# An Optimization Model for the Placement of Mobile Stroke Units

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**Abstract.** Mobile Stroke Units (MSUs) are specialized ambulances that can diagnose and treat stroke patients; hence, reducing the time to treatment for stroke patients. Optimal placement of MSUs in a geographic region enables to maximize access to treatment for stroke patients. We contribute a mathematical model to optimally place MSUs in a geographic region. The objective function of the model takes the tradeoff perspective, balancing between the efficiency and equity perspectives for the MSU placement. Solving the optimization problem enables to optimize the placement of MSUs for the chosen tradeoff between the efficiency and equity perspectives. We applied the model to the Blekinge and Kronoberg counties of Sweden to illustrate the applicability of our model. The experimental findings show both the correctness of the suggested model and the benefits of placing MSUs in the considered regions.

**Keywords:** Optimization · MILP · Time to Treatment · Mobile Stroke Unit (MSU) · MSU Placement

## 1 Introduction

A stroke refers to when a blood clot or a bleeding interrupts the blood circulation inside the brain, and stroke is a main global reason for death and permanent disability [1]. There are three main stroke types, each requiring specific treatment. To assure providing the correct stroke treatment, a computed tomography (CT) scan is required to identify which type of stroke the patient is suffering. Ischemic strokes are most common, and they occur when blood clot(s) impede blood circulation in the brain. Treatment for ischemic stroke patients typically involves thrombolysis and, in specific cases, thrombectomy.

Early treatment is known to be crucial for the successful recovery of stroke patients [2]. However, it is often difficult to treat stroke patients immediately since the patient typically cannot be diagnosed and treated until the ambulance delivers him/her to an acute hospital.

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T. Guarda et al. (Eds.): ARTIIS 2023, CCIS 1935, pp. 297–310, 2024.
https://doi.org/10.1007/978-3-031-48858-0_24

One effective way to decrease the time to treatment for stroke patients involves the utilization of Mobile Stroke Units (MSUs). An MSU is a specialized ambulance equipped with advanced medical equipment, including a CT scanner, that enables the ambulance personnel to diagnose and administer thrombolysis on stroke patients while the patient is still in the MSU. As a result, MSUs have the potential to reduce the time to treatment by eliminating the time required for the transportation and diagnosis of the patient at the hospital. However, due to the high operational expenses associated with MSUs, only a limited number of MSUs can be placed in a geographic region. Therefore, when introducing MSUs, it becomes essential to strategically place them in order to provide a timely service for residents living in a region.

There are a number of studies focusing on identifying the optimal locations for placing MSU(s) within a region to enhance stroke care. These studies mainly explore two perspectives regarding where to place MSUs: efficiency and equity. The term efficiency refers to placing MSUs so that they provide access to treatment in the shortest possible time for most patients in a region, for example, in urban areas [3, 4]. Equity emphasizes the placement of MSUs in a manner that ensures equal access to healthcare services regardless of the geographic location of the patients, for example, in rural areas [5]. Phan et al. [3] introduce a data-driven approach that utilizes the Google application programming interface to determine the best possible placement for an MSU within the Sydney area. Rhudy Jr. et al. [4] use geospatial analysis to optimize service delivery for stroke patients in Memphis by studying the distribution of an MSU throughout the city. Dahllöf et al. [6] propose an expected value optimization approach to determine the best placement for an MSU in Sweden's Skåne county, aiming to assess the potential advantages of placing an MSU for urban and rural residents respectively. Amouzad Mahdiraji et al. [7] utilize an exhaustive search approach to optimally place MSUs in southern Sweden with the aim of balancing the efficiency and equity perspectives for the placement of MSUs.

The aim of the current study is to introduce a mathematical optimization model in the form of a mixed integer linear programming (MILP) model to identify the best locations of MSUs within a geographic region. Mathematical optimization has been demonstrated to be an effective technique to solve complex problems in a wide range of domains, such as emergency medical services (EMS). Due to the computational complexity of emergency vehicle placement problems, it is vital to build efficient mathematical models to represent the key characteristics of the MSU placement problem. However, no existing research directly addresses the mathematical formulation of the MSU placement problem. The objective function of the presented MILP model expresses a tradeoff between the efficiency and equity perspectives, aiming to provide maximum population coverage as well as equal service for the inhabitants of a region; however, considering the chosen tradeoff between the two perspectives. A scenario study is conducted in two counties of southern Sweden to show the correctness and advantages of our proposed model, where we solve the model to identify the optimal placements for different numbers of MSUs.

