HM-QCNN: Hybrid Multi-branches Quantum-Classical Neural Network for Image Classification

Haowen Liu[1], Yufei Gao[1(✉)], Lei Shi[1], Lin Wei[1], Zheng Shan[2], and Bo Zhao[2]

[1] School of Cyber Science and Engineering, Songshan Laboratory, Zhengzhou University, Zhengzhou, China
{yfgao,shilei}@zzu.edu.cn

[2] State Key Laboratory of Mathematical Engineering and Advanced Computing, Zhengzhou, China

Abstract. Quantum machine learning has been developing in recent years, demonstrating great potential in various research domains and promising applications for pattern recognition. However, due to the constraints of quantum hardware, the input qubits are restricted caused by small circuit size, and the fuzziness in all dimensions caused by the features that are difficult to be effectively mined. Besides, previous studies focus on binary classification, but multi-classification received little attention. To address the difficulty in multi-classification, this paper proposed a hybrid multi-branches quantum-classical neural network (HM-QCNN) that utilizes a multi-branch strategy to construct the convolutional part. The part consists of three branches to extract the features of different scales and morphologies. Two quantum convolutional layers apply quantum CRZ gates and rotational gates to design a random quantum circuit (RQC) with 4 qubits and full qubits measurements. The experiments on three public datasets (MNIST, Fashion MNIST, and MedMNIST) demonstrate that HM-QCNN outperforms other prevalent methods with accuracy, precision, and convergence speed. Compared with the classical CNN and the hybrid neural network without multi-branches, HM-QCNN reached 97.40% and improved the accuracy of classification by 6.45% and 1.36% on the MNIST dataset, respectively.

Keywords: Quantum machine learning · Multi-classification · Hybrid quantum neural network · Medical images

1 Introduction

As quantum computing improves by leaps and bounds, the development of quantum algorithms that uses noisy intermediate-scale quantum (NISQ) to perform useful computational tasks is entering a boom period [1]. In this stage, quantum machine learning (QML) is a promising applications of quantum computing in the era of NISQ, which attempts to use quantum hardware to achieve computational acceleration or better performance for tasks in machine learning, while random quantum circuits (RQC) provide

X. Yang et al. (Eds.): ADMA 2023, LNAI 14177, pp. 139–151, 2023.
https://doi.org/10.1007/978-3-031-46664-9_10

a prospective path [2–4]. Compared with classical machine learning, QML algorithms based on RQC have two potential advantages, i.e., greater expressiveness [5] and more computational power [6, 7], which originate from the superposition principle of quantum mechanics.

Recently, inspired by CNNs, quantum convolutional neural networks (QCNNs) have been proposed. These networks employed both classical and quantum hardware, and encapsulated parts of complex neural networks in quantum devices to exploit the superposition and entanglement of quantum systems, thus speeding up computation [8]. The central idea is to implement a quantum convolutional layer by applying shallow RQC, and the corresponding feature mapping is implemented by measuring the output quantum state of the RQC. The output of the quantum convolutional layer is classical data and thus can be directly adapted to the structure in CNNs, while also exploiting the capabilities of hardware of the current NISQ.

A proliferation of studies using QCNNs for binary classification, and an increasing number of research scholars devote themselves to studying the task of pattern recognition on images. The research on multi-classification is further complex because the distinction between multiple categories needs to be considered, and the classifier needs to make additional decisions. Therefore, for the multi-classification task, a framework combining classical computer and quantum hardware is introduced, which has been widely used in recent QML studies [9, 10], and the classifier needs to make more decisions, studies on multi-classification are more complex and fewer than binary classification. For the multi-classification task, a framework combined with classical computers and quantum hardware is introduced, which has been widely used in recent QML studies, and helps to explore the potential computational power of the NISQ computer. As shown in Fig. 1, it can be divided into two parts: the encoding model and the HNN model. The former is responsible for processing the input data, and the latter is the module for training.

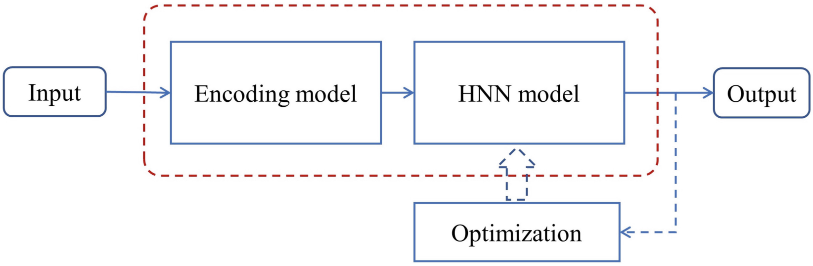

Fig. 1. Framework of the quantum convolutional neural network

The contributions of the current study are summarized in the following four folds:

1. The proposed HM-QCNN introduces multiple branches to construct networks, which implemented by RQCs, and two different scales of convolution kernels, in order to learn the syncretic features.
2. To verify the applicability of the model to the multi-classification, the experiments are conducted both on natural image datasets (MNIST, Fashion MNIST) and medical image dataset (MedMNIST).

