A Wear Depth Characterization Method Based on Fractal Order Taylor Expansion of Measured Normal Displacements

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Abstract. Normal displacements are measured by sensors on current wear apparatus to represent wear depth. In practice, measured normal displacements are interfered by actual working condition factors such as thermal expansion and mechanical vibration, resulting in a puzzle of relationship between wear depth and measured normal displacements. This paper provides a characterization method of such relationship under mixed lubrication condition. By introducing Taylor expansion to measured normal displacements, Archard model and dissipation model are unified as the first order of such Taylor expansion. A non-integer fractal order D is determined to give the function relationship between wear depth and measured normal displacements. The fractal order D is verified to be a characterization parameter of measurement quality based on a series of wear experiments. Further analysis proves that the fractal order D is capable to be a prediction parameter of wear depth, the prediction errors are below 15%. These works are expected to make contributions for online wear monitoring in engineering.

Keywords: Wear Depth Characterization · Fractal Order · Taylor Expansion · Normal Displacement · Measurement

1 Introduction

In Tribology studies, a wear process is generally accompanied by measurement processes for wear characterization [1–3]. Conventionally, this characterization is indirectly but continuously realized by measuring friction coefficient, temperature, sliding velocity and normal load during wear process [4, 5], or directly but discontinuously realized by measuring sample height, weight and surface topography before and after wear process [6]. Online wear monitoring with direct and continuous measurement is a significant project in engineering.

With development of displacement sensor technology, such as laser [7–9], eddy current [10–12] and sound emission [13–15], normal displacement in wear process can be directly and continuously measured. On this basis, wear depth is expected to be directly obtained from measured normal displacement. However, the relationship between wear depth and measured normal displacement is unclear so that online wear

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monitoring has not been reached, since there are interference factors such as thermal expansion, mechanical vibration, mechanical deformation, assembly error and other accidental errors [16–18].

Thus, this paper provides a method to characterize the relationship between wear depth and measured normal displacement in a view of mathematical form. The measured normal displacement is presented with Taylor expansion to formally unify Archard model and dissipation model. A non-integer fractal order D is further determined to characterize the relationship between measured normal displacement and actual wear depth. Experiments are conducted to verify the function of D, including measurement quality characterization and wear depth prediction.

2 Methodology

2.1 Taylor Expansion

In practice, the measured normal displacement H could not represent the actual wear depth w_h directly since there are interference factors. So that a function f is expected to be found to connect H and w_h.

$$w_h = f(H) \tag{1}$$

Besides wear depth, wear volume is generally acquired in practice. The wear volume can be obtained through relationship between wear depth and width of wear track. The wear volumes of rotating or oscillating tribo-pairs and linear reciprocating tribo-pairs are shown as Eqs. (2) and (3) respectively.

$$w_v = \int_0^{\Omega} \int_0^{w_h} b(z) \mathrm{d}z \mathrm{d}\theta \tag{2}$$

$$w_v = \int_0^{L} \int_0^{w_h} b(z) \mathrm{d}z \mathrm{d}x \tag{3}$$

The normal displacement $H(t)$ at moment t is determined as the difference value between the measured value $h(t)$ and the beginning value $h(0)$ of the laser sensor.

$$H(t) = h(t) - h(0) \tag{4}$$

Herein, such normal displacement $H(t)$ is regarded as the Taylor expansion of function $H(Y(t))$ where $Y(t)$ is a time extension quantity.

$$H(t) = H(0) + \sum_{n=1}^{M} \frac{1}{n!} \frac{\mathrm{d}^n H}{\mathrm{d}Y^n}|_0 (Y(t) - Y(0))^n + o(Y(t) - Y(0)) \tag{5}$$

In practice, the $H(t_i)$ and $Y(t_i)$ are discretely obtained with specific time interval Δt, t_i is accumulation $i\Delta t$ of i time intervals. Thus, Eq. (5) becomes a discrete form. Generally, the $H(i\Delta t)$ is approximated by the first order expansion $H_1(i\Delta t)$ expanded at zero point where $H(0)$ and $Y(0)$ are zeros, the high order expansions are ignored.

$$H_1(i\Delta t) = \frac{\Delta H}{\Delta Y}|_0 \sum_{i=1}^{N} \Delta Y(i\Delta t) = \frac{\Delta H(0)}{\Delta Y(0)} \sum_{i=1}^{N} \Delta Y(i\Delta t) \tag{6}$$