The subsequent sections of the paper are outlined as follows. We review the related work in Sect. 2. In Sect. 3, we present the MSU placement problem with a tradeoff between the efficiency and equity perspectives. In Sect. 4, we present our optimization model for the described problem. The scenario study is presented in Sect. 5, which is followed by an analysis of experimental results and a discussion. Eventually, we conclude the paper in Sect. 6.

## 2 Related Work

Previous studies in EMS use MILP for problems related to ambulance routing and placement, ambulance fleet allocation, crew scheduling, and resource allocation, ultimately leading to better patient outcomes and resource utilization [8]. As an example, Tavakoli et al. [9] propose a mathematical model for the strategic placement of ambulances, aiming to improve the response time of EMS in Fayetteville, North Carolina. Røislien et al. [10] use mathematical modeling to explore the optimal locations for air ambulance sites in Norway. Their approach utilizes high-resolution population data to estimate the number of required sites to provide service within 30 and 45 min for different shares of the population. Leknes et al. [11] present a MILP model to address the strategic and tactical problems of placing ambulance sites in heterogeneous regions. The authors examined the model in an urban-rural area in Norway. Akdoğan et al. [12] utilize queuing theory and a MILP model to locate emergency vehicles on fully connected networks. The MILP model aims to reduce the average response time of EMS according to an approximate queuing model.

In another study, Tlili et al. [13] propose a mathematical model to improve EMS transportation during disaster situations. The authors use a genetic algorithm for the ambulance routing problem to reduce time-sensitive treatment delays during urgent situations involving congested traffic compounds. Acuna et al. [14] contribute an ambulance placement optimization model to decrease patients' waiting times, time to treatment, and emergency department overcrowding in a county in Florida. The model considers disparities and fairness in placing ambulance services to emergency departments. Wan S. et al. [15] use a 0–1 MILP model to represent the location of distribution centers in massive emergencies, applied in a case study of earthquake response logistics in Chengdu, China. The proposed bi-objective model considers both the total transportation cost and the coverage level of emergency supplies.

While numerous research studies focus on the mathematical modeling of ambulance location problems, no previous study explicitly contributes to the mathematical formulation of the MSU placement problem. To address this gap, in this paper, we present a MILP model to represent the MSU placement problem.

## 3 MSU Placement Problem

As mentioned earlier, when placing MSU(s) in a geographic region, we need to take the impacts of the MSU locations into account to assure that the inhabitants of different parts of a region receive maximum benefit. In this section, we describe the MSU placement problem and how MSUs can be placed in a region considering the tradeoff between the efficiency and equity perspectives.

In our companion study [7], we demonstrate how different placements of MSUs would impact individuals living in different parts of a region. In particular, we propose an objective function that could be used in an optimization model to tradeoff between the efficiency and equity perspectives, and hence, allows placing MSUs so that most people living in a region are expected to receive more equitable service and shorter time to treatment. In addition, we employ the concept of the expected time to treatment to capture the value of the corresponding measure for each perspective. It should be noted that the expected time to treatment for a stroke patient denotes the expected time until the patient gets treatment either at a hospital or inside an MSU. In a previous study [16], we present how to calculate the expected time to treatment for patients in different subregions of a geographic region, considering that both a regular ambulance and an MSU can be dispatched.

The efficiency perspective refers to placing MSUs in a region to ensure a higher proportion of the population is expected to receive treatment at an earlier time. Using this perspective, the MSUs are placed close to highly populated regions, that is, in or near the urban areas. The efficiency perspective can be measured by the weighted average time to treatment (WATT). The expected time to treatment for individuals located in each subregion of a larger region is multiplied by the share of stroke cases expected to take place in the corresponding subregion; the sum of these values yields the WATT. We can use the WATT as an objective function in an optimization model for the MSU placement problem that considers the efficiency perspective.

The equity perspective refers to placing MSUs where the people who live far from the medical centers (for example, hospitals) benefit most, that is, people living in or close to rural areas. The range measure can be utilized to model the equity perspective, aiming to minimize the time difference between the expected times to treatment for patients who are located in different subregions of the studied region. The focus of an optimization problem corresponding to the equity perspective is to identify the MSU placements that minimize range.

In our companion study [7], we also introduce a tradeoff function that is established based on the WATT and range. It is shown that the tradeoff function enables to balance between the efficiency and equity perspectives to optimally place MSUs. In an optimization problem for placing MSUs in a region, the tradeoff perspective aims to find the locations of MSUs that minimize the tradeoff function.

It should be highlighted that in the formulated optimization problem, we only consider, for each perspective, the placement of MSUs in the existing ambulance sites in a geographic region.

## 4 Optimization Model

We here present our MILP model, which represents the key characteristics of the MSU placement problem. Our optimization model aims to minimize the tradeoff function that enables to identify the optimal locations for MSU(s) that can provide highly equitable service and reduced time to treatment for residents within a region.

