Vibration of Internal Gear in Planetary Gear Trains Under Moving Load

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Abstract. This paper investigates the vibration of an internal ring gear within a planetary gear train. The motivation for this research comes from the the wind industry, in which the ring gear becomes more and more flexible with the increase of the power scale. In this paper, the gear is modeled as an thin-walled elastic ring which is connected to the frame with finite number of constraints, such as bolts or pins. The meshing force between the planet gear and the ring gear is modeled as a sinusoidal function of time acting along the line of action. By assuming the mode shape and taking advantage of the orthogonality of the mode shapes, the governing equation of motion in the form of ordinary differential equation is obtained, and solved numerically. The results are compared against published work, and it shows they agree very well.

Keywords: Internal gear · Ring · Vibration · Moving force

1 Introduction

Planetary gear trains (PGT) are widely used in various industrial and military applications. Research on its dynamics and vibration started in late 1970s, and many publications can be found in scientific journals and conference proceedings. Review on these research works were made by Yang and Dai [1], and Cooley and Parker [2]. In the published works, the lumped parameter models are most often used. PGTs are also widely adopted in wind power systems. In such systems, the internal gear within the PGTs becomes more and more flexible with the never-stop increase in power and size. This fact makes the lumped parameter modeling in PDT dynamics unrealistic. In addition, the meshing force between the planet gear and the ring gear(internal gear) moves constantly along the circumference, bringing the moving load effect into the dynamics. In this case, two factors, namely the flexibility of the internal gear, and the moving gear meshing load, call for careful consideration in the dynamics modeling. This is the motivation of this paper.

The paper is organized as below: Sect. 1 presents the problem statement, and introduces a very brief literature review. The dynamic model is developed

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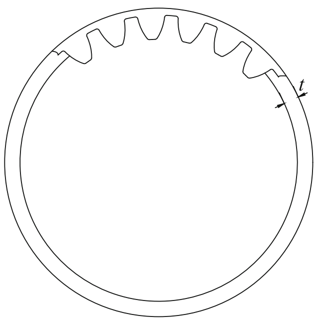
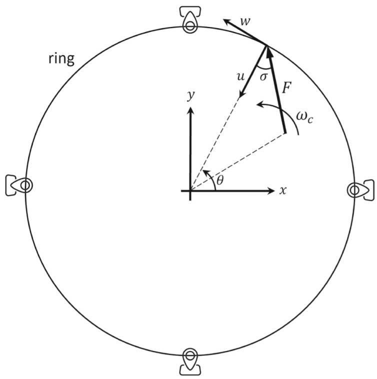
in Sect. 2. In Sect. 3 the solution strategy in presented and simulation results are outlined. A case study is presented in Sect. 4. The conclusions are drawn in Sect. 5.

2 Dynamic Model

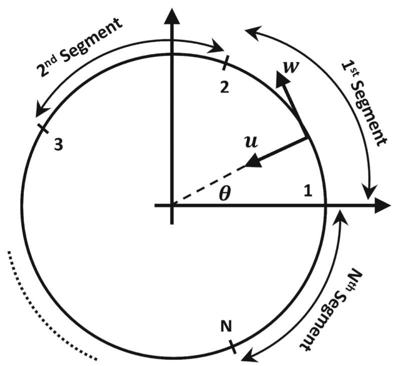
The internal gear is simplified as an thin-walled elastic ring. The thickness of the ring is taken as the thickness measured from the pitch circle of the gear as shown in Fig. 1(a). In Fig. 1(b), the ring is represented by its center line, and the constrains in the form of bolts or pins are shown. The displacements of a specific point on the center line are represented by a tangential component w and a radial component u. The meshing force is along the line of action and shown as F. The constrain supports, generally 3 or 4, divide the whole ring into several segments as shown in Fig. 1(c). In general the division is made equally.



