





Robust Optimization for Operating Room Scheduling with Uncertain Surgical Durations: Impact of Risk-Aversion on Delay

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Abstract. We introduce a robust optimization model for scheduling operating rooms with uncertain surgical durations. The model addresses multiple operating rooms and surgical procedures. In the numerical analysis, we verify the influence of the risk-averse tendency on the schedule. The schedules created by the robust optimization are compared with those of stochastic programming. The results suggest that robust optimization avoids long delays, and obtains a solution faster than stochastic programming. In specific control conservative, robust optimization exhibits the same performance as stochastic programming. The robust optimization model is more effective for operating room managers who desire to obtain an accurate solution quickly.

Keywords: Operations research in health service · Operating room scheduling · Robust optimization · Stochastic programming

1 Introduction

Hospital management is critical to improving the quality of service to patients [19]. Surgeries account for most of the hospital revenue and expenditure [15, 18]. Efficient surgical management is required to achieve optimal hospital management. By clarifying the cost structure underlying operating room time, Dexter and Macario revealed that improved operating room scheduling can effectively reduce costs [8]. Creating a robust operating room schedule is effective in managing the operating room.

In the flow of the operating room scheduling, the surgeon and patient decide the surgery date through mutual agreement. The surgeon then reports the

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surgery date and estimated duration of surgery to the operating room manager. The manager decides when and in which operating room to perform the surgery based on information such as the estimated duration of surgery and department.

It is the issue of operating room management that surgery is often not performed according to the schedule. The quality of service to patients is affected because of the waiting time occurrence owing to the delay from the scheduled end time of surgery. Surgical duration is uncertain, influenced by the patient's condition, lack of information on the preoperative diagnosis, and the surgeon's skill. The challenge is to cope with the uncertainty of the surgical duration.

The manager desires to avoid a risk of delay, with surgery being delayed significantly from the scheduled end time. Long delays lead to increased overtime for surgical staff, not only increasing costs, but also reducing staff satisfaction. In operating room scheduling, it is necessary to consider decision-making to avoid the risk of delay.

In this study, we propose a robust optimization model that considers surgical procedures and minimizes delay from the regular opening time of the operating room. After calculating the delay for uncertain surgical duration parameter sets in the numerical analysis and comparing the performance of the proposed model to those of the stochastic programming model, we verify whether risk avoidance tendencies are reflected in schedules. In the proposed model, we consider multiple operating rooms that could not be considered in Namba et al. [14].

From the numerical results, three important points are obtained as follows:

- Robust optimization tends to avoid long delays.
- Robust optimization, which has only information on the 10th and 90th percentile duration of the scenario, exhibits the same performance as stochastic programming, which has complete information on the scenario in specific control conservative.
- Robust optimization obtains a solution faster than stochastic programming because the number of variables and constraints in robust optimization is smaller than in stochastic programming.

The remainder of the paper is organized as follows: Sect. 2 provides a literature review. Section 3 introduces the problem setting and robust optimization model of operating room scheduling. In Sect. 4, we describe the data used in the study and report the results of our numerical experiments. We conclude the paper in Sect. 5 with comments regarding matters for future exploration.

2 Literature Review

Operating room scheduling has been studied extensively [5, 10, 20]. A few studies have proposed a stochastic model for operating room scheduling [2, 9, 17]. Batun et al. [4] presented a two-stage stochastic mixed-integer programming model with uncertain surgical duration. Addis et al. [1] proposed the operating room rescheduling with uncertain patient arrival and surgical duration.

Kamran et al. [16] approached the operating room scheduling problem with different formulations of stochastic programming. Ito et al. [11,12] proposed a stochastic programming model for scheduling an operating room using the conditional value-at-risk (CVaR). The CVaR expresses the risk-aversion of the manager towards the risk that the surgical duration estimated by the surgeon could be significantly delayed. Another technique that reflects delayed risk aversion is robust optimization. Bandi and Gupta [3] developed a new criterion and a robust optimization approach for staffing and operating room scheduling problems under uncertain case mix and case lengths. Denton et al. [6] proposed an operating room scheduling model with robust optimization to address the uncertain surgical duration.

Our work is somewhat related to Denton et al. [6], but is particularly different in that their study did not address the sequence of surgeries in the operating room. It is important to consider the sequence of surgeries within the operating room [7]. The manager is making efforts regarding the order of surgeries, e.g., surgeries belonging to the same department consecutively perform when arranging surgical equipment and adjusting schedules.