We let $I = \{1, \ldots, m\}$ denote the index set over ambulances sites, where $m$ is the total number of ambulance sites, and $N$ is the number of MSUs to place. The aim is to place a fixed number of MSUs at the existing ambulance sites in a geographic region. It is assumed that there is always at least one regular ambulance available at each ambulance site. We also assume that there is always an ambulance or an MSU available for dispatch when it is required, and that the placed MSU(s) have no limitation concerning driving distance, and that they can provide service throughout the whole region.

We further assume that the studied region is divided into a non-overlapping set of subregions, denoted by $R = \{1, \ldots, n\}$, where $n$ is the total number of subregions. We also assume that all inhabitants located in subregion $r \in R$ are in the same location, for example, in the centroid of $r$.

We let $t_r^{RA}$ be the shortest time to treatment using a regular ambulance located in any ambulance site $i \in I$ for subregion $r \in R$, $t_{ir}^{MSU}$ be the expected time to treatment for a patient located in subregion $r \in R$ using an MSU located in site $i \in I$, and $Q_r$ be the share of stroke incidents within the studies region that is expected to take place in subregion $r \in R$ ($\sum_{r \in R} Q_r = 1$). Please note that the $t_r^{RA}$:s ($r \in R$), $t_{ir}^{MSU}$:s ($i \in I$, $r \in R$), and $Q_r$:s ($r \in R$) are input parameters, and hence can be calculated beforehand.

In order to formulate the MILP model, we need the following decision variables:

- $x_i \in \{0, 1\}$, ($i \in I$) is a binary decision variable such that:

$$\mathrm{x_i} = \begin{cases} 1 \; \textit{if there is an MSU in site } i \in I, \\ 0 \; \textit{Otherwise}. \end{cases} \tag{1}$$

- $y_{ir}^{MSU}$ is the expected time to treatment for a patient in subregion $r \in R$ using an MSU in site $i \in I$. This variable is assigned a large value, $M$, if there is no MSU placed in site $i \in I$.
- $y_r^{MSU}$ is the shortest expected time to treatment for a patient in subregion $r \in R$ using any of the placed MSUs.
- $y_r$ is the shortest expected time to treatment for a patient in subregion $r \in R$ using either an MSU or a regular ambulance.
- $u^{max}$ is the longest expected time to treatment for any subregion $r \in R$.
- $u^{min}$ is the shortest expected time to treatment for any subregion $r \in R$.

The tradeoff function $z$, presented in Eq. (2), is the objective function for our MILP model. The objective function has two components: the first one is the WATT as a measure for the efficiency perspective, and the second one is the range (time difference between subregions with the shortest and longest expected time to treatments) as a measure for the equity perspective.

$$\min z = \sum\nolimits_{r=1}^{R} (1 - w) y_r Q_r + w\left(u^{max} - u^{min}\right), \tag{2}$$

In Eq. (2), $w \in [0, 1]$ is the weight employed to control the effects of the efficiency and equity perspectives. For example, we here assume $w = 0.5$ to let each of the terms have an equal impact on the tradeoff function.

The optimal solution of our model is subject to the following constraints:

$$y_{ir}^{MSU} = x_i t_{ir}^{MSU} + M(1 - x_i), i \in I, r \in R, \tag{3}$$

$$y_r^{MSU} = \min_{i \in I}\left\{y_{ir}^{MSU}\right\}, i \in I, r \in R, \tag{4}$$

$$y_r = \min\left\{y_r^{MSU}, t_r^{RA}\right\}, r \in R, \tag{5}$$

$$u^{max} \geq y_r, r \in R, \tag{6}$$

$$u^{min} \leq y_r, r \in R, \tag{7}$$

$$\sum\nolimits_{i \in I} x_i = N. \tag{8}$$

We use constraint sets (3)-(5) to obtain the values of the $y_r$, which is the shortest expected time to treatment for any subregion $r \in R$ using either an MSU or a regular ambulance. The constraint in Eq. (3) assigns $t_{ir}^{MSU}$ to $y_{ir}^{MSU}$ if there is an MSU available in site $i$ for a patient in subregion $r$. However, if no MSU is located in site $i$, it instead assigns a large value $M$ to the $y_{ir}^{MSU}$. $M$, which is a parameter in our optimization model, is a sufficiently large constant value. For example, $M$ can be set to any value larger than the longest expected time to treatment for any subregion $r$ and any ambulance site $i$, that is, $M > \max_{r \in R} t_r^{RA}$.