3. Compared with previous approaches, HM-QCNN achieves better performance of accuracy, precision, and convergence speed.
4. To the best of our knowledge, this study is the first to explore the effectiveness of QNNs on medical images.

The remainder of the paper is organized as follows. Recent works related to QML and QCNNs are reviewed and summarized in Sect. 2. And Sect. 3 describes the encoding model and proposed HM-QCNN architecture in detail. The experiments of this work are presented in Sect. 4, comparing and demonstrating the various performances of HNN for image classification, and discussing the results. In Sect. 5 conclusions are drawn and directions for future work are suggested.

2 Related Work

The current volume of data is growing at an overwhelming rate, and the computational power required by machine learning algorithms increases with the data, which is gradually becoming limited for classical machine learning. And with the computational potential of quantum computers exceeding that of any classical computer, QML as a research frontier in AI has emerged as a prospective solution to the challenge of increasing data volumes [11]. QML has received a lot of attention in recent years, including quantum autoencoders [12, 13], quantum Boltzmann machines [14], quantum generative adversarial learning [7, 15, 16], and quantum kernel methods [17, 18].

Among them, lots of studies focused on the applications of QML in classification tasks, such as Edward Grant et al. [19] concluded that more expressive circuits have better accuracy and established hierarchical quantum circuits for binary classification of classical datasets IRIS and MNIST. Moreover, Yang et al. [20] organized SRA images into a data tensor and proposed a deep sparse tensor filter network for image classification.

In addition, motivated by the learning capability of CNNs and the potential power of QML, the hybrid quantum-classical neural network framework has emerged as a promising approach for classification tasks. Liu et al. [21] designed a hybrid quantum-classical convolutional neural network (QCCNN) that is friendly to current NISQ computers in terms of quantum bits and circuit depth, adapting to quantum computing to enhance the process of feature mapping while retaining the nonlinearity and scalability of classical CNN. Wei et al. [22] presented a quantum convolutional neural network (QCNN), which greatly reduces the computational complexity compared to classical. And applied it for image processing with numerical simulations for spatial filtering and edge detection. Finally, the model was verified on MNIST to have some robustness in image recognition. Cong et al. [23] analyzed the performance of the QCNN beyond existing methods and demonstrated that it could accurately identify quantum states associated with a one-dimensional topological phases. Francesco et al. [24] proposed a network model based on a variational circuit that reduces the circuit depth required for data encoding, using quantum neural networks for classification methods on recent quantum hardware. MacCormack et al. [9] offered the branching quantum convolutional neural network bQCNN inspired by QCNN with higher expressiveness.

Most of the existing studies are focused on the tasks of pattern recognition and image binary classification, the solution to multi-classification problems through quantum neural networks is still being explored, and the research on the recognition and classification of traditional natural images is also deficient. This work explored and designed a network model based on a quantum convolution filter fabricated by RQC combining a quantum convolution layer with a traditional network model structure for the multi-classification problems of handwritten digits and some natural images.

3 Method

3.1 Encoding Model

Quantum encoding is a process of converting classical information into quantum states, which is a very important step in the process of solving classical problems using quantum algorithms. Most encoding methods could be seen as parameterized circuits acting on and the parameters are determined by the classical information. The task of the encoding model in the framework is to map classical morphological data to quantum states in Hilbert space, and here three different encoding methods will be presented to achieve this transformation.

The first and most efficient in spatial terms method is to encode classical data in superimposed amplitudes by associating the normalized input data with the probability amplitudes of the quantum states, called the amplitude encoding method (AE) [25]. This approach encodes an N-dimensional classical vector x to a quantum state with n quantum bits, where $n = log_2(N)$ and $|x = \sum_i^N x_i|i$. Here $|i$ is a set of computational bases in Hilbert space and needs to satisfy $|x|^2 = 1$. However, depending on the quantum classifier used, the computational cost of preparing the data to quantum form will offset the speedup obtained in the classification process in general.