(a) Internal gear



(b) Constrained internal gear



(c) Segments of internal gear

Fig. 1. Modeling of internal gear

For simplicity, the following two assumptions are made

1. The effects of rotary inertia and shear deformations are neglected.
2. The center line of the ring remains inextensible.

With the assumption of inextensibility, the following relation exists.

$$u = \frac{\partial w}{\partial \theta} \tag{1}$$

where θ is the position angle shown in Fig. 1(c).

Then the vibration of the ring can be represented by only the tangential component w as below:

$$\frac{\partial^6 w(\theta,t)}{\partial \theta^6} + 2\frac{\partial^4 w(\theta,t)}{\partial \theta^4} + \frac{\partial^2 w(\theta,t)}{\partial \theta^2} + \frac{R^4 \rho A}{EI}\frac{\partial^2}{\partial t^2}\left[\frac{\partial^2 w(\theta,t)}{\partial t^2} - w(\theta,t)\right]$$
$$= \frac{R^4}{EI}\left(\frac{\partial f}{\partial \theta} - p\right)\delta(\theta - \omega_c t) \tag{2}$$

where f and p are the meshing force component along the radial and tangential directions, respectively. ω_c is the moving angular velocity of the meshing force. In the case of PGT, it is the rotation velocity of the carrier. δ is the Dirac delta function. It is used to represent the concentrated meshing force. E and ρ are the elastic modulus and the density of the material. A is the area of the cross section.

For the sake of brevity, the derivation of the equation is not presented here. It can be found in classical texts on continuous system vibration, such as the paper by Chidamparam [3] and the book by Rao [4].

At a support θ_s, the boundary and continuous conditions can be developed easily based on the form of constrains. If the supports are pins, the following conditions apply.

$$u|_{\theta_s} = 0 \tag{3}$$

$$w|_{\theta_s} = 0 \tag{4}$$

$$\frac{\partial^2 w}{\partial \theta^2}|_{\theta_s} = \frac{\partial^2 w}{\partial \theta^2}|_{\theta_s + \frac{2\pi}{n}} \tag{5}$$

$$\frac{\partial^3 w_{n+1}}{\partial \theta^3}|_{\theta_s} = \frac{\partial^3 w_n}{\partial \theta^3}|_{\theta_s + \frac{2\pi}{n}} \tag{6}$$

where n is the number of the constrains and θ_s is the angle position of the constraint. The subscripts n and $n+1$ indicate the adjacent two segments of the ring.

The meshing force between the planet gear and the internal ring gear is simplified as a sinusoidal function with the meshing frequency.

$$F = F_s + F_0 \cos(\omega_m t + \theta_0) \tag{7}$$

where F_s and F_0 are the mean meshing force and the amplitude of the varying force, respectively.

3 Solution Strategy

Equation (2) is treated by introducing the following two relations.

$$w(\theta,t)=\sum_{j=1}^{\infty}\left[\Psi_j(\theta)q_j(t)\right] \tag{8}$$

$$q_j(t)=D_j e^{i\omega_j t} \tag{9}$$

where $i=\sqrt{-1}$.

Inserting Eqs. (8) and (9) into Eq. (2), and retaining only the first term give

$$\frac{d^6\Psi}{d\theta^6}+2\frac{d^4\Psi}{d\theta^4}+\left(1-\frac{R^4\rho A\omega^2}{EI}\right)\frac{d^2\Psi}{d\theta^2}+\frac{R^4\rho A\omega^2}{EI}=0 \tag{10}$$

To this point, the partial differential equation (PDE) is transformed into the form of ODE.