3 Operating Room Scheduling

3.1 Robust Optimization

We propose a robust optimization model for the operating room scheduling problem under uncertain parameter sets; the worst-case that results in maximum total surgical duration. The operating room scheduling determines the allocations to an operating room and surgery procedures.

Surgeries are limited to elective surgeries with prior consent between the patient and surgeon; thus, we did not consider the interruption of emergency surgery. In addition, all operating rooms are treated with the same function. We define surgical duration as the difference between a patient's entry times and when the patient leaves the operating room.

The formulation of the maximum surgical duration problem, which is considered the main problem, is presented in Sect. 3.2. The formulation of the operating room scheduling problem is shown as follows:

Notation

Index sets.

J : Set of surgeries.

D : Set of departments.

E_d ($d \in D$): Set of surgeries belonging to the department d .

M : Set of operating rooms.

Parameters.

d_m ($m \in M$): Regular opening time of the operating room.

n_d ($d \in D$): Number of surgeries in department d , $n_d = |E_d|$.

$\overline{p_j}, p_j (j \in J)$: Upper and lower bounds on the duration for surgery j .

τ : Control conservative. Set how conservatively you want to control the worst-case scenario from the decision-maker's perspective. This represents the number of surgeries for which the upper bound of the surgical duration is reached.

Variables.

$p_j (j \in J)$: Duration of surgery j .

$c_j (j \in J)$: Finishing time of surgery j .

$t_m (m \in M)$: Delay from the regular opening time of the operating room.

$z_{ij} (i, j \in J, i \neq j)$: Binary variable for surgery precedence, where $z_{ij} = 1$ if surgery i is processed before surgery j , $z_{ij} = 0$ otherwise.

$x_{mj} (m \in M, j \in J)$: Binary variable for surgery assignment to the operating room, where $x_{mj} = 1$ if in operating room m , surgery j is assigned, $x_{mj} = 0$ otherwise.

$\gamma_{mij} (m \in M, i, j \in J, i \neq j)$: Linearized binary variables, where $\gamma_{mij} = 1$ if in operating room m , surgery i precedes surgery j , $\gamma_{mij} = 0$ otherwise.

$\theta_m (m \in M)$: Linearized binary variables, where $\theta_m = 1$ if surgeries l and k in the department d perform in operating room m , $\theta_m = 0$ otherwise.

$\alpha, \beta_j (j \in J)$: Dual variables.

Formulation

Minimize

$$\sum_{m \in M} t_m \quad (1)$$

subject to

$$\sum_{i \in J} p_j x_{mj} \leq d_m + t_m, \quad \forall m \in M, \quad (2)$$

$$\sum_{m \in M} x_{mj} = 1, \quad \forall j \in J, \quad (3)$$

$$z_{ij} + z_{ji} = 1, \quad i \neq j, \forall i, j \in J, \quad (4)$$

$$z_{ij} + z_{jk} + z_{ki} \leq 2, \quad i \neq j, j \neq k, k \neq i, \forall i, j, k \in J, \quad (5)$$

$$\sum_{j \in J} j x_{(m-1)j} \geq \sum_{j \in J} j x_{mj}, \quad m = 2, \dots, |M|, \quad (6)$$

$$\sum_{m \in M} m x_{mi} - \sum_{m \in M} m x_{mj} = (|M| - 1) z_{ji}, \quad i \neq j, \forall i, j \in J, \quad (7)$$

$$\sum_{i \in E_d} \sum_{j \in J} z_{ij} = \frac{n_d(n_d + 1)}{2}, \quad \forall d \in D, \quad (8)$$

$$\sum_{m \in M} \theta_m \geq 1, \quad (9)$$

$$\sum_{j \in J} (p_j - \underline{p}_j) \geq \alpha \tau + \sum_{j \in J} (\overline{p}_j - \underline{p}_j) \beta_j, \quad (10)$$

$$\frac{1}{\overline{p}_j - \underline{p}_j} \alpha + \beta_j \geq 1, \quad \forall j \in J, \quad (11)$$

$$\underline{p}_j \leq p_j \leq \overline{p}_j, \quad \forall j \in J, \quad (12)$$

$$\gamma_{mij} + 1 \geq z_{ji} + x_{mi}, \quad \forall m \in M, i \neq j, \forall i, j \in J, \quad (13)$$

$$1 - x_{ml} - x_{mk} - \theta_m \geq 0, \quad \forall l, k \in E_d, \forall m \in M, \forall d \in D, \quad (14)$$