The constraint in Eq. (4) takes the minimum over the expected times to treatment for the possible MSU locations. The minimum operation is used to assign the shortest expected time to treatment using an MSU for a patient in subregion $r$. In the optimization model, the constraint $y_r^{MSU} = \min_{i \in I}\{y_{ir}^{MSU}\} = \min\{y_{1r}^{MSU}, \ldots, y_{mr}^{MSU}\}$ is modeled as an ordered sequence of $(|I|-1)$ minimum operations, each having two components. For this purpose, we introduce a set of positive help variables $p_{ir}^{MSU}$, $i \in \{1, \ldots, |I|-1\}$, $r \in R$, which are used in the following way:

$$p_{1r}^{MSU} = min\left\{y_{1r}^{MSU}, y_{2r}^{MSU}\right\}, \tag{9}$$

$$p_{2r}^{MSU} = min\left\{p_{1r}^{MSU}, y_{3r}^{MSU}\right\},$$

$$\ldots$$

$$p_{(\lceil I \rceil-1)r}^{MSU}\text{h} = min\left\{p_{(\lceil I \rceil-2)r}^{MSU}, y_{|I|r}^{MSU}\right\}.$$

In turn, each of these $(|I|-1)$ minimum operations are represented using six constraints in our optimization model.

To model each of the minimum operations (including two components), we also need one binary variable. We let binary help variable $s_{ir}^{MSU}$, $i \in \{1, \ldots, |I|-1\}$, $r \in R$ be used in the $i$:th minimum operation in this sequence for subregion $r$.

The first of the minimum operations $p_{1r}^{MSU} = min\left\{y_{1r}^{MSU}, y_{2r}^{MSU}\right\}$, determining the minimum between $y_{1r}^{MSU}$ and $y_{2r}^{MSU}$ is modeled using the following (six) constraints.

$$y_{2r}^{MSU} - y_{1r}^{MSU} \leq M s_{1r}^{MSU}, \tag{10}$$

$$y_{1r}^{MSU} - \mathrm{y}_{2r}^{MSU} \leq M\left(1 - s_{1r}^{MSU}\right),$$

$$p_{1r}^{MSU} \leq y_{1r}^{MSU},$$

$$p_{1r}^{MSU} \leq y_{2r}^{MSU},$$

$$p_{1r}^{MSU} \geq y_{1r}^{MSU} - M\left(1 - s_{1r}^{MSU}\right),$$

$$p_{1r}^{MSU} \geq y_{2r}^{MSU} - Ms_{1r}^{MSU}$$

The $i$:th ($2 \leq i \leq |I| - 1$) of the minimum operations $p_{ir}^{MSU} = min\left\{p_{(i-1)r}^{MSU}, y_{(i+1)r}^{MSU}\right\}$, determining the minimum between $p_{(i-1)r}^{MSU}$ and $y_{(i+1)r}^{MSU}$, is modeled using the following (six) constraints. Please note that there are in total $|I| - 2$ such constraint sets for each subregion $r$.

$$y_{(i+1)r}^{MSU} - p_{(i-1)r}^{MSU} \leq Ms_{ir}^{MSU}, \tag{11}$$

$$p_{(i-1)r}^{MSU} - y_{(i+1)r}^{MSU} \leq M\left(1 - s_{ir}^{MSU}\right),$$

$$p_{ir}^{MSU} \leq p_{(i-1)r}^{MSU},$$

$$p_{ir}^{MSU} \leq y_{(i+1)r}^{MSU},$$

$$p_{ir}^{MSU} \geq p_{(i-1)r}^{MSU} - M\left(1 - s_{ir}^{MSU}\right),$$

$$p_{ir}^{MSU} \geq y_{(i+1)r}^{MSU} - Ms_{ir}^{MSU}.$$

Then, we use the constraint in Eq. (12) (one for each $r \in R$) to acquire $y_r^{MSU}$. Please note that this constraint is needed in order to be consistent with the constraint set (11).

$$y_r^{MSU} = p_{(|I|-1)r}^{MSU}, r \in R. \tag{12}$$

The constraint $y_r = \min\left\{y_r^{MSU}, t_r^{RA}\right\}$ shown in Eq. (5) captures the minimum value between $y_r^{MSU}$ and $t_r^{RA}$ for the patients located in subregion $r$. In the optimization model, this constraint is modeled using the following six constraints, where $v_r$, $r \in R$ is a binary help variable:

$$y_r^{MSU} - t_r^{RA} \leq Mv_r, r \in R, \tag{13}$$

$$t_r^{RA} - y_r^{MSU} \leq M\left(1 - v_r\right), r \in R,$$

$$y_r \leq t_r^{RA}, r \in R,$$

$$y_r \leq y_r^{MSU}, r \in R,$$

$$y_r \geq t_r^{RA} - M\left(1 - v_r\right), r \in R,$$

$$y_r \geq y_r^{MSU} - Mv_r, \, r \in R.$$

The constraints $u^{max} \geq y_r$ and $u^{min} \leq y_r$ in Eqs. (6) and (7) capture the longest and shortest expected time to treatment for any subregion $r$ and for any MSU in site $i$. The value of $u^{max} - u^{min}$ in the objective function, see Eq. (2), refers to the range measure.