Another simpler approach is basic encoding, where the data is encoded onto the substrate of a quantum state. Each classical data vector will be encoded in each quantum bit, with the two fundamental states 0 and 1 will be considered as $|0\rangle$ and $|1\rangle$ of the quantum bit. This type of encoding method transforms a binary string of length n into a quantum state $|x\rangle = |i_x\rangle$ with n quantum bits, and is therefore inefficient in terms of space, yet efficient in terms of time [25].

The third encoding method is angle encoding, which employs quantum rotation gates to encode classical information x. The angle of these quantum gates is determined by the classical information. $|x\rangle = \overset{n}{\otimes} R(x_i)|0^n\rangle$, here any one of Rx, Ry, and Rz can be used as R. Usually the number of quantum bits is equal to the classical information dimension.

As the experimental framework shown in Fig. 1, this paper tried each of the above three methods in the encoding module to compare and analyze their performance in the multi-classification task. Among them, the basic encoding applied X gate and angle encoding used Ry gate rotating around the y-axis. All of them are constructed by RQC, whose circuits are shown in the Fig. 2.

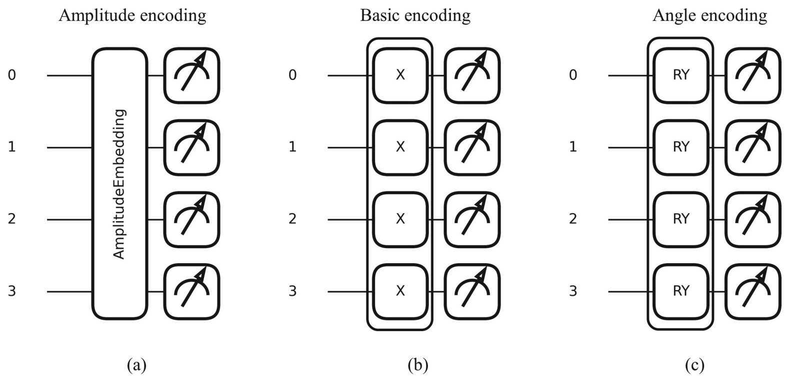

Fig. 2. Different encoding circuits

3.2 Hm-Qcnn

After encoding the classical data into quantum states, different gate operations are employed to each qubit corresponding to these data to form a quantum convolutional layer. In most previous works, the network is a quantum convolutional replacement of one traditional convolutional layer in the traditional network structure so that the whole structure contains at least one quantum convolution. The hybrid network structure applied in this work is based on hybrid computation, which consists of two parts, quantum and classical networks. The quantum part is responsible for the quantum convolution and the classical network part uses the convolutional and fully connected layers with the classical CNN structure. Here, unlike previous works, three branches are constructed in the HM-QCNN model, as shown in Fig. 3, two of which are quantum convolutional layers composed of quantum circuit and the other is a conventional convolutional layer.

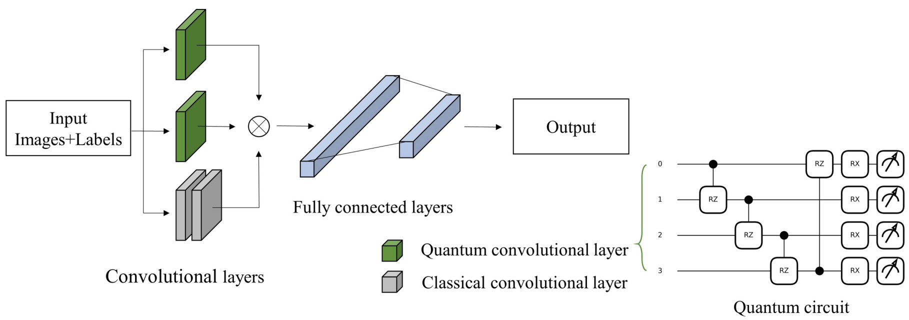

Fig. 3. HM-QCNN architecture

The main point of convolution layers is utilized filter to analyze all patches of images. This concept has been further developed in the background of quantum computing. The difference between classical and quantum convolution is that quantum circuits can produce complex kernels to extract meaningful features, which are difficult to handle by classical convolution. Quantum convolution is used as small RQCs to compute convolution operations, and match noisy mesoscale quantum hardware, with the advantage that it can work with a shallow depth quantum circuit and few quantum bits. The two quantum convolution layers in HM-QCNN are computed by applying RQC to respectively build kernel_size of 4 with stride 2 and kernel_size of 2 with stride 2 as the main part for convolutional filters, which employs a series of unitary transform and measurements connected by wires (qubits). The present model adopts pennyLane to initialize and simulate four qubits, i.e., the constructed RQC consists of four qubits. As depicted in Fig. 3, in the quantum convolutional filter, first a two-qubit CRZ quantum gate operation is employed, in other words, CRZ quantum gates are operated on each pair of adjacent qubits, which enables to capture of the relevant information on the same layer of the network. Then the RX quantum rotational gates are applied to operate on each qubit, embedding valid information into the quantum system. The final measurement phase, also known as the decoding phase, refers to the conversion of the quantum data into classical form [26]. Pauli matrix can be used as a measurement method, unlike other works with single qubit measurements, all-qubit are measured in this work, taking expectations by using Pauli-Z measurements for each qubit to obtain enough hidden information from the quantum system. The results of measurement are not yet direct representations of the predicted labels and therefore need to be further input to the classical network for processing.

