



# An Improved Model for Road-Tyre Interaction

Kamran Hashmi<sup>1</sup> and Anirban Guha<sup>2</sup>(✉)

<sup>1</sup> Vehicles Research and Development Establishment, DRDO, Ahmednagar, India

<sup>2</sup> Department of Mechanical Engineering, Indian Institute of Technology, Bombay, India  
anirbanguha1@gmail.com

**Abstract.** Tyres comprise of both rigid and flexible components. Difficulty in judging the flexibility and material properties makes it one of the more difficult-to-model components in the passenger-vehicle-suspension-tyre-road system. Attempts to arrive at a good model have resulted in the need to make a compromise between model capability (prediction accuracy) and modelling complexity. The commercially available FTire model is usually accepted to be the benchmark for prediction accuracy. This work proposes a modelling approach which can give a tyre model of much lower complexity but outperforms the prediction accuracy of FTire. This was achieved by a combination of a version of the rigid ring tyre model and the enveloping model. Thereafter, by a systematic sensitivity analysis, the model was further simplified to arrive at a model of appropriate level of complexity which is almost as good as the original model. This work provides guidelines for development of simple but accurate tyre models which will be easy to integrate with vehicle, suspension and road models.

**Keywords:** Road-tyre interaction · Rigid ring tyre model · Enveloping model · Genetic algorithm · Ride comfort

## 1 Introduction

The tyre of a ground vehicle has three primary functions: absorbing shock and vibration due road irregularity, supporting weight and generating forces for cornering, traction, and braking. The road-tyre interaction greatly influences vehicle behaviour in terms of comfort and handling and depending on the situation, their contribution ranges from 50 to 90% [1]. Thus, developing a tyre model which can accurately reproduce the reaction forces generated during the interaction of a tyre with an imperfection in the road is crucial for predicting the vehicle handling properties and ride comfort. The large number of tyre models developed so far have been classified into four categories by Pacejka [2] based on their modelling approach. By contrast, Li et al. [3] classified tyre models based on their application, namely models for handling and stability analysis, ride comfort analysis and road load analysis. These models have reported varying levels of accuracy in their ability to trace real life or experimental conditions. The commercially available FTire model is generally accepted to be accurate. However, the ability to formulate a tyre model which achieves the right and sufficient balance between accuracy and modelling complexity is

still an open problem. This paper is an attempt in that direction. It proposes a modelling method inspired by some previous attempts reported in literature. It then shows how the complexity can be reduced while keeping the accuracy within acceptable limits.

## 2 Formulation of the Tyre Model

### 2.1 Introduction

A version of the rigid ring tyre model has been formulated and integrated with the enveloping model. The latter—formulated using two elliptical cams—enables effective modelling of obstacles with shorter wavelengths. The mass of tyre assembly is appropriately apportioned to three mass elements—rim, rigid ring and tread. So, the mass of the tyre is assumed to be trifurcated into three major elements—beads, sidewalls, and tread band. The bead mass represents supporting portion of tyre which anchors the tyre to the wheel rim. Therefore, the mass associated with the beads is assumed to be integral to the rim mass during computations. The mass associated with tread band is considered the mass of rigid ring. The sidewall mass represents the connecting mass between beads and tread band. The mass of sidewall is apportioned half to rim mass and half to rigid ring mass during computations. The mass of tread associated with contact patch is considered as a tread mass element. Rim mass and rigid ring mass are connected through three sets of springs and dampers, one each in horizontal ( $K_{tyre\_x}$  &  $C_{tyre\_x}$ ), vertical ( $K_{tyre\_z}$  &  $C_{tyre\_z}$ ) and rotational ( $K_{tyre\_\theta}$  &  $C_{tyre\_\theta}$ ) directions. Rigid ring mass and tread mass are connected through two sets of springs and dampers, one in radial direction ( $K_{vr}$  &  $C_{vr}$ ) and another in circumferential direction ( $K_{cr}$  &  $C_{cr}$ ). This tyre model has nine degrees of freedom. Its schematic representation is shown in Fig. 1. Hereafter, the model is explained with its parameters and equations.