Finally, the constraint $\sum_{i \in I} x_i = N$ defined by Eq. (8) specifies the number of MSUs to be placed in a region.

## 5 Scenario Study

In this section, we describe the application of our proposed optimization model to two counties in southern Sweden. We then describe the experimental results.

### 5.1 Scenario Description

To evaluate the efficacy of the presented optimization model, we apply it to the Blekinge and Kronoberg counties of Sweden, which are parts of Sweden's southern healthcare region (SHR). The SHR covers an area of 16,622 km$^2$ and encompasses four counties: Skåne, Blekinge, Halland, and Kronoberg. The SHR has 49 municipalities, and its population was 1,687,000 in 2018. In Sweden, over 21,000 stroke incidents occur annually, with 3,900 cases reported in SHR [17]. In SHR, there are 39 ambulance sites and 13 acute hospitals equipped with CT scanners. Using the standard solvers, for example, Gurobi, we realized that it would be difficult to solve the model for large problem instances, that is, the entire SHR. Therefore, we decided to test the model with two counties of SHR. Table 1 represents the demographic and geographic statistics for each county of SHR. Figure 1 shows an overview of SHR, where each green triangle (referred to by a specific circled number) and each purple circle corresponds to an ambulance site and an acute hospital, respectively. The borders of the Blekinge and Kronoberg counties are represented in red and blue, respectively. As shown in Fig. 1, the ambulance sites in Blekinge are in Karlshamn (id: 15), Karlskrona (id: 16), Olofström (id: 26), Ronneby (id: 29), and Sölvesborg (id: 30), and ambulance sites in Kronoberg are in Älmhult (id: 1), Alvesta (id: 3), Lenhovda (id: 21), Lessebo (id: 22), Ljungby (id: 23), Markaryd (id: 25), Tingsryd (id: 34), and Växjö (id: 36).

We considered the same input data and assumptions as we did in our companion study [7]. In particular, we utilized the demographic data and stroke data for 2018 collected from Statistics Sweden [18] and Sweden's southern healthcare region committee [19], respectively. In our data, each county of SHR was divided into a set of non-overlapping subregions, each equaling to $1 \times 1$ km$^2$ and indicated by $r \in R$ so that the union of all subregions $\bigcup_{r \in R} r$ equals to the corresponding county of SHR. The demographic data included the number of inhabitants for each subregion $r \in R$ and each of the 21 assumed age groups, that is $\{[0, 4), [4, 8), \ldots, [95, 99), [100, \infty)\}$. In addition, the stroke data included the number of stroke cases for each age group in each county of SHR. Using the provided data, we calculated $Q_r$, indicated in Sect. 4, for each subregion $r \in R$, obtained by dividing the expected number of stroke cases in subregion $r$ by the total expected number of stroke cases in the SHR.

**Table 1.** Demographic and geographic data of each county of SHR.

| County | Population | Number of municipalities | Number of subregions | Number of ambulance sites | Number of hospitals |
|---|---|---|---|---|---|
| Blekinge | 134,188 | 5 | 1,959 | 5 | 2 |
| Halland | 133,025 | 3 | 1,603 | 4 | 1 |
| Kronoberg | 198,903 | 8 | 4,233 | 8 | 2 |
| Skåne | 1,221,074 | 33 | 8,827 | 22 | 8 |