In Archard model [19–21], such time extension quantity is the relative sliding distance of tribo-pairs S. The reciprocal of original distance $\Delta S(0)$ corresponds to the coefficient K, the original displacement $\Delta H(0)$ corresponds to the ratio of normal load F and soft material hardness p.

$$H_1(i\Delta t) = \frac{1}{\Delta S(0)}\Delta H(0)\sum\nolimits_{i=1}^{N}\Delta S(i\Delta t) = K\frac{F}{p}\sum\nolimits_{i=1}^{N}\Delta S(i\Delta t) = w_h(i\Delta t) \quad (7)$$

In dissipation model [22–25], such time extension quantity is the wear work W. The ratio of original displacement $\Delta H(0)$ and work $\Delta W(0)$ corresponds to the linear coefficient B of dissipation wear model.

$$H_1(i\Delta t) = \frac{\Delta H(0)}{\Delta W(0)}\sum\nolimits_{i=1}^{N}\Delta W(i\Delta t) = B\sum\nolimits_{i=1}^{N}\Delta W(i\Delta t) = w_h(i\Delta t) \quad (8)$$

Thus, such first order Taylor expansion $H_1(i\Delta t)$ is the unified form of common wear models. So that, ideally, wear depth w_h could be obtained from measured normal displacement H by the function $f(H) = H_1(i\Delta t)$.

2.2 Fractal Order

In practice, the mentioned function $f(H) = H_1(i\Delta t)$ could not accurately correspond to the actual wear depth w_h since there are interference factors such as thermal expansion and mechanical vibration, these interference factors have not been eliminated in current wear test technology. So that a non-integer fractal order D between 0 and 1 order is introduced, intending to provide an accurate function relationship between measured normal displacement H and actual wear depth w_h. The D order Taylor extension H_D is shown as Eq. (9).

$$H_D(t) = \frac{1}{D!}\frac{\mathrm{d}^D H}{\mathrm{d}Y^D}|_0 Y(t) = \frac{1}{\Gamma(D+1)}\frac{\mathrm{d}^D H}{\mathrm{d}Y^D}|_0 Y(t) = w_h(t) \quad (9)$$

In a discrete form, assuming the wear process is relatively steady so that the time extension quantity Y increases with equal ΔY, and the measured normal displacement H increases with average $\overline{\Delta H}$. N points are collected in this process. Since D is between 0 and 1, $\Delta^D H$ is approximately represented by $\overline{\Delta H}$.

$$H_D(i\Delta t) = \frac{1}{\Gamma(D+1)}\frac{\overline{\Delta H}}{\Delta Y^D}(N\Delta Y)^D = \frac{N^D}{\Gamma(D+1)}\overline{\Delta H} = w_h(i\Delta t) \quad (10)$$

Thus, the function f between measured normal displacement H and actual wear depth w_h is obtained, shown as Eq. (11).

$$w_h = f(H) = \frac{N^D}{\Gamma(D+1)}\overline{\Delta H} \quad (11)$$

Equation (11) is determined by measured data numbers N and fractal order D. Fractal order D is considered to be an intrinsic characterization parameter of measurement quality.

$$D = 0, w_h = \overline{\Delta H} \quad (12)$$

$$D = 1, w_h = N\overline{\Delta H} \tag{13}$$

As shown in Eqs. (12) and (13), when D reaches 0, the total wear depth w_h will be equal to the $\overline{\Delta H}$ in each step, indicating that the measurement quality is too low to recognize the accumulation of wear depth; while when D reaches 1, the total wear depth w_h will be the sum of $\overline{\Delta H}$ in each step, indicating that the measurement quality is rather high so that the accumulation process of wear depth is measured step by step.

3 Experiments and Results

3.1 Experiments

In order to obtain primary data of normal displacement and wear depth, wear experiments are conducted on standard tester and simulation apparatus with tribo-pair samples. The materials and lubrication condition are selected according to typical mechanical seals applied on aero-engines. The experiment process is shown in Fig. 1.