$$x_{ml} - \theta_m \geq 0, \quad \forall l \in E_d, \forall m \in M, \forall d \in D, \quad (15)$$

$$t_m \geq 0, \quad \forall m \in M, \quad (16)$$

$$\alpha \geq 0, \quad (17)$$

$$\beta_j \geq 0, \quad \forall j \in J, \quad (18)$$

$$z_{ij} \in \{0, 1\}, \quad i \neq j, \forall i, j \in J, \quad (19)$$

$$x_{mj} \in \{0, 1\}, \quad \forall m \in M, \forall j \in J, \quad (20)$$

$$\gamma_{mij} \in \{0, 1\}, \quad \forall m \in M, i \neq j, \forall i, j \in J, \quad (21)$$

$$\theta_m \in \{0, 1\}, \quad \forall m \in M. \quad (22)$$

In the above formulation, the objective function (1) minimizes the total delay from the regular closing time of the operating room. Constraint (2) determines delay based upon the surgical duration and regular closing time of operating room m . Constraint (3) ensures that only one surgery is performed at a

time. Constraints (4) and (5) are partial circuit constraints for surgery assignments. Constraints (6) and (7) prevent symmetry in surgery assignments. Constraint (8) ensures that surgeries in the same department are performed in succession. Note that the constraint has a hidden constraint; surgeries in the same department are assigned last of the operating room. Constraint (9) ensures that surgeries in the same department are performed in the same operating room. The right-hand side of constraint (10) is the objective function value of the dual problem, and constraint (11) is the constraint for the dual problem. Here, the dual problem is the complement problem of maximizing the total duration of the surgery in the next section. Constraint (12) limits the upper and lower bounds of surgical duration. Constraint (13) is a constraint on the linearization variable γ_{mij} . Constraint (14) and (15) are constraints on the linearization variable θ_m . Constraints (16)–(18) are non-negative constraints on variables t_m, α, β_j . Constraints (19)–(22) are binary variable constraints on variables $z_{ij}, x_{mj}, \gamma_{mij}$ and θ_m .

3.2 Surgical Duration Uncertainty

Real-world surgical durations are often subject to uncertainties. A robust optimization model with uncertainty may be more suitable and reasonable for decision-making. We assume that the uncertain surgical duration \tilde{q}_j for surgery j is with respect to the uncertainty set, without assumptions on distribution. This assumption eliminates the need for accurate distribution information and enables the scheduling using only limited information, such as the average, minimum, and maximum values of data. Variable \tilde{q}_j is defined $\tilde{q}_j = p_j - \bar{p}_j, \forall j \in J$. The formulations are as follows:

Maximize

$$\sum_{j \in J} \tilde{q}_j \tag{23}$$

subject to

$$\sum_{j \in J} \left(\frac{\tilde{q}_j}{\bar{p}_j - \underline{p}_j} \right) \leq \tau, \tag{24}$$

$$0 \leq \tilde{q}_j \leq \bar{p}_j - \underline{p}_j, \quad \forall j \in J. \tag{25}$$

The objective function (23) maximizes the total surgical duration. Constraint (24) limits worst-case scenarios by conservative τ . Worst-case scenarios represent the number of surgeries for which the upper bound of the surgical duration is reached. Constraint (25) defines the possible range of uncertain surgical duration \tilde{q}_j .

4 Numerical Analysis

4.1 Data

In the following analysis, we used a dataset based on Ito et al. [13]. We solved the scheduling problem of five operating rooms and eleven surgeries using Gurobi 9.5.1. We compared the schedule created using the robust optimization model with that created using the stochastic programming model. The computational equipment is an Intel(R) Core (TM) i7-7500U CPU 2.90 GHz 8.00 GB.

The time from the operating room opening to the time when surgery j should be completed, d_m is 8 h or 480 min. It is desirable that all surgeries be completed within the regular opening time d_m . There are ten departments, i.e., $|D| = 10$. Surgeries 1 and 2 were in the same department. Table 1 shows the expected value $\mathbb{E}_j[p_j]$ and standard deviation σ_j of the duration of surgery j . The upper \overline{p}_j and lower \underline{p}_j bounds on the surgical duration used in robust optimization were the 10th and 90th percentiles of the duration in surgical scenarios. The conservative τ varied from 0 to 10 with 1. We assumed that the occurrence probability of the 1000 scenarios used in the stochastic programming model followed a uniform distribution, and the surgical duration in each scenario followed a log-normal distribution. We used the stochastic programming model proposed by Ito et al. [12].

