Simulation Processes in Business and Economics: Fundamentals of the Monte Carlo Simulation

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Abstract Simulations, especially Monte Carlo simulations, are a common tool in operations research. They enable the study of complex scenarios, evaluation of cost-effectiveness, risk mitigation, and efficient testing of improbable situations. Monte Carlo simulations facilitate decision-making by creating realistic experimental models for various operations research purposes.

1 Fundamentals of Simulation Processes

1.1 Definition and Purpose

Fig. 1 Purpose of Simulations. Source: Own representation according to Lipinski et al. (2017) [2].

The purpose of simulations is to gain knowledge that can be transferred to real processes [1]. 1 It is irrelevant here whether, within the framework of the simulation, the

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systems are real or imaginary. The result of the simulation describes an experimental model with realistic dynamic processes and serves as a basis for decision-making. As can be seen in Figure 1, there are different purposes for simulations. They can be used, for example, to assess cost-effectiveness, and reduce security risks and complexity. Simulations have the advantage that situations can be tested which are considered impossible in practice. In many cases, simulations help to save time because they are faster in execution than real-time tests. Simulations can be helpful for a baseline systematic presentation. Simulations describe in the broadest sense the preparation, execution, and evaluation of simulations with a simulation model [2].

Figure 2 shows a classification model for simulations. In this model, simulations can be divided into the situation, complication, question and answer (SCQA) model - also known as the minto-pyramid principle. With regard to the first dimension of SCQA, the situation contains the description of the status quo, followed by a problem in the situation, which is further explained within the com-plication. Additionally, the complication generates the tension in the respective situation. The third dimension, the question, will emerge within the complication and lead to possible answers. There are two different ways to check the answers resulting from the previous steps. Firstly, the answers can be tested in real experi-ments. Secondly - for the reasons mentioned above - they can also be tested with a simulation model. After the test results have been substantiated, an adequate decision can be made which leads to a solution [3].

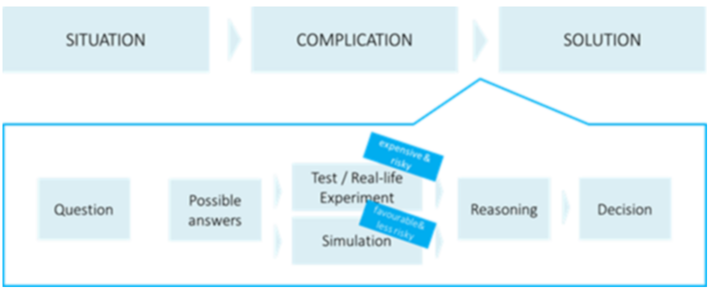

Fig. 2 Classification in the SCQA Model. Source: Own representation according to Minto (2002) [3].

1.2 Workload

A further advantage of simulations is that they can be adjusted in time, which has a temporal advantage in the case of very slow or very fast systems - e.g., slow motion of evolutionary processes or similar. Simulations are also useful if the complexity of the real system does not allow for observations or if the economic profitability is not given. An important area of application is also the training of personnel for complex systems such as pilot training in a flight simulator [2].

1.3 Fields of Application

Three main areas in which simulations are used can be distinguished as scientific cases, technical cases, and economic cases. This work focuses on economic issues, but the main areas of application are meteorology, medicine, mobility, traffic planning, and physics. The focus in these specific areas depends on various variables. On the one hand, corporate planning is geared toward the economic context, and simulations are primarily used for decision-making [4]. On the other hand, the focus in medical education is on the scientific context. In these cases, it is used primarily for research purposes [5].

1.4 Types of Simulations

Basically, two types of simulations can be distinguished: the deterministic simulation and the stochastic simulation. In a deterministic simulation, the input and output variables are clearly defined [6]. This means that for each input parameter, an output parameter exists in a 1:1 relationship. It is assumed that all components and the interrelationships are fully predictable. However, it should be noted that in most cases it is not realistic to set clearly defined input and output parameters in a complex system [7]. In contrast to deterministic simulation, stochastic simulation works with random variables. This results in a dependence on random variables. A stochastic simulation performs many iterations based on random variables and produces a different output in each iteration in a one-to-n (1:n) relationship [6]. Depending on the random variables, the input parameters are not reproducible. A variant of a stochastic simulation is the Monte Carlo simulation, which generates random numbers to simulate a random process [6].

2 Monte Carlo Simulations

2.1 Introduction

The Monte Carlo method is an example of one of the stochastic simulations described in the first chapter. Sobol describes it as a numerical technique for solving mathematical problems by modeling random numbers [8]. It uses mathematical principles such as probability theory, the law of large numbers, the central limit theorem, and the sampling method with a large number of samples. One of the most common Monte

Carlo simulations is the approximation of the expected value via the Monte Carlo evaluator in mathematical models with undefined or not clearly defined integrals [9].

A Monte Carlo simulation is a computer-based method for assessing future events and supports decision-makers in evaluating various options [10].

