bio-Sisal2, where sisal FRP composite was bonded over the three faces of the RC beam, the failure pattern is a mixed composite failure of flexure-shear tensile cracking and shear - diagonal cracking, the overall shear-cracks in the model is considerably lower than that of the Model ShearRC1 in the shear zone, indicating the strength-stiffness contribution of the Bio-Sisal FRP to the RC beam. From Fig. [6\(](#page-8-0)c) i.e. Model GFRP3, where GFRP composite was bonded over the three faces of the RC beam, the failure pattern mild shear-tensile cracking and mild shear - diagonal cracking, the overall shear-cracks in the model is considerably lower than that of the Model ShearRC1 as well as Model Bio-Sisal2, in the respective shear zones, indicating the strength-stiffness contribution of the GFRP to the RC beam. As GFRP has higher ultimate tensile strength over Bio-Sisal FRP, so strength-stiffness contribution is also higher in terms of restorative effect. But for both the models Model Bio-Sisal2 and Model GFRP3, overall the shear-diagonal tensile cracking pattern is much lesser than that of the Model ShearRC1. Also the ultimate failure load as obtained from the NLFEM of analysis is as presented in Table [1.](#page-9-8)

Fig. 6. Diagonal shear-tensile cracking and respective vector-plots of the FEM models (a) NLFEM generated crack-vector-plot of Model ShearRC1, (b) NLFEM generated crack-vector-plot of Model Bio-Sisal2, (c) NLFEM generated crack-vector-plot of Model GFRP3.

NLFEM of analysis and respective beam models	Thickness of FRP (mm) and ultimate tensile strength of FRP(MPa)	Maximum shear failure load at convergence criteria of load (KN)	Predominant failure pattern	Enhancement in shear load
Model ShearRC1		98 KN	Heavy diagonal shear tensile cracking	
Model Bio-Sisal 2	3.98mm 234MPa	172 KN	Diagonal shear tensile cracking alongside mild flexure-shear cracking	0.75
Model GFRP3	1.4 _{mm} 678MPa	184 KN	Mild diagonal shear tensile cracking alongside mild flexure- shear cracking	0.88

Table 1. Results of the restorative NLFEM of analysis

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