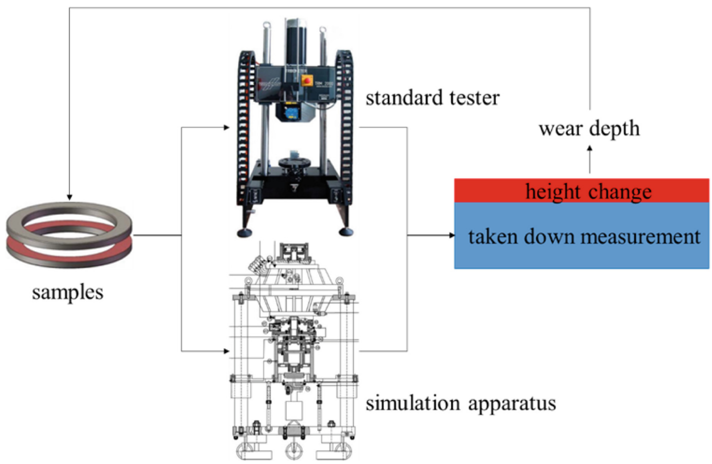


Fig. 1. Schematic diagram of the wear experiments.

On standard TRM 1000 (Wazau, Berlin) tester, GH4169 with Cr_2O_3 coating is selected as rotating ring material, T482 is selected as static ring material. These samples are lubricated with Mobil jet oil II. To ensure that the samples work under mixed lubrication condition, the Stribeck curve which gives the relationship between lubrication condition and working condition parameters are measured before experiments. The working condition parameters are set within mixed lubrication range and maintained during the experiments: the sliding velocity is 0.36 m/s, the normal load is 0.15 MPa and the temperature is 30 °C. The real-time normal displacements are collected by laser sensor on the tester every 0.1s, while the wear depth is obtained by taken-down measurement every 30min. Heights of 12 points on the static ring are measured before and after each experiment period to obtain corresponding wear depth. Experiments on the standard tester are repeated twice.

On a 1:1 working condition simulation apparatus of aero-engine mechanical seals, the GH4169 and T482 materials are tested under sliding velocity 111.42 m/s, normal load 0.15 MPa and temperature 150 °C. The samples are also lubricated with Mobil jet oil II under mixed lubrication. The normal displacements are measured by an eddy current sensor on the apparatus every 1s, the wear depth is measured by taken-down measurement for 5 times during an experiment.

3.2 Results

The fractal orders *D* of the experiment results are calculated by substituting their w_h and $\overline{\Delta H}$ into Eq. (11), shown in Fig. 2.

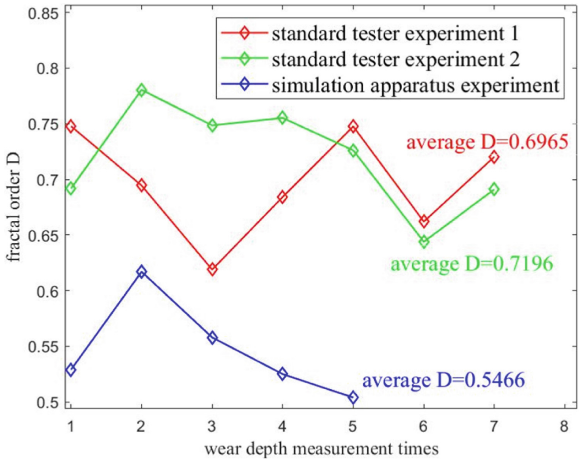
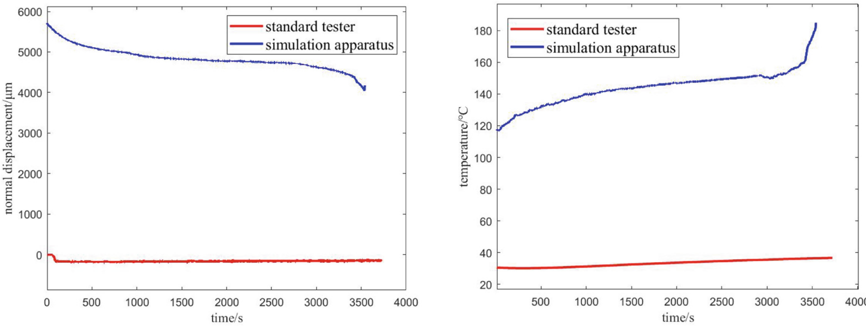


Fig. 2. Fractal orders D of experiment results.

During the wear experiments, the values of *D* are always ranged within 0 and 1. Such result conforms to the determination of *D*, which is a non-integer expansion order between the 0 order expansion and 1 order expansion of measured normal displacement Taylor expansion. The result also reveals that the fractal order *D* could be applied on different wear testers which work under different working conditions and measure with different sensors.