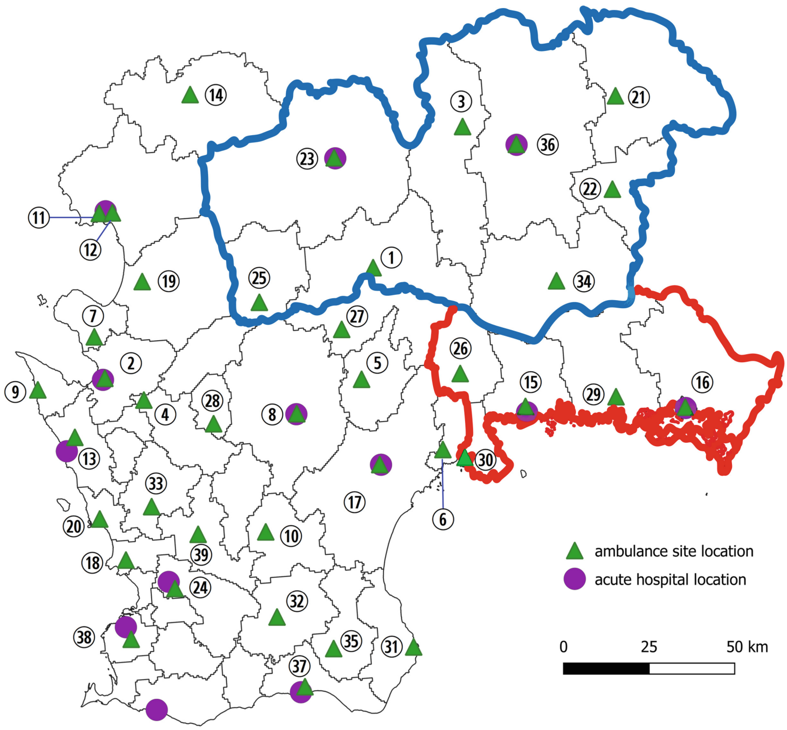


**Fig. 1.** Overview of the Sweden's southern healthcare Sweden (SHR). The purple circles and green triangles show the locations of acute hospitals and ambulance sites, respectively. The circled numbers indicate the corresponding ambulance site IDs. The borders of the Blekinge and Kronoberg counties are shown in red and blue, respectively. (Color figure online)

In the scenario study, we aimed to identify the optimal locations of different numbers of MSUs in either Blekinge or Kronoberg using the proposed optimization model. In the experiments, we took into consideration that every ambulance site within the region could potentially serve as a location for placing an MSU. In addition, for all experiments, we measured the results using only the expected time to treatment. We also compared the experimental results of placing MSU(s) with the experimental results of the baseline, representing the current situation in the SHR, where there are only regular ambulances across all 39 ambulance sites in the SHR.

We solved the problem in Gurobi[1] 10.0.0, which uses the barrier and simplex algorithms to solve continuous relaxations of mixed-integer models and continuous models. In all experiments, we solved the described problem using the barrier and simplex algorithms. All of the code was written in Jupyter Notebook using Python on a computer with 32-gigabyte memory (RAM) and a Core(TM) i7-8650U CPU 1.90 gigahertz Intel(R) processor.

### 5.2 Experimental Results

As mentioned above, we applied our model to different parts of the SHR, that is, Blekinge and Kronoberg counties. To demonstrate the functionality of our optimization model and to explore how large problem instances can be solved using this approach, we initially tried to apply it to a smaller county of SHR, that is, Blekinge, which is a smaller region and which has a lower number of ambulance sites compared to the entire SHR. We, then, applied the model to a broader region, that is, Kroboberg county, with a higher number of ambulance sites. The reason that we did not represent the application of the model to the complete SHR, which is a large area, is that it was challenging and time-consuming to optimally solve our proposed model for such a large region with the corresponding large amount of input data.

The experimental results for the Blekinge county are presented in Table 2. We considered two situations regarding the number of ambulance sites that are available for placing MSUs in Blekinge: Situation 1) the number of available ambulance sites corresponds to the number of ambulance sites in Blekinge; Situation 2) the number of available ambulance sites corresponds to the number of ambulance sites in Blekinge + all ambulance sites located in the neighborhood of Blekinge. As can be seen the Fig. 1, there are six ambulance sites close to Blekinge, where the two nearest ambulance sites are Bromölla (id: 6) and Tingsryd (id: 34).

Since there are 5 ambulance sites in the Blekinge county, in the experiments, we solved the problem for placing 1 to 4 MSUs. According to Table 2, by adding the number of MSUs, the tradeoff value decreases in comparison with the baseline (where there is no MSU in Blekinge). The results also demonstrate that by using MSU(s) in Blekinge, the values of the tradeoff, WATT, range, and average time to treatment are expected to decrease compared to the baseline. In particular, by placing two MSUs in Blekinge, it is possible to make the treatment available within an hour for all inhabitants living in Blekinge.

When we solved the problem considering Situation 1 (only ambulance sites in Blekinge), the Simplex and Barrier algorithms produced the same results for each MSU placement. For Situation 1, we pointed out the minimum execution time between Simplex and Barrier in Table 2.

For Situation 2, where we considered 11 ambulance sites (5 ambulance sites in Blekinge and 6 neighborhood ambulance sites), the Gurobi solver using the Simplex algorithm had difficulty in solving the problem for placing of 1 to 4 MSUs. We instead

[1] Available: https://www.gurobi.com.

decided to only consider the 2 nearest ambulance sites to the existing ambulance sites of Blekinge and perform the experiments with 7 ambulance sites, shown in the third row of each MSU placement in Table 2. However, using the Barrier algorithm, Gurobi could solve the problem considering 11 ambulance sites in a feasible amount of time. According to the Gurobi documentation[2], the reason is probably that the Barrier algorithm is more efficient for complex models with large size. The results of the Barrier algorithm for Situation 2 are presented in parentheses in the third row for each MSU placement.

