Geometric Accuracy Innovative Design Method for Machine Tool

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Abstract. This paper presents a geometric accuracy innovation design method for the moving component layer of machine tools, which can characterize the intrinsic connection between machine tool geometric errors and machine tool spatial errors, so as to effectively ensure the spatial accuracy of machine tools after accuracy design. Firstly, the spatial error model of the machine tool is constructed using the screw theory. Secondly, a geometric accuracy innovation design method is proposed with the sensitivity as the design weight and the spatial accuracy as the constraint. Then, an intelligent population optimisation algorithm combined with Monte Carlo is used to optimally solve the design results of the optimal geometric accuracy. Finally, the validity and universality of the proposed geometric accuracy innovation design method is verified through simulation, which can ensure that more than 95.25% of the simulation position points in the workspace of the vertical machining center can meet the spatial accuracy of less than 15μm.

Keywords: Spatial accuracy · Accuracy innovation design method · Optimisation algorithms · Geometric accuracy

1 Introduction

Spatial accuracy is an important performance indicator of machine tools. There are many factors affecting spatial accuracy, including force error [1], geometric error [2], thermal error [3] and dynamic error [4], among which geometric error has the greatest impact on the machining accuracy of the machine tool, up to 40%. In general, machine tools mainly improve spatial accuracy through accuracy design. The accurate error model and reasonable geometric accuracy design are essential to improve machine tool machining accuracy.

Scholars have conducted extensive research on the error model and have proposed many modelling methods, including the error vector method [5, 6], the mechanistic modelling method [7], the multi-body system theory [8] and the D-H method [9]. Tan [10] used the multi-body system theory to establish a static accuracy model of the machine tool machining posture, and explored the laws of geometric errors that affect machining accuracy. In recent years, the screw theory has been widely used, providing

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new ideas for error identification and error compensation. The use of screw theory does not require the establishment of local coordinate systems, reducing the errors introduced in the modelling process.

In terms of the machine tool accuracy design object, scholars mainly focus on the linear errors. Huang [11] approximates the angle error as the linear error and mirrors the geometric error to the part error for machine tool accuracy design. Such approximation may cause inaccurate accuracy design results. Moreover, it can be seen from the machine tool error models constructed in the literature [11, 12] that angle errors have a greater impact on the spatial error than linear errors which need to be further controlled in the machine tool accuracy design.

In terms of the machine tool accuracy design methods, scholars have transformed the machine tool accuracy design problem into an accuracy allocation problem. Fan [13] studied the lowest cost reliability accuracy allocation scheme under the same reliability of the whole grinding machine. These studies provide effective solutions for the accuracy design, however, the accuracy of the allocation model is questionable as the manufacturing cost model or reliability model is almost empirically developed. In addition, the machine tool cost-tolerance model needs to be adjusted to different types of machine tools.

In view of the above problems, this paper proposes a geometric accuracy design method for machine tool with spatial accuracy as a constraint and geometric error sensitivity as a weight. The rest of the paper is organized as follows. The mapping relationship between the spatial error and the geometric error is constructed based on the screw theory in Sect. 2. In Sect. 3, a design model for motion geometric accuracy is established in combination with the sensitivity analysis of the geometric error, which is solved and optimized using a combination of Monte Carlo and optimisation algorithms. In Sect. 4, simulations are carried out on a vertical machining center as an example to verify effectiveness, and Sect. 5 gives the conclusions.

2 Machine Tool Spatial Error Model

Due to the influence of geometric errors of moving parts, a certain deviation exists between the point of the tool tip and the point of the workpiece to be machined during actual machining. The research object is a vertical machining center. There are six position dependent geometric errors (PDGEs) for each axis. The parallel axis has 1 position error, 2 straightness errors and 3 angle errors (roll, pitch and yaw). For example, the error of x axis is $\delta_x(x)$, $\delta_y(x)$, $\delta_z(x)$, $\varepsilon_x(x)$, $\varepsilon_y(x)$, $\varepsilon_z(x)$. The rotation axis has 1 axial error, which represents the linear offset, 2 radial errors, 1 angle positioning error and 2 tilt errors, For example, the error of B axis is $\delta_x(b)$, $\delta_y(b)$, $\delta_z(b)$, $\varepsilon_x(b)$, $\varepsilon_y(b)$, $\varepsilon_z(b)$. In addition, there are position independent geometric errors (PIGEs) between the moving parts, see Table 1. The screw expression of geometric errors in machine tools can be found in literature [12].