The classical convolution layer is the key and important layer to extract features in the part of CNNs, which performs the convolution operation on the input features with kernels. Features are extracted from the images and map them to the next layer as complex features. The traditional convolutional layer branches in this model consist of two convolutional kernels of sizes 1 and 4 with strides 1 and 2, respectively. After these operations, the outputs of these three branches are concatenated and input to two fully-connected layers for classifying, and leakyReLU are utilized as the activation function to finally obtain the predicted results for the input images. The fully-connected layer is the second part of the CNN structure, that performs the classification process by applying weights to predict the classes. Classical CNN network with the equivalent structure and hybrid quantum neural network (QUANV1 – CONV1 – FC1 – FC2) are compared in this experiment.

In the learning phase, the cross-entropy loss is utilized as the loss function, and Adam is adopted as the optimizer for parameter optimization. During training, the network model is updated with parameters by backpropagation to minimize the error between the output results and the real results.

4 Experiments

4.1 Experiments Setting

Three independent models are compared in this paper, the proposed HM-QCNN, classical CNN with the equivalent structure, and HNN without multiple branches, i.e. HNN (w/o multi). Accuracy, precision, recall, and F1 score are used as evaluation metrics to assess classification performance of the model.

Setup. The experimental environment used in this work is Python 3.8, PyTorch 1.12.0, CUDA 11.6, batch size set to 32, the learning rate of 0.5, with a total of 50 epochs trained. Numerical simulations of the experiments are performed with PennyLane [27].

Datasets. Experiments are conducted on three public datasets MNIST, Fashion MNIST and MedMNIST. Different triple-classification tasks are performed on different datasets in this work. For example, the MNIST dataset is randomly generated in three experiments, the first experiment contains numbers {1,7,9}, the second task kept numbers {3,5,8}, and the third performed classification experiments on numbers {2,4,6}, which are described as E1, E2, and E3, respectively. Similarly, three tasks were generated on the Fashion MNIST dataset: the first task retained "T-shirt/top" "Trouser" and "Pullover"; the second task classified "Dress" "Coat" and "Sandal"; the third task kept the data of "Shirt", "Sneaker" and "Bag", which are denoted as E4, E5, and E6, accordingly.

4.2 Results and Discussion

This section discusses the performance of HM-QCNN on image multi-classification tasks. The experimental results demonstrate that the proposed model can be used to solve many types of image classification problems, and good results can be obtained not only on handwritten digital images, but also on natural images.

Three independent models are tested in the experiment with accuracy, precision, recall and F1-score as evaluation indicators shown in Table 1. The number of optimal performances is bolded. On MNIST dataset, HM-QCNN achieves 95.73%, 93.75%, 97.40% accuracy, which are 2.08%, 9.27% and 6.45% higher than the classical CNN model, and better than the HNN (w/o multi) model by 2.6%, 2.6%, and 1.36%, respectively. The optimal result is presented in E3 with accuracy, precision, recall and f1 scores of 97.40%, 97.40%, 97.39% and 97.40%. Moreover, on Fashion MNIST dataset, HM-QCNN performs slightly poor than HNN (w/o multi) in E4 and E5, but achieves 98.44%, 98.47%, 98.46% and 98.46% for accuracy, precision, recall and f1-score in E6, which is the best result among these methods.

Similarly, as shown in Fig. 4, the experimental data after model training are visualized with the t-SNE technique. Combined with the results in Table 1 to analyze, the data of E2 is not aggregated and also corresponds to the lower accuracy in the table. Compared with other groups of experiments, considering the reason of which, the original data of E2 is more disorganized, the results after classification are relatively poor.