**Table 2.** Experimental results for the Blekinge county. NoAAS: number of available ambulance sites for placing MSUs; NoM: number of MSUs to place in the county; algorithm: algorithm used to solve the problem; MSU IDs: found optimal MSU site IDs, denoted by numbers within the square brackets; Ex. Time: Execution time (in seconds); Tr.: objective function corresponding to tradeoff value (in hour); Ra.: range (in hours); ATT: average time to treatment (in hour); WATT: weighted average time to treatment (in hour); and ES: exhaustive search.

| NoAAS | NoM | Algorithm | MSU IDs | Ex. Time | Tr. | Ra. | WATT | ATT |
|---|---|---|---|---|---|---|---|---|
| Baseline | - | - | - | - | 1.39 | 1.44 | 1.34 | 1.61 |
| 5 & 11 | 1 | ES | [29] | - | 1.09 | 1.16 | 1.01 | 1.11 |
| 5 | 1 | Simplex & Barrier | [29] | 22 | 1.09 | 1.16 | 1.01 | 1.11 |
| 7 (11) | 1 | Simplex (Barrier) | [29] | 42 (45) | 1.09 | 1.16 | 1.01 | 1.11 |
| 5 & 11 | 2 | ES | [15,16] | - | 0.87 | 0.89 | 0.84 | 0.97 |
| 5 | 2 | Simplex & Barrier | [15,16] | 572 | 0.87 | 0.89 | 0.84 | 0.97 |
| 7 (11) | 2 | Simplex (Barrier) | [15,16] | 978 (1008) | 0.87 | 0.89 | 0.84 | 0.97 |
| 5 | 3 | ES | [15,16,29] | - | 0.81 | 0.83 | 0.79 | 0.93 |
| 5 | 3 | Simplex & Barrier | [15,16,29] | 466 | 0.81 | 0.83 | 0.79 | 0.93 |
| 7 (11) | 3 | Simplex (Barrier) | [15,16,34] | 1243 (1127) | 0.82 | 0.79 | 0.84 | 0.95 |
| 11 | 3 | ES | [15,16,34] | - | 0.82 | 0.79 | 0.84 | 0.95 |
| 5 | 4 | ES | [15,16,26,29] | - | 0.78 | 0.81 | 0.75 | 0.89 |
| 5 | 4 | Simplex & Barrier | [15,16,26,29] | 24 | 0.78 | 0.81 | 0.75 | 0.89 |
| 7 (11) | 4 | Simplex (Barrier) | [15,16,26,34] | 1003 (995) | 0.81 | 0.81 | 0.80 | 0.91 |
| 11 | 4 | ES | [15,16,26,34] | - | 0.81 | 0.81 | 0.80 | 0.91 |

In Table 2, the comparison of the results of Situation 1 and Situation 2 shows that the identified MSU locations are equal when placing 1 and 2 MSUs. However, the identified MSU locations are different when placing 3 and 4 MSUs, where the corresponding tradeoff values of Situation 1 are smaller than Situation 2.

[2] Available: https://www.gurobi.com/documentation/

According to Table 2, in Situation 1 and Situation 2, the highest execution times are recorded when placing 2 MSUs (572 s) and 3 MSUs (1243 s for Simplex and 1127 s for Barrier), respectively.

In order to verify the optimal solutions and optimal objective function values obtained using our optimization model, we compared the output of our model with the exhaustive search, proposed in our companion paper [7], for placing 1, 2, 3, and 4 MSUs in Blekinge, presented in Table 2. In all MSU placements and Situations, the identified solutions and objective function values are the same both for our proposed model and for the exhaustive search.