Using the screw theory to establish the machine spatial error model. The structure of the machine tool is shown in Fig. 1. The machine tool reference coordinate system origin is the workspace boundary point, which corresponds to the machine origin. The machine is divided into a workpiece chain and a tool chain, both of which have a motion transformation matrix ideally as:

$$g_{ow} = e^{\hat{\xi}_Y \cdot y} \cdot e^{\hat{\xi}_B \cdot \theta_B} \cdot g_{ow}(0) \tag{1}$$

$$g_{ot} = e^{\hat{\xi}_Z \cdot z} \cdot e^{\hat{\xi}_X \cdot x} \cdot g_{ot}(0) \tag{2}$$

where $g_{ow}(0)$ and $g_{ot}(0)$ represent the initial motion matrix of the workpiece and tool relative to the machine reference coordinate system with no motion of each axis, respectively.

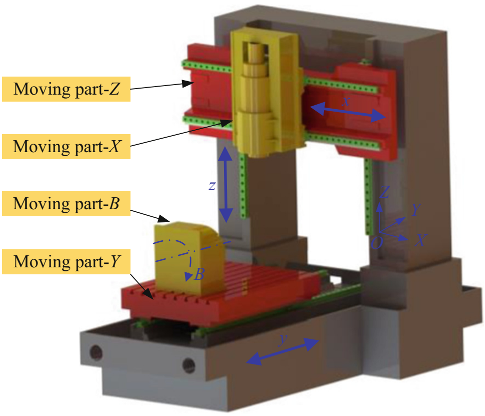

Fig. 1. The structure of the machine tool.

Table 1. Position independent geometric errors (PIGEs).

Symbols	Geometric significance
S_{zx}	Squareness error between z, x axes
S_{zy}	Squareness error between z, y axes
S_{xy}	Squareness error between x, y axes
S_{xb}	Squareness error between x, b axes
S_{zb}	Squareness error between z, b axes

The corresponding errors are brought into the kinematic transformation matrix of the tool and workpiece chains to obtain the forward kinematic model under actual conditions:

$$g_{ow}^{e} = (e^{\hat{\xi}_{eY}^{I} \cdot \theta_{eY}^{I}} \cdot e^{\hat{\xi}_Y \cdot y} \cdot e^{\hat{\xi}_{eY}^{D} \cdot \theta_{eY}^{D}}) \cdot (e^{\hat{\xi}_{eB}^{I} \cdot \theta_{eB}^{I}} \cdot e^{\hat{\xi}_{eB}^{D} \cdot \theta_{eB}^{D}} \cdot e^{\hat{\xi}_B \cdot \theta_B}) \cdot g_{ow}(0) \tag{3}$$

$$g_{ot}^{e} = (e^{\hat{\xi}_Z \cdot z} \cdot e^{\hat{\xi}_{eZ}^{D} \cdot \theta_{eZ}^{D}}) \cdot (e^{\hat{\xi}_{eX}^{I} \cdot \theta_{eX}^{I}} \cdot e^{\hat{\xi}_X \cdot x} \cdot e^{\hat{\xi}_{eX}^{D} \cdot \theta_{eX}^{D}}) \cdot g_{ot}(0) \tag{4}$$

where $e^{\hat{\xi}_{ei}^{D}\cdot\theta_{ei}^{D}}$ and $e^{\hat{\xi}_{ei}^{I}\cdot\theta_{ei}^{I}}$ denote the screw expression of PDGEs and PIGEs for each axis, respectively.

Combining Eq. (3) and Eq (4), the spatial error model of the machine tool can be obtained:

$$g_{wt}^{e} = g_{wo}^{e} \cdot g_{ot}^{e} = (g_{ow}^{e})^{-1} \cdot g_{ot}^{e} \tag{5}$$

Neglecting the infinitesimal of the second order and above in the matrix, Eq. (5) simplified to obtain the mapping relationship between the machine tool spatial error and the geometric error.

$$\begin{bmatrix} \Delta \boldsymbol{P} \\ \Delta \boldsymbol{V} \end{bmatrix} = \begin{bmatrix} \boldsymbol{S}_{pl}, & \boldsymbol{S}_{pa} \\ \boldsymbol{O}, & \boldsymbol{S}_{va} \end{bmatrix} \begin{bmatrix} \boldsymbol{E}_{l} \\ \boldsymbol{E}_{a} \end{bmatrix} \tag{6}$$

where $\Delta \boldsymbol{P}$ and $\Delta \boldsymbol{V}$ represent the spatial linear error and the spatial angle error in the x, y and z directions, respectively. $\boldsymbol{E}_l$ denotes the vector consisting of the linear geometric error in each axis. $\boldsymbol{E}_a$ denotes the vector consisting of the angle geometric error in each axis and PIGEs. $\boldsymbol{S}_{pl}$ and $\boldsymbol{S}_{pa}$ denote the matrix of mapping coefficients between the spatial linear error and the linear and angle geometric error, respectively. $\boldsymbol{S}_{va}$ denotes the matrix of mapping coefficients between the spatial angle error and the angle geometric error.