The fractal orders *D* on standard tester are ranged from 0.6189 to 0.7478 and from 0.6439 to 0.7805, they fluctuate around the average values 0.6965 and 0.7196. The fractal orders *D* on simulation apparatus are ranges from 0.504 to 0.617, they fluctuate around the average value 0.5466. These *D* on simulation apparatus are less than *D* on standard tester, since more severe thermal expansions and mechanical vibrations caused by higher temperature and sliding velocity on the apparatus, shown as Fig. 3.

The measured normal displacements on simulation apparatus reach millimeter level while the measured normal displacements on standard tester are micron level, the temperatures on simulation apparatus are around 150 °C and rise to over 180 °C in the end



(a) Normal displacements. (b) Temperatures.

Fig. 3. Measured normal displacements and temperatures of the experiments.

while the temperatures on the standard tester maintain around 30 °C. These more severe interference factors reduces the measurement qualities, representing by D. Such comparison result reveals that D is capable to be a characterization parameter of measurement quality.

Additionally, the testing condition is defined by the Hersey parameter G of Stribeck curve, shown as Eq. (14). Such G is related to the sliding velocity v, the normal pressure P and lubricant viscosity η related to temperature T with a Barus viscosity-temperature equation, shown as Eq. (15).

$$G = \frac{\eta(T)v}{P} \tag{14}$$

$$\eta(T) = \eta_0 \exp(\beta(T - T_0)) \tag{15}$$

For the standard tester, the range of G is 72.6523 to 87.1547 with temperature range 30.2 °C to 36.7 °C, the sliding velocity is 0.36m/s and the normal pressure is 0.15 MPa. For the simulation apparatus, the range of G is 344.8269 to 2380.4 with temperature range 116.9–185°C. Thus, the proposed method is applicable in both ranges with relatively low and high testing conditions for the specifically studied materials GH4169 with Cr_2O_3 coating and T482. The practicability of the proposed method for more materials under more testing conditions will be validated in further investigations [26, 27].

4 Discussions

For the experiments conducted in this study, the fractal order D could be applied to predict wear depth by substituting D, measured N and $\overline{\Delta H}$ in Eq. (11). As shown in Sect. 3.2, the fractal orders D during the experiment process maintain around their average values. So that the wear depths of experiment 1 on standard tester are predicted with the average D. The comparison of predicted wear depths and measured wear depths are shown in Fig. 4.

The errors between predictions and actual results are less than 15%. Thus, it is verified that the fractal order is capable to be a prediction parameter of wear depth.

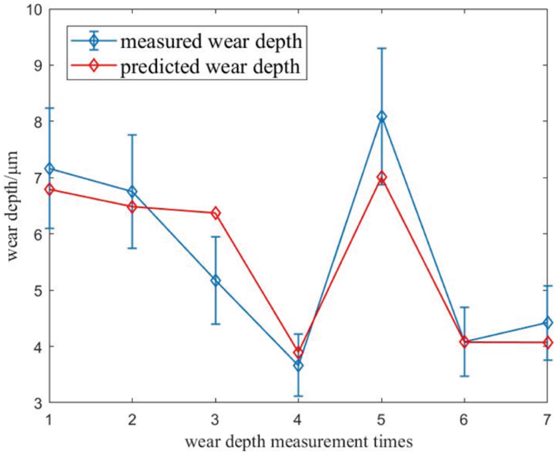


Fig. 4. Comparison of predicted wear depths by D and actual results.

Under relative steady conditions where the fractal orders D are approximately equal, online wear monitoring could be realized by an estimated D and real-time measured normal displacements. Such wear depth is predicted under mixed lubrication condition, the proposed method will also be tested under unlubricated conditions in the future to give more precise prediction since lubrication starvation scenario occurs in practice.

5 Conclusions

This paper characterizes the relationship between wear depth and measured normal displacement under mixed lubrication condition by introducing a fractal order Taylor expansion. Several conclusions are drawn as follows:

(1) Archard model and dissipation model are formally unified by introducing Taylor expansion to measured normal displacement. These wear models are regarded as the first order expansion of measured normal displacement function to relative sliding distance and wear dissipation energy.
(2) A non-integer fractal order D is determined to characterize the relationship between measured normal displacement and actual wear depth. Wear depth and average measured normal displacement are connected by a simplified Taylor expansion with variants fractal order D and measured data number N.
(3) The functions of D are proved by a series of wear experiment. Experiment results reveal that D is capable to be the characterization parameter of measurement quality and the prediction parameter of wear depth. D also shows the potential to realize online wear monitoring.

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