**Table 3.** Experimental results for the Kronoberg county. The abbreviations are the same as in Table 2.

| NoAAS | NoM | Algorithm | MSU IDs | Ex. Time | Tr. | Ra. | WATT | ATT |
|---|---|---|---|---|---|---|---|---|
| Baseline | - | - | - | - | 1.65 | 1.84 | 1.45 | 1.78 |
| 8 | 1 | ES | [3] | - | 1.32 | 1.53 | 1.11 | 1.31 |
| 8 | 1 | Simplex (Barrier) | [3] | 2195 (133) | 1.32 | 1.53 | 1.11 | 1.31 |
| 8 | 2 | ES | [23,36] | - | 1.09 | 1.24 | 0.94 | 1.13 |
| 8 | 2 | Simplex | [23,36] | 2667 | 1.09 | 1.24 | 0.94 | 1.13 |
| 8 | 3 | ES | [21,23,36] | - | 1.01 | 1.10 | 0.91 | 1.10 |
| 8 | 3 | Simplex | [21,23,36] | 3481 | 1.01 | 1.10 | 0.91 | 1.10 |
| 8 | 4 | ES | [21,23,34,36] | - | 0.99 | 1.10 | 0.87 | 1.02 |
| 8 | 4 | Simplex | [21,23,34,36] | 4099 | 0.99 | 1.10 | 0.87 | 1.02 |
| 8 | 5 | ES | [21,23,25,34,36] | - | 0.97 | 1.10 | 0.83 | 0.98 |
| 8 | 5 | Simplex | [21,23,25,34,36] | 2340 | 0.97 | 1.10 | 0.83 | 0.98 |

In Table 3, we present the results of applying our model to the Kronberg county. In the experiments, we assumed that only ambulance sites in Kronberg can be used for placing MSUs. Considering the complexity of solving the model, we, further, assumed that it is relevant to solve the problem of placing 1 to 5 MSUs in Kronoberg.

According to Table 3, by adding the number of MSUs, the tradeoff value decreases in comparison with the baseline (where there is no MSU in Kronoberg). In Table 3, the results also show that by placing MSU(s) in Kronoberg, the values of the tradeoff, WATT, range, and average time to treatment are expected to reduce compared to the baseline. Especially, placing 5 MSUs in Kronoberg would potentially provide treatment within an hour for all inhabitants living there.

It can be observed in Table 3 that when we solve the problem of placing one MSU in Kronoberg, the Simplex and Barrier algorithms produce the same results. However, when placing more than one MSU in Kronoberg, the Barrier algorithm had difficulties in finding feasible solutions for placing 2 to 5 MSUs. Alternatively, using the Simplex algorithm, Gurobi could solve the problem for different numbers of MSUs within a feasible amount of time. As mentioned above, the reason appears to be that the Barrier algorithm tends to be quicker when handling large complex models, but it exhibits greater numerical sensitivity. On the other hand, the simplex algorithm is generally less affected by numerical issues. According to Table 3, the highest execution time (4099 s) is recorded when placing 4 MSUs in Kronoberg.

Similar to Table 2, we compared the output of the presented optimization model with the exhaustive search for placing different numbers of MSUs in Kronoberg, presented in Table 3. As can be seen for all MSU placements, the identified solutions and objective function values are the same both for our proposed model and for the exhaustive search.

From the conducted experiments, we could explore to what extent large problem instances can be solved using our optimization model, and in that way, we could learn about the limits of using the Gurobi solver to solve the described problem.

## 6 Conclusions

We have presented a MILP model for the optimal placement of MSUs in a geographic region. The objective function of our optimization model is a tradeoff function proposed in our prior study [7], used to tradeoff the equity and efficiency perspectives for the MSU placement problem while aiming to provide shorter time to treatment and equal service for residents living in a region. To evaluate our optimization model, we conducted a scenario study to place MSUs in the Blekinge and Kronoberg counties of Sweden. Applying the model to smaller counties provided us the opportunity to assess the model's functionality and performance on a more manageable scale before scaling it up to larger problem instances. In the presented model, the time needed to identify an optimal solution for the given problem instances indicated the complexity of the MSU placement problem. The experimental results, supported by the results of the exhaustive search approach presented in previous research [7], indicated that the proposed optimization model is able to find the optimal MSU locations concerning the defined objective function and constraints. The results of the experiments also showed that using our proposed optimization model for the MSU placement problem enabled to cut down the expected time to treatment for most residents compared to the baseline. From the experimental results, we concluded that by placing 2 and 5 MSUs in Blekinge and Kronoberg, respectively, it is likely to achieve access to treatment within an hour for all inhabitants living there, which is often considered an important goal.

The focus of the current paper was on validating the correctness of the proposed optimization model and illustrating the possible idea of placing MSU(s) in a region using the tradeoff perspective. As mentioned above, solving large problem instances, for example, the SHR, is computationally expensive, in particular for standard optimization solvers. For future work, we plan to investigate the use of heuristics to solve large problem instances within a reasonable time frame.

**Acknowledgment.** This work was partially supported by the Kamprad Family Foundation for Entrepreneurship, Research & Charity, Sweden's Southern Regional Healthcare Committee, and the Wallenberg AI, Autonomous Systems and Software Program – Humanities and Society (WASP-HS), funded by the Marianne and Marcus Wallenberg Foundation.

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