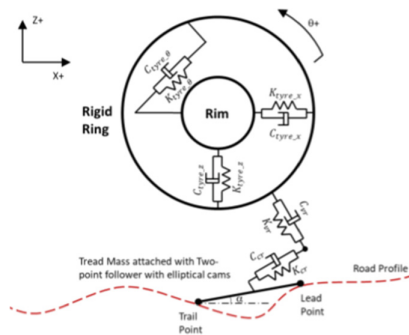
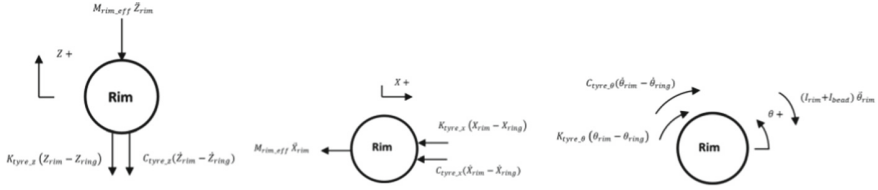


Fig. 1. The proposed model: a combination of rigid ring and enveloping models

### 2.2 Equations of Motions

The free body diagrams of the rim mass are given in Figs. 2, 3 and 4. The Newton’s method is used for deriving the equations of motion.



**Fig. 2.** Free body diagram for the rim in the three directions

The equations of motion for the rim mass are mentioned in equations 1–3.

$$-M_{rim\_eff} \ddot{X}_{rim} - K_{tyre\_x} (X_{rim} - X_{ring}) - C_{tyre\_x} (\dot{X}_{rim} - \dot{X}_{ring}) = 0 \quad (1)$$

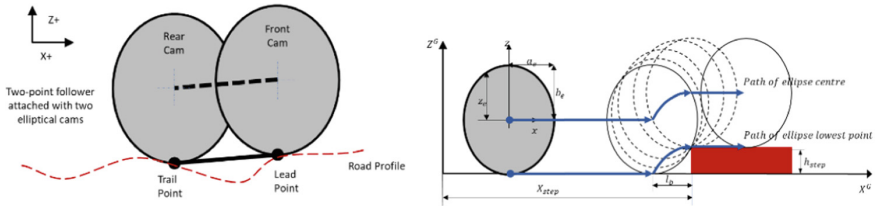
$$-M_{rim\_eff} \ddot{Z}_{rim} - K_{tyre\_z} (Z_{rim} - Z_{ring}) - C_{tyre\_z} (\dot{Z}_{rim} - \dot{Z}_{ring}) = 0 \quad (2)$$

$$-(I_{rim} + I_{bead}) \ddot{\theta}_{rim} - K_{tyre\_theta} (\theta_{rim} - \theta_{ring}) - C_{tyre\_theta} (\dot{\theta}_{rim} - \dot{\theta}_{ring}) = 0 \quad (3)$$

Similar equations were formulated for the rigid ring and and tread masses.

### 2.3 Enveloping Model

The enveloping model is formulated using two elliptical cams and it is attached to the tread mass. This enables effective modelling of obstacles with shorter wavelengths. Figure 3 shows the schematic diagram of the enveloping model and the method of path generation for the elliptical cam.



**Fig. 3.** The enveloping model

The dimensions of the elliptical cams are defined by shape parameters ellipse length ( $a_e$ ), ellipse height ( $b_e$ ) and ellipse exponent ( $c_e$ ). The distance between the two elliptical cams is  $l_s$  and length of basic curve followed by elliptical cam is denoted by  $l_b$ . Schmeitz [4] worked out dimensionless parameters for estimating  $a_e$ ,  $b_e$ ,  $c_e$  and  $l_s$  in terms of the length of the contact patch ( $2a$ ) and effective rolling radius ( $R_{eff}$ ). Equation 4 defines the value of  $z_e$  in local co-ordinates. The road profile is defined in the global co-ordinates. The equation for the curve generated by the elliptical cam in global co-ordinates is defined by Eqs. 5–7.

$$\left(\frac{x}{a_e}\right)^{c_e} + \left(\frac{z}{b_e}\right)^{c_e} = 1 \quad (4)$$

$$Z = 0, \text{ if } X \leq -l_b + X_{step} \tag{5}$$

$$Z = h_{step} - b_e + \left| b_e \left( 1 - \left( \frac{|X - X_{step}|}{a_e} \right)^{c_e} \right)^{\frac{1}{c_e}} \right|, \text{ if } -l_b + X_{step} \leq X \leq X_{step} \tag{6}$$

$$Z = h_{step}, \text{ if } X \geq X_{step} \tag{7}$$

### 3 Simulation and Validation of Rigid Ring Tyre Model

The rigid ring tyre model is modelled as combination of four sub-models—obstacle, tread, rigid ring and rim. The obstacle model is the interface between the road and tyre models. It defines the enveloping behaviour of the tyre. The other three models address the tyre characteristics. The physical parameters of tyre like dimensions, mass and inertia used in this work are same as reported by Frey [5]. Figure 4 illustrates the enveloping characteristics for the cleat input. A cleat of height 9.5 mm and length 19 mm is considered as input to the obstacle model. A two-point follower with elliptical cams traverses on the cleat. The laboratory test results of a tyre for the cleat input as reported by Frey [5] are used as a reference for experimental data in this work.