Assuming the solution of Eq. (10) has the following form:

$$\Psi(\theta)=A_1e^{\Upsilon_1\theta}+A_2e^{\Upsilon_2\theta}+A_3e^{\Upsilon_3\theta}+A_4e^{\Upsilon_4\theta}+A_5e^{\Upsilon_5\theta}+A_6e^{\Upsilon_6\theta} \tag{11}$$

and taking advantage of the following orthogonality.

$$\int_0^{\theta_f}\left[\frac{d\Psi_i}{d\theta}\frac{d\Psi_j}{d\theta}+\Psi_i\Psi_j\right]=\begin{cases}0 & \text{if } i\neq j\\ 1 & \text{if } i=j\end{cases} \tag{12}$$

With all the above, the solution of the equation can be represented in the form below.

$$\begin{aligned}\ddot{q}_k+\omega_k^2q_k&=Q_{0k}\sin(P_{0k}t)+Q_{0k}\cos(P_{0k}t)+Q_{1k}\sin(P_{1k}t)\\&\quad+Q_{2k}\cos(P_{1k}t)+Q_{3k}\sin(P_{2k}t)+Q_{4k}\cos(P_{2k}t)\end{aligned} \tag{13}$$

The coefficients in this equation are as below:

$$Q_{0k}=\sqrt{\frac{2}{k^2+1}}\frac{\omega_m\cos\sigma F_0}{\omega_c\rho A\sqrt{\theta_f}} \tag{14}$$

$$Q_{1k}=\frac{G_k}{2}\left(\sin\theta_0+\cos\theta_0\right)+\frac{H_k}{2}\left(\sin\theta_0+\cos\theta_0\right) \tag{15}$$

$$Q_{2k}=\frac{G_k}{2}\left(\cos\theta_0+\sin\theta_0\right)-\frac{H_k}{2}\left(\cos\theta_0+\sin\theta_0\right) \tag{16}$$

$$Q_{3k}=\frac{G_k}{2}\left(\sin\theta_0-\cos\theta_0\right)+\frac{H_k}{2}\left(\sin\theta_0+\cos\theta_0\right) \tag{17}$$

$$Q_{4k}=\frac{G_k}{2}\left(\sin\theta_0+\cos\theta_0\right)+\frac{H_k}{2}\left(\cos\theta_0-\sin\theta_0\right) \tag{18}$$

$$P_{0k}=\frac{k\omega_c\pi}{\theta_f} \tag{19}$$

$$P_{1k} = \omega_m - \frac{k\omega_c\pi}{\theta_f} \quad (20)$$

$$P_{2k} = \omega_m + \frac{k\omega_c\pi}{\theta_f} \quad (21)$$

4 Case Study

A symmetrically pin-supported internal spur gear ring is studied. The gear is connected to the frame through 4 pins equally separated. The parameters used in the simulation are given in Table 1.

Table 1. Parameters of simulation

Definition	Parameter	Value	Unit
Outer radius of ring	R_0	0.520	m
Inner radius of ring	R_i	0.500	m
Density	ρ	7800	kg/m^3
Thickness	t	0.02	m
Face width	b	0.110	m
Young's modulus	E	210	GPa
Carrier rotational speed	ω_c	12.57	rad/s
Meshing force angular speed	ω_m	3.5	rad/s
Meshing force amplitude	F_0	50	N
Mean meshing force	F_s	75	N
Meshing phase angle	θ_0	0	rad
Pressure angle	σ	24.6	^{o}