The advantage of the Monte Carlo method is that it allows the modeling of uncertain situations, plays it a hundred or a thousand times on a computer, and estimates the probabilities of the occurrence of a certain situation (most Monte Carlo simulations use at least 1,000 runs) [10].

The method also illustrates the consequences of moderate decisions. The procedure simulates random numbers for each factor involved. The combination of different random numbers for the numerous factors involved yields the result of the scenario. This allows mathematical problems to be solved - in contrast to deterministic methods - with random numbers for certain parameters [11].

The first known Monte Carlo simulation was used by the scientist Georges Louis Leclerc de Buffon in the 18th century. He wanted to calculate the probability that a needle thrown by him would end up in a defined pattern. So, he threw the needle several times, then noted and analyzed the results. This test is known as a precursor to the Monte Carlo simulations [12].

The most famous founding father of the Monte Carlo simulation is Stanislaw Ulam, who was a Polish-American scientist. In 1946 he applied the method for the first time together with John von Neumann, a Hungarian-American computer scientist. They used Monte Carlo simulations during the Second World War to calculate neutron diffusion. Later, these simulations contributed to the development of the atomic bomb. The name "Monte Carlo Method" was chosen for reasons of secrecy and is based on the names of the famous casino in the Monte Carlo district of Monaco [13].

As shown in Figure 3, the process of using a Monte Carlo simulation is divided into five steps: developing the model logic, defining the input distribution, determining the number of iterations, running the simulation, and analyzing the results.

The process starts with the first step of developing the model logic, which means that the user determines which parameters influence the other variables (usually based on mathematical formulas or models). Examples of input variables are sales or variable costs, and those for output parameters are risk factors or present values. Furthermore, the user of the Monte Carlo method is able to determine the strength of the influence. At least one parameter is required, but usually, at least two or more parameters are included in the simulation.

The second step is to define the input distribution. This means defining which possible results in the parameters can assume and which probable distribution results from the combination of the parameters. Normally, a moderate value in the middle of the scale is much more likely than an extreme value.

In the next step, the user sets the number of iterations that the computer program should execute or repeat steps one and two. Since the simulation is based on the law of large numbers and the central limit theorem, it is very important that the number of iterations is high enough to obtain reliable data. Various experts recommend

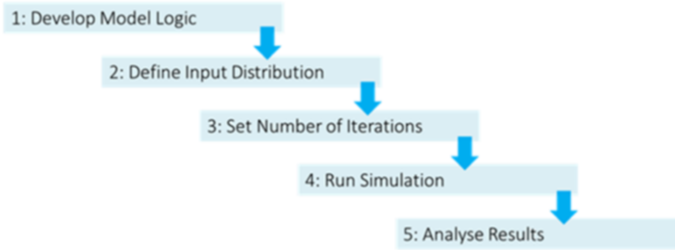

Fig. 3 Five Steps of a Monte Carlo Simulation. Source: Own representation according to Gleißner (2017) [14].

performing the simulation at least 1,000 times, whereas Haufe (Ed.) recommends repeating the Monte Carlo simulation at least 10,000 times to obtain reliable data.

It depends, above all, on the model observed and the uncertainties contained therein. The more uncertainties there are, the more iterations are required to obtain reliable results. Overall, a higher number of iterations usually leads to a higher quality of the Monte Carlo method.

The fourth step is to run the simulation. The computer program determines random numbers for the parameters from the first step using the distribution from the second step and converts them into a result. This is repeated according to the number of iterations specified in step three. At the end of this step, there are many hundreds or thousands of results for the same scenario.

These results are analyzed in step five. From the distribution of the results, the user gets the probability of a certain event, e.g., for the probability that the net present value of an investment will be positive [14]. Supported by the computer program, it is also possible to assign the results to different scenarios. In the above example, you can determine the probability that the net present value of an investment will be [14].

The following case study is a simple example demonstrating how a Monte Car-lo simulation with the Microsoft Office Excel Add-In @Risk from Palisade works.

2.2 Case Study - Project Management

The case study supports the planning process of a project where specific tasks must be done, and the end date of the project has to be determined. In this example, specific tasks must be done in this project. Some tasks can only be started when another task has been completed. The specific duration of a task is in most cases not known, so a distribution of the expected duration of every task can be defined.

With the help of Risk, the project is simulated and returns a distribution of the simulated end dates. This result could be used by a project manager for the determination of a project end date considering the probability of the project duration.

The significance of the result highly depends on the accuracy of the estimated task durations. Weekends and other influences have been excluded in this case study.

Step 1: Develop Model Logic

The start day is the only mandatory known input in this example. The other input values are the project task durations. A task can only start the day after the previous task is completed. If no necessary previous task exists, the task can be started at the beginning of the project. The formula for the end date of every single task (i) is computed by:

$$\text{EndDate}_i = (\text{IF}(\text{Prev}_i != NULL, \text{StartDate}, \text{EndDate}_{\text{Prev}} + 1)) + \text{Duration}_i \quad (1)$$

The simulation result corresponds to the end date of the last task of the project, so the formula for the project end date is:

$$\text{ProjectEnd} = \max(\text{EndDate}) \quad (2)$$

The project consists of five tasks shown in Figure 4. Some tasks are dependent on another task. The start day of the project is set for Jan. 6th, 2020.