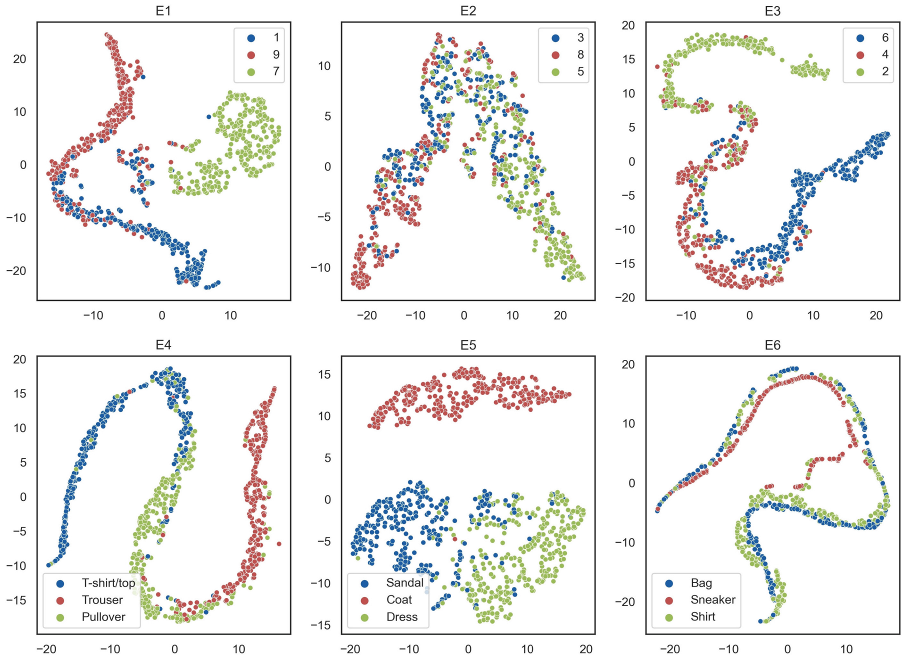

Fig. 4. Visualization with t-SNE of experimental datasets after training

Meanwhile, as shown in Fig. 5, it can be clearly seen that the performance of HM-QCNN is superior to classical CNN and the HNN without multiple branches, and all the classification accuracy can reach more than 93%. It indicates that the proposed HM-QCNN can effectively improve network performance and better solve classification problems in images. In addition, from the comparison of running time in the Fig. 6, it can be seen that the proposed model can significantly reduce training time and speed up convergence, which will help to classify images faster in practical applications. However, the gap between the execution time of HNN and the classical CNN is large, and the reason for this is the experiments are conducted with quantum numerical simulation, which speed cannot reach the real quantum computing hardware. Moreover, the speed is also affected by the limitation on the input qubits. However, in the future, with the development of quantum hardware, more qubits can be used to process images, thus improving the performance of the HNN.

Table 1. Performance evaluation of experiments

Experiment		Model	Acc. (%)	Pre. (%)	Re. (%)	F1-score (%)
MNIST		CNN	93.65	93.66	93.48	93.55
	E1	HNN (w/o multi)	93.13	92.95	92.03	92.97
		HM-QCNN	**95.73**	**95.66**	**95.64**	**95.65**
		CNN	84.48	84.54	84.51	84.52
	E2	HNN (w/o multi)	91.15	91.13	91.24	91.26
		HM-QCNN	**93.75**	**93.75**	**93.73**	**93.74**
		CNN	90.94	90.95	90.96	90.94
	E3	HNN (w/o multi)	96.04	96.10	96.03	96.05
		HM-QCNN	**97.40**	**97.40**	**97.39**	**97.40**
Fashion MNIST		CNN	94.06	94.08	94.01	94.02
	E4	HNN (w/o multi)	**94.17**	**94.30**	**94.19**	**94.24**
		HM-QCNN	93.95	93.65	93.65	93.65
		CNN	94.48	94.48	94.47	94.47
	E5	HNN (w/o multi)	**96.98**	**96.99**	**96.97**	**96.97**
		HM-QCNN	95.83	95.84	95.83	95.83
		CNN	97.40	97.43	97.50	97.46
	E6	HNN (w/o multi)	96.25	96.35	96.39	96.36
		HM-QCNN	**98.44**	**98.47**	**98.46**	**98.46**