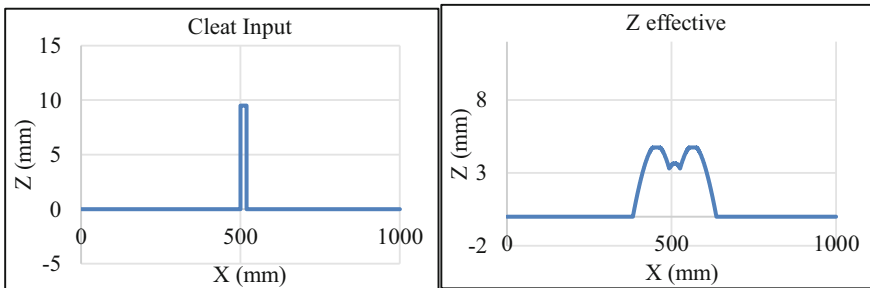


Fig. 4. Cleat input and its enveloping behavior with the elliptical cams

#### 3.1 Estimation of Model Parameters

The parameters for the rigid ring tyre model (i.e., stiffness and damping coefficients of the springs and dampers) are obtained by minimization of the squared sum of the difference between the analytical model response and experimental values reported by Frey [5] (Eq. 8). The terms have been chosen to capture the most important features of the response. An attempt to capture more features gave unsatisfactory results. Genetic algorithm was used for this optimization. The ten variables are  $K_{tyre\_x}$ ,  $C_{tyre\_x}$ ,  $K_{tyre\_z}$ ,  $C_{tyre\_z}$ ,  $K_{tyre\_\theta}$ ,  $C_{tyre\_\theta}$ ,  $K_{vr}$ ,  $K_{cr}$  and  $C_{cr}$ . Spring constants were bound between 1 and  $10^8$  units and damper coefficients between 1 and  $10^4$  units.

$$\begin{aligned}
 f = & \left[ \max_{0 < t < 0.5} F_{x\_test} - \max_{0 < t < 0.5} F_{x\_sim} \right]^2 + \left[ \min_{0 < t < 0.5} F_{x\_test} - \min_{0 < t < 0.5} F_{x\_sim} \right]^2 \\
 & + \left[ \min_{0.25 < t < 0.5} F_{x\_test} - \min_{0.25 < t < 0.5} F_{x\_sim} \right]^2 + \left[ \max_{0 < t < 0.5} F_{z\_test} - \max_{0 < t < 0.5} F_{z\_sim} \right]^2 \\
 & + \left[ \min_{0.22 < t < 0.26} F_{z\_test} - \min_{0.22 < t < 0.26} F_{z\_sim} \right]^2 + \left[ \max_{0.22 < t < 0.26} F_{z\_test} - \max_{0.22 < t < 0.26} F_{z\_sim} \right]^2
 \end{aligned} \tag{8}$$

### 3.2 Simulation Results

Figure 5 shows a comparison of horizontal and vertical forces predicted by the model proposed in this work (RRT10) and the experimental results reported by Frey [5]. The figures also show the predictions of FTire and the model reported by Frey [5]. The superiority of RRT10 over FTire and Frey’s model is obvious. In particular, the superiority of RRT10 over the commercially well accepted FTire was very encouraging. This prompted an attempt to simplify RRT10 to identify the simplest model of this nature which can outperform FTire.

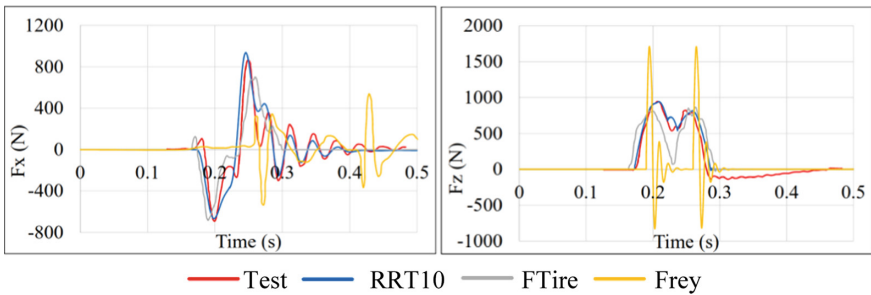


Fig. 5. Horizontal and vertical forces predicted by different models vs test results

## 4 Sensitivity Analysis and New Model Development

Each of the ten parameters of RRT10 was increased by 50% and its effect on horizontal and vertical forces was noted.  $C_{tyre\_θ}$ ,  $K_{tyre\_θ}$  and  $C_{tyre\_z}$  were found to have the least effect the forces. Thus, two more models were developed—RRT8 by deleting  $C_{tyre\_θ}$  and  $K_{tyre\_θ}$  and RRT7 by deleting  $C_{tyre\_θ}$ ,  $K_{tyre\_θ}$  and  $C_{tyre\_z}$ . . A comparison with RRT10 (Fig. 6) led to the acceptance of RRT8.