Known Inputs	
Start Date	06.01.2020

Uncertain Inputs				
Task Name	**Start**	**Previous**	**Duration**	**End**
Task 1	06.01.2020		10	16.01.2020
Task 2	17.01.2020	Task 1	12	29.01.2020
Task 3	06.01.2020		9,666667	15.01.2020
Task 4	30.01.2020	Task 2	5	04.02.2020
Task 5	05.02.2020	Task 4	3	08.02.2020

Output	
Project end date	08.02.2020

Fig. 4 Case Study - Model Logic. Source: Own representation.

Step 2: Define Input distribution

The uncertain parameters are the estimated durations of the defined tasks. Considering that the project has different types of tasks, every task needs its own parameter distribution for the duration. For this example, some available distributions in @Risk are used for the different parameters, displayed in Figure 5.

Task	Distribution type	Parameters	@Risk Command
Task 1	Normal	Mean = 10; StdDev = 2	RiskNormal(10;2)
Task 2	Triangle	Min = 10; Most Propably = 11; Max = 15	RiskTriang(10;11;15)
Task 3	Triangle	Min = 8; Most Propably = 9; Max = 12	RiskTriang(8;9;12)
Task 4	Uniform	Min = 4; Max = 6	RiskUniform(4;6)
Task 5	Fix value	Value = 3	3

Fig. 5 Case Study - Distributions for the Task Durations. Source: Own representation.

For task 5, no distribution is used and the duration is defined as a fixed value, so this task will take 3 days in any case. Task 4 will take a random time between 4 and 5 days.

Step 3 and 4: Set the Number of Iterations / Run Simulation

The number of iterations is set to 10,000, so the project will be simulated 10,000 times (see Figure 6). Then the simulation can be executed via the "Start Simulation" button.

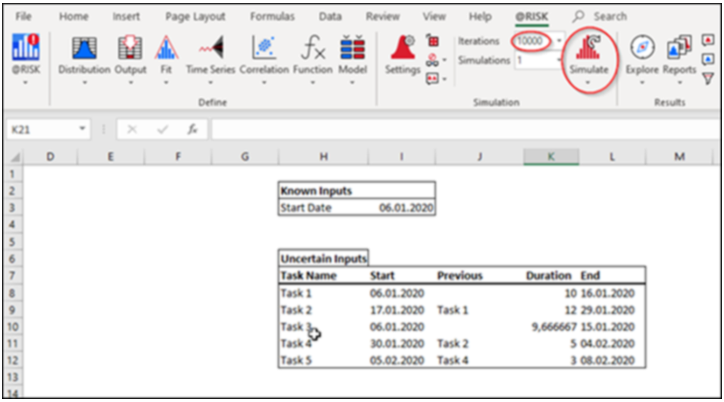

Fig. 6 Case Study – Iterations. Source: Own representation by using @Risk (Excel Add-In) (2020) [15].

Step 5: Analyse Results

As shown in Figure 7, the approximate end date of the project is the 8th of February, based on the density of the simulated project ends.

Table 1 shows the main statistic values of the simulation. In about 5% of the executed simulations, the tasks have been ready after the 11th of February, and another 5% of the simulations determined a project ended before the 4th of February. All simulation results are between Jan. 31st and Feb. 16th, 2020.

With this result, the project management has decision support for estimating or determining the project end, which was generated by the Monte Carlo simulation.

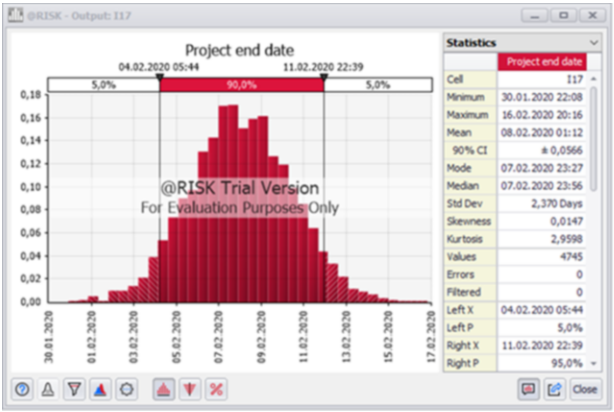

Fig. 7 Case Study - Determined Project End Date. Source: Own representation by using @Risk (Excel Add-In) (2020) [15].

Table 1 Case Study – Conclusion

STATISTIC	VALUE	INTERPRETATION
Minimum	30.01.2020	The day of minimum duration of the project.
Maximum	16.02.2020	The day of maximum duration of the project.
Mean	08.02.2020	The calculated mean value of all simulations (with the highest probability).
5^{th} Percentile	04.02.2020	5% probability that the project lasts at least this day.
95^{th} Percentile	11.02.2020	95% probability that the project lasts longer than this day.

Source: Own representation by using @Risk (Excel Add-In) (2020) [15].

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