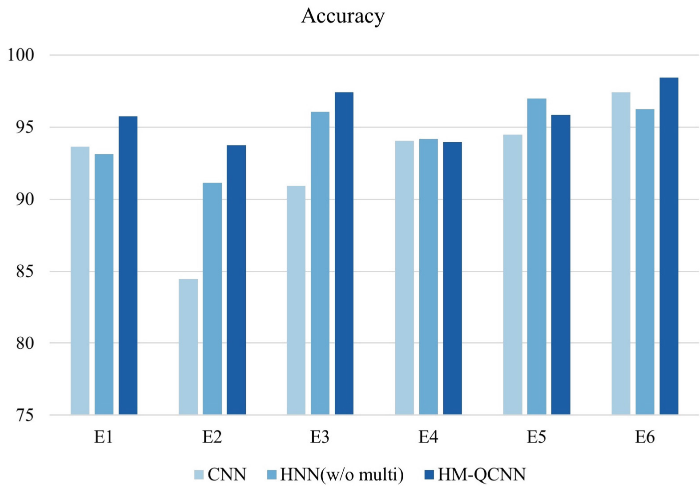

Fig. 5. Classification accuracy of different experiments

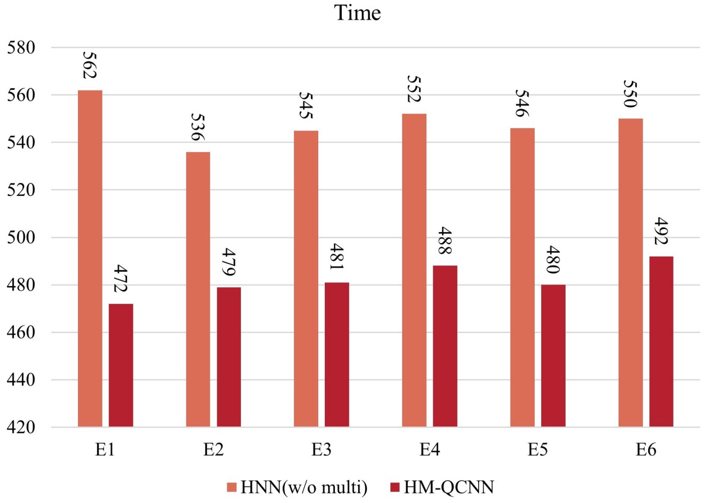

Fig. 6. Comparison of the execution time

In addition, experiments were also conducted on different encoding methods for triple-classification on MNIST and visualized the training results on the {1,7,9} sub-dataset. The training accuracy of the three different encoding methods is depicted in Fig. 7(a), and the training loss curves of the three different encoding methods with continuous reduction are shown in Fig. 7(b). The figure demonstrates that the angle encoding converges faster and achieves higher accuracy with smaller loss values. While amplitude encoding converges slower, but the training accuracy exceeds basic encoding to reach 99.98% at 25 epochs. Therefore, in this experiment, the angle encoding method works better.

The HM-QCNN model is also tested on the MedMNIST- breastMNIST dataset, with an accuracy of 73.08% on both the training set and testing set. Although the accuracy is not as good as on the other two datasets, this is because biomedical images have more special characteristics compared with other natural images. On the one hand, medical images have higher noise and lower contrast, which may affect the performance of the model. On the other hand, medical images represent structures inside the human body, and the morphology and other features of these structures vary greatly from case to case, which requires higher generalizability of the model. However, the HM-QCNN model still offers the prospect of application for tasks such as classification and diagnosis in medical images. The model can be improved in the future to enhance the generalization performance and improve the analysis of medical images.