Thereafter, a quarter car model was integrated with RRT10, RRT8 and FTire and tested on two obstacles—the previously defined cleat and a sinusoidal bump obstacle of height 102 mm and length 305 mm. The two represent obstacles smaller and larger than the tyre’s contact patch respectively. The plots of forces at the rim (Fig. 7) clearly show the superiority of RRT10 and RRT8 in their ability to generate more realistic smoother results compared to the rapidly varying output of FTire which is difficult to explain.

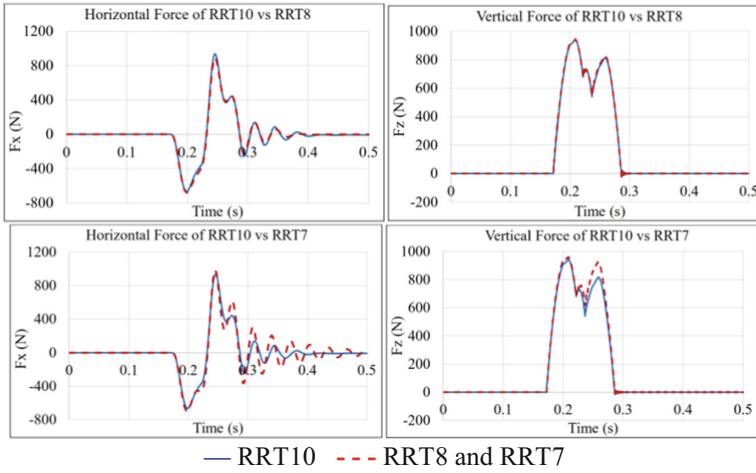


Fig. 6. Comparison of RRT10 with RRT8 and RRT7

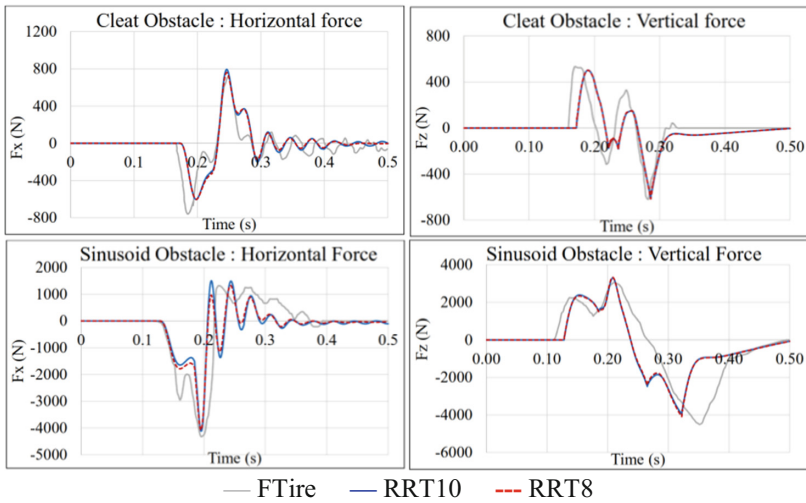


Fig. 7. Comparison of quarter car model integrated RRT10 and RRT8 with FTire

### 5 Conclusion

This work establishes a methodology for creating tyre models of lower level of complexity but with better prediction capability than a commercially available model—FTire. The combination of rigid ring tyre models with enveloping models, though inspired by literature, shows the ability to outperform other well-known tyre models. This led to the possibility of further simplifying the model by deleting the least important components which were identified through a sensitivity analysis exercise. Finally, the ability of the model to integrate well with a quarter car model was also demonstrated. This work may

inspire more innovative approaches in tyre modelling by combining the best aspects of models reported in literature.

## References

1. Uil, R.T.: Tyre Models For Steady-State Vehicle Handling Analysis. Eindhoven University of Technology, Eindhoven (2007)
2. Pacejka, H.B.: Tire and Vehicle Dynamics, Elsevier (2012)
3. Li, B., Yang, X., Yang, J.: Tire model application and parameter identification—a literature review. *JSAE Int. J. Passeng. Cars—Mech. Syst.* **7**(1), 231–243 (2014)
4. Schmeitz, A.: A semi-empirical three-dimensional model of the pneumatic tyre rolling over arbitrary uneven road surfaces. Doctoral Thesis, TU Delft (2004)
5. Frey, N.: Development of a Rigid Ring Tire Model and Comparison Among Various Tire Models for Ride Comfort Simulations. Clemson University, Tiger Prints (2009)