4.2 Results

Figure 1 shows the results of total delay under different τ of robust optimization. The total delay of the stochastic programming is shorter than that of the robust optimization for all control conservatives because the surgical scenarios in the stochastic programming are used. The surgical scenario refers to a combined set of the duration of surgery. From Fig. 1, when the control conservative τ is 6, the total delay of the robust optimization shows a value equivalent to that of the

Table 1. Expected value and standard deviation of surgical duration (min).

Surgery j	$\mathbb{E}_j[p_j]$	σ_j
1	230	141
2	235	146
3	236	70
4	225	70
5	271	66
6	230	117
7	242	131
8	242	131
9	245	134
10	242	138
11	241	142

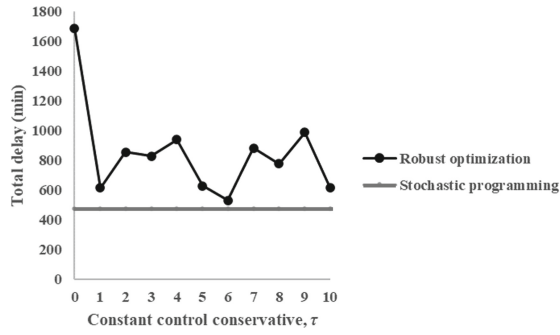


Fig. 1. Results of total delay.

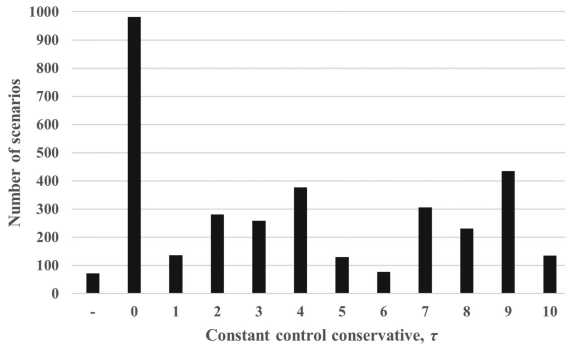


Fig. 2. Number of schedule scenarios with an excessive delay of 1000 min or more, ‘-’: result obtained by stochastic programming.

stochastic programming. Figure 2 shows the number of schedule scenarios with an excessive delay of 1000 min or more. From Fig. 2, when the control conservative τ is 6, the number of schedule scenarios with an excessive delay also shows an equivalent value in robust optimization and stochastic programming. In the above results, the robust optimization, which has only information on the 10th and 90th percentile duration of the scenario, exhibits the same performance as the stochastic programming, which has complete information on the scenario in specific control conservative. The robust optimization does not require the estimation of accurate distribution.

Table 2 provides the CPU times required to obtain robust optimization and stochastic programming solutions for a single instance as the constant control conservative varies. From Table 2, the CPU time of the robust optimization is shorter than that of the stochastic programming in all cases. From the above results, robust optimization can obtain a solution faster than stochastic programming because the number of variables and constraints in robust optimization is smaller than in stochastic programming. The robust optimization involves 1,311 constraints and 759 variables, while the stochastic programming comprises 11,271 constraints and 5,742 variables.

Table 2. CPU times for robust optimization and stochastic programming solutions (seconds).

Model		τ										
		0	1	2	3	4	5	6	7	8	9	10
Robust optimization		0.10	0.11	0.10	0.20	0.10	0.09	0.09	0.10	0.15	0.12	0.12
Stochastic programming	380.38											

5 Conclusion

In this study, we proposed a robust optimization model that minimizes the delay from the regular closing time of the operating room. The proposed model considers multiple operating rooms and surgical procedures. We also verified whether the risk-averse tendency is reflected in the schedule. The numerical analysis suggests that robust optimization models tend to avoid long delays. From the numerical analysis, robust optimization exhibits the same performance as stochastic programming, which has complete information on the scenario in specific control conservative. Robust optimization obtains a solution faster than stochastic programming because the number of variables and constraints in robust optimization is smaller than in stochastic programming. The robust optimization model is more effective for operating room managers who desire to obtain an accurate solution quickly.

In future work, we will clarify the effect of optimizing the surgical sequence on delay reduction. For this purpose, we define delay from the planned surgery end time and modify a part of the proposed model.