4.1 Natural Frequencies and Mode Shapes

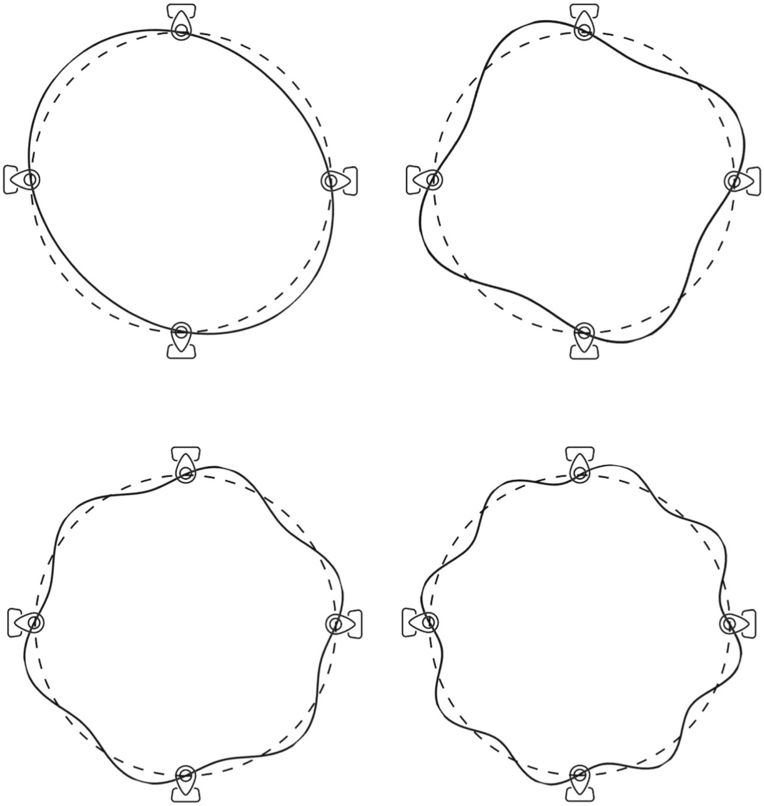
The first 4 natural frequencies are obtained. To validate, the results are compared against existing publication [5]. The results are given in Table 2.

Table 2. Natural frequencies

Method	ω_1 (Hz)	ω_2 (Hz)	ω_3 (Hz)	ω_4 (Hz)
Current work	98.26	222.64	532.06	628.20
[5]	96.63	273.83	523.83	847.62

Clearly the results obtained by the two method agree very well.

The vibration mode shapes corresponding to the 4 natural frequencies are given in Fig. 2



(c) 3^{rd} mode (d) 4^{th} mode

Fig. 2. Mode shapes

4.2 Dynamic Coefficient

The ratio of the maximum dynamic deflection to the static deflection of the middle of any the segment can is defined as the dynamic coefficient. It is often used as a measure of the moving load effect. The effect is shown in Fig. 3.

It can be seen clearly that the dynamic coefficient is way bigger than unit, indicating the moving load has a significant effect on the dynamics. In addition, the maximum deflection appears later then the mid point of the segment. This is reasonable under the moving load condition.

5 Conclusion

This paper investigates the dynamics of an internal gear within a PGT to the moving meshing load. The gear is modelled as an thin-walled elastic ring. The meshing force is represented as a sinusoidal force along the line of action. The

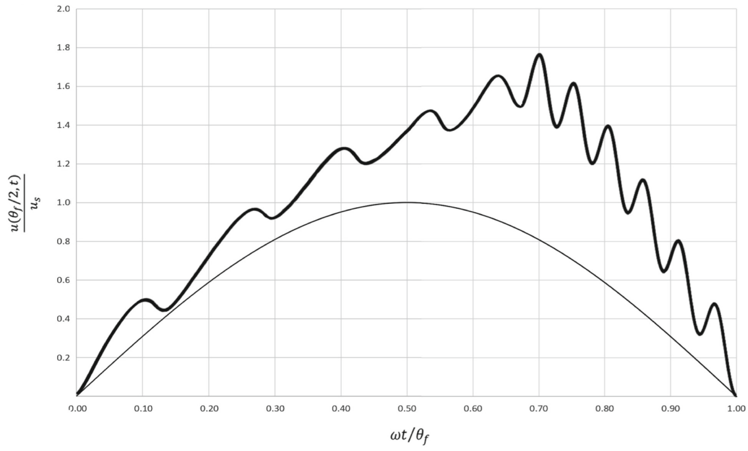


Fig. 3. Deflection

free response is first studied, and the first 4 orders of natural frequencies and mode shapes are obtained. Comparison is made against published work. It is found that the moving load has a significant effect on the deflection of the gear; thus, the effect on dynamics is not negligible.

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