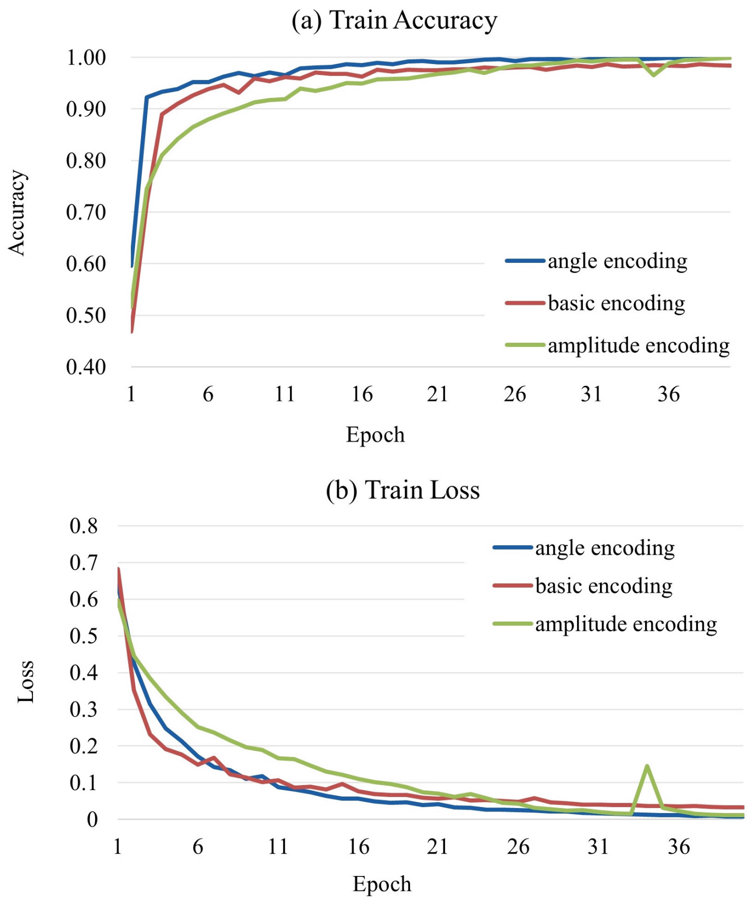

Fig. 7. Visualization of the learning curve for different encodings

5 Conclusion

To effectively improve the efficiency of classical CNNs while ensuring accuracy and precision, this paper develops the structure of hybrid quantum neural networks with multi-branch by constructing parameterized quantum circuits. And conducts some experiments for multi-classification tasks. The results indicate that the HM-QCNN model achieves better accuracy in both MNIST and Fashion MNIST and outperforms the HNN without branches in terms of execution time.

In the NISQ era, due to the limitations of the quantum hardware for the input qubits, the size of natural images is too large for existing devices, so relevant operations like dimensionality reduction are required before inputting to the model, which may adversely affect the model performance. However, in the near future, as the algorithms continue to be explored, lower qubit algorithms suitable for quantum hardware will be studied and designed.

Furthermore, future work will aim to expand the diagnostic classification research to more complex medical images. The potential of hybrid quantum neural networks for various tasks in medical imaging will also be explored, including disease diagnosis, lesion region localization, and tumor segmentation.

Acknowledgements. This work was supported in part by the National Key R&D Program of China (2020YFB1712401), the Nature Science Foundation of China (62006210), the Key Scientific and Technology Project of Henan Province of China (221100210100, 221100211200, 221100210600), the Key Project of Collaborative Innovation in Nanyang (22XTCX12001), the Research Foundation for Advanced Talents of Zhengzhou University (32340306), Preresearch Project of Songshan Laboratory (YYJC022022001), and Supported project by Songshan Laboratory (232102210154).

References

1. Lü, Y., Gao, Q., Lü, J., Ogorzałek, M., Zheng, J.: A quantum convolutional neural network for image classification. In: 2021 40th Chinese Control Conference (CCC), pp. 6329–6334 (2021). https://doi.org/10.23919/CCC52363.2021.9550027
2. Benedetti, M., Lloyd, E., Sack, S., Fiorentini, M.: Parameterized quantum circuits as machine learning models. Quantum Sci. Technol. **5**(1) (2019) https://doi.org/10.1088/2058-9565/ab5944
3. Liu, Y., Wang, D., Xue, S., Huang, A.: Variational quantum circuits for quantum state tomography. Phys. Rev. A. **101**(5) (2020). https://doi.org/10.1103/PhysRevA.101.052316
4. McClean, J.R., Romero, J., Babbush, R., Aspuru-Guzik, A.: The theory of variational hybrid quantum-classical algorithms. New J. Phys. **18**(2) (2016). https://doi.org/10.1088/1367-2630/18/2/023023
5. García-Pérez, G., Rossi, M.A. C., Maniscalco, S.: IBM Q experience as a versatile experimental testbed for simulating open quantum systems. NPJ Quantum Inf. **6**(1) (2020). https://doi.org/10.1038/s41534-019-0235-y
6. Du, Y., Hsieh, M.-H., Liu, T., Tao, D.: Expressive power of parametrized quantum circuits. Phys. Rev. Res. **2**(3) (2020). https://doi.org/10.1103/PhysRevResearch.2.033125
7. Lloyd, S., Weedbrook, C.: Quantum generative adversarial learning. Phys. Rev. Lett. **121**(4) (2018). https://doi.org/10.1103/PhysRevLett.121.040502
8. Trochun, Y., Stirenko, S., Rokovyi, O., Alienin, O.: Hybrid classic-quantum neural networks for image classification. In: 2021 11th IEEE International Conference on Intelligent Data Acquisition and Advanced Computing Systems: Technology and Applications (IDAACS), pp. 968–972 (2021). https://doi.org/10.1109/idaacs53288.2021.9661011
9. MacCormack, I., Delaney, C., Galda, A., Aggarwal, N., Narang, P.: Branching quantum convolutional neural networks. Phys. Rev. Res. **4**(1) (2022). https://doi.org/10.1103/PhysRevResearch.4.013117
10. Henderson, M., Shakya, S., Pradhan, S., Cook, T.: Quanvolutional neural networks: powering image recognition with quantum circuits. Quantum Mach. Intell. **2**(2) (2020). https://doi.org/10.1007/s42484-020-00012-y
11. Hur, T., Kim, L., Park, D.K.: Quantum convolutional neural network for classical data classification. Quantum Mach. Intell. **4**(1) (2022). https://doi.org/10.1007/s42484-021-00061-x
12. Romero, J., Olson, J.P., Aspuru-Guzik, A.: Quantum autoencoders for efficient compression of quantum data. Quantum Sci. Technol. **2**(4) (2017). https://doi.org/10.1088/2058-9565/aa8072