References

1. Addis, B., Carello, G., Grosso, A., Tánfani, E.: Operating room scheduling and rescheduling: a rolling horizon approach. *Flex. Serv. Manuf. J.* **28**, 206–232 (2016)
2. Akiyama, R., Ito, M., Takashima, R., Hoshino, K.: Stochastic programming model for elective surgery planning: an effect of emergency surgery. In: *Proceedings of 11th International Conference on Operations Research and Enterprise Systems*, pp. 231–235 (2022)
3. Bandi, C., Gupta, D.: Operating room staffing and scheduling. *Manuf. Serv. Oper. Manag.* **22**(5), 869–1106 (2020)
4. Batun, S., Denton, B.T., Huschka, T.R., Schaefer, A.J.: Operating room pooling and parallel surgery processing under uncertainty. *INFORMS J. Comput.* **23**(2), 220–237 (2011)
5. Cardoen, B., Demeulemeester, E., Beliën, J.: Operating room planning and scheduling: a literature review. *Eur. J. Oper. Res.* **201**(3), 921–932 (2010)
6. Denton, B.J., Miller, A.J., Balasubramanian, H.J., Huschka, T.R.: Optimal allocation of surgery blocks to operating rooms under uncertainty. *Oper. Res.* **58**, 802–816 (2010)
7. Denton, B.J., Viapiano, A.V.: Optimization of surgery sequencing and scheduling decisions under uncertainty. *Health Care Manag. Sci.* **10**(1), 13–24 (2007)

8. Dexter, F., Macario, A.: Applications of information systems to operating room scheduling. *Anesthesiology* **85**, 1232–1234 (1996)
9. Gerchak, Y., Gupta, D., Henig, M.: Reservation planning for elective surgery under uncertain demand for emergency surgery. *Manag. Sci.* **42**(3), 321–334 (1996)
10. Guerriero, F., Guido, R.: Operational research in the management of the operating theatre: a survey. *Health Care Manag. Sci.* **14**, 89–114 (2011)
11. Ito, M., Kobayashi, F., Takashima, R.: Minimizing conditional-value-at-risk for a single operating room scheduling problems. In: *Proceedings of International Multi-Conference of Engineers and Computer Scientists 2018*, vol. 2, pp. 968–973 (2018)
12. Ito, M., Kobayashi, F., Takashima, R.: Risk averse scheduling for a single operating room with uncertain durations. In: Ao, S.-I., Kim, H.K., Castillo, O., Chan, A.H., Katagiri, H. (eds.) *IMECS 2018*, pp. 291–306. Springer, Singapore (2020). https://doi.org/10.1007/978-981-32-9808-8_23
13. Ito, M., Hoshino, K., Takashima, R., Suzuki, M., Hashimoto, M., Fujii, H.: Does case-mix classification affect predictions?: a machine learning algorithm for surgical duration estimation. *Healthc. Anal.* **2**, 100119 (2022)
14. Namba, Y., Ito, M., Takashima, R.: A robust optimization for a single operating room scheduling problem with uncertain durations. In: *Proceedings of the 12th International Conference on Operations Research and Enterprise Systems*, pp. 180–184 (2023)
15. Jackson, R.: The bushiness of surgery. *Health Manag. Technol.* **23**(7), 20–22 (2002)
16. Kamran, M.A., Karimi, B., Dellaert, N.: Uncertainty in advance scheduling problem in operating room planning. *Comput. Ind. Eng.* **126**, 252–268 (2018)
17. Lamiri, M., Xie, X., Dolgui, A., Grimaud, F.: A stochastic model for operating room planning with elective and emergency demand for surgery. *Eur. J. Oper. Res.* **185**, 1026–1037 (2008)
18. Macario, A., Vitez, T.S., Dunn, B., McDonald, T.: Where are the costs in peri-operative care? Analysis of hospital costs and charges for inpatient surgical care. *Anesthesiology* **83**, 1138–1144 (1995)
19. Suzuki, A.: Analytics approach to the improvement of the management of hospitals. In: Sinha, B.K., Bagchi, S.B. (eds.) *Strategic Management, Decision Theory, and Decision Science*, pp. 247–256. Springer, Singapore (2021). https://doi.org/10.1007/978-981-16-1368-5_15
20. Zhu, S., Fan, W., Yang, S., Pei, J., Pardalos, P.M.: Operating room planning and surgical case scheduling: a review of literature. *J. Comb. Optim.* **37**, 757–805 (2019)