13. Ding, Y., Lamata, L., Sanz, M., Chen, X., Solano, E.: Experimental implementation of a quantum autoencoder via quantum adders. Adv. Quantum Technol. **2**(7–8) (2019). https://doi.org/10.1002/qute.201800065
14. Jain, S., Ziauddin, J., Leonchyk, P., Yenkanchi, S., Geraci, J.: Quantum and classical machine learning for the classification of non-small-cell lung cancer patients. SN Appl. Sci. **2**(6) (2020). https://doi.org/10.1007/s42452-020-2847-4
15. Pandian, A., Kanchanadevi, K., Mohan, V.C., Krishna, P.H. and Govardhan, E.:Quantum generative adversarial network and quantum neural network for image classification. In: 2022 International Conference on Sustainable Computing and Data Communication Systems (ICSCDS), pp.473–478 (2022). https://doi.org/10.1109/icscds53736.2022.9760943
16. Jonathan Romero, A.A.-G.: Variational quantum generators: generative adversarial quantum machine learning for continuous distributions. Adv. Quantum Technol. **4**(1) (2020). https://doi.org/10.1002/qute.202000003
17. Patrick Rebentrost, M.M., Lloyd, S.: Quantum support vector machine for big data classification. Phys Rev Lett. **113**(13) (2014). https://doi.org/10.1103/10.1103/PhysRevLett.113.130503113.130503
18. Havlíček, V., et al.: Supervised learning with quantum-enhanced feature spaces. Nature **567**(209–212) (2019). https://doi.org/10.1038/s41586-019-0980-2
19. Grant, E., Benedetti, M., Cao, S., Hallam, A.: Hierarchical quantum classifiers. NPJ Quantum Inf. **4**(1) (2018). https://doi.org/10.1038/s41534-018-0116-9
20. Yang, S., Wang, M., Feng, Z., Liu, Z., Rundong, L.: Deep sparse tensor filtering network for synthetic aperture radar images classification. IEEE Trans. Neural Netw. Learn. Syst. **29**, 3919–3924 (2018). https://doi.org/10.1109/TNNLS.2017.2688466
21. Liu, J., Lim, K. H., Wood, K. L., Huang, W.: Hybrid quantum-classical convolutional neural networks. Sci. China Phys. Mech. Astron. **64**(9) (2021). https://doi.org/10.1007/s11433-021-1734-3
22. Wei, S., Chen, Y., Zhou, Z., Long, G.: A quantum convolutional neural network on NISQ devices. AAPPS Bull. **32**(1) (2022). https://doi.org/10.1007/s43673-021-00030-3
23. Cong, I., Choi, S., Lukin, M.D.: Quantum convolutional neural networks. Nat. Phys. **15**(1273–1278) (2019). https://doi.org/10.1038/s41567-019-0648-8
24. Tacchino, F., Barkoutsos, P. K., Macchiavello, C., Gerace, D.: Variational learning for quantum artificial neural networks. In: 2020 IEEE International Conference on Quantum Computing and Engineering (QCE), pp.130–136 (2020). https://doi.org/10.1109/qce49297.2020.00026
25. Jian, Z., Zhao-Yun, C., Xi-Ning, Z., Cheng, X.: Quantum state preparation and its prospects in quantum machine learning. Acta Phys. Sin. **70**(14) (2021). https://doi.org/10.7498/aps.70.20210958
26. Yuki Takeuchi, T.M.: Quantum computational universality of hypergraph states with Pauli-X and Z basis measurements. Sci. Rep. **13585**(9) (2019). https://doi.org/10.1038/s41598-019-49968-3
27. Bergholm, V., Izaac, J., Schuld, M., Gogolin, C., Ahmed, S.: PennyLane: automatic differentiation of hybrid quantum-classical computations. ArXiv. (2022). https://doi.org/10.48550/arXiv.1811.04968