3 Innovation Design Methods for Motion Geometric Accuracy with Spatial Accuracy Constraints

3.1 Motion Geometric Accuracy Design Model

The sensitivity of the geometric error for the machine tool indicates the range of variation of the spatial error to the response of the range of variation of the geometric error, the specific expression is

$$\mu_j = \frac{\overline{\Delta r_j} - \underline{\Delta r_j}}{\overline{\Delta E_j} - \underline{\Delta E_j}} \tag{7}$$

where $\overline{\Delta r_j}$ and $\underline{\Delta r_j}$ denote the upper and lower limits of the variation of the spatial error caused by error j, respectively. $\overline{\Delta E_j}$ and $\underline{\Delta E_j}$ denote the upper and lower limits of the variation of geometric error j, respectively.

Machine tool spatial errors are directional and for each direction the sensitivity of the geometric error can be defined separately as:

$$\mu_{x,j} = \frac{\overline{\Delta r_{x,j}} - \underline{\Delta r_{x,j}}}{\overline{\Delta E_{x,j}} - \underline{\Delta E_{x,j}}} \tag{8}$$

$$\mu_{y,j} = \frac{\overline{\Delta r_{y,j}} - \underline{\Delta r_{y,j}}}{\overline{\Delta E_{y,j}} - \underline{\Delta E_{y,j}}} \tag{9}$$

$$\mu_{z,j} = \frac{\overline{\Delta r_{z,j}} - \underline{\Delta r_{z,j}}}{\overline{\Delta E_{z,j}} - \underline{\Delta E_{z,j}}} \tag{10}$$

Meanwhile, the spatial error of the machine tool is the result of the combined effect of multi-directional errors, defining the comprehensive sensitivity of the machine tool as:

$$\mu_{c,j} = \sqrt{\sum_{i=x,y,z}^{3} (\mu_{i,j})^2, j = 1, 2, \cdots, 29} \tag{11}$$

According to Eq. (6) and Eq. (11) can be obtained from the machine tool spatial linear error to the sensitivity of each linear geometric error as:

$$\mu_{pl,i,j} = \begin{cases} 1 \ S_{pl,i,j} \neq 0 \\ 0 \ S_{pl,i,j} = 0 \end{cases}, i = x, y, z, j = 1, 2, \cdots, 12 \tag{12}$$

Similarly, the sensitivity of the machine tool spatial angle error to each angle geometric error is as follow:

$$\mu_{vr,i,j} = \begin{cases} 1 \ S_{vr,i,j} \neq 0 \\ 0 \ S_{vr,i,j} = 0 \end{cases}, i = x, y, z, j = 1, 2, \cdots, 17 \tag{13}$$

The motion geometric accuracy design model consists of design variables, objective function and constraints, which need to be determined separately.

(a) Design variables

Different angle errors have different effects on the spatial accuracy of the machine. In addition, the probability of each geometric error in any interval is the same, assuming that the angle error is:

$$E_{aj} \sim U\left(-t_{aj}\ t_{aj}\right), j = 1, 2, \cdots, 17 \tag{14}$$

where t_{aj} and $-t_{aj}$ represent the upper and lower range of angle error, t_{aj} can be calculated to obtain the accuracy design values for each angle error. The design variables are thus defined as $\boldsymbol{t} = \left[\, t_{a1}\ t_{a1}\ \cdots\ t_{a17} \,\right]$.

(b) Design objective function

Traditional machine tool accuracy design may result in the design variables being taken too loosely or too strictly. In order to avoid these situations reducing the difficulty of machining and assembling the machine, the accuracy design process needs to relax each geometric error to its maximum permissible feasible domain. Using the comprehensive sensitivity to determine the design weights for the geometric accuracy of the motion, i.e.

$$\lambda_j = \frac{\mu_{c,j}}{\sum_{j=1}^{17} \mu_{c,j}} \tag{15}$$

The greater the error sensitivity the smaller the range of error variations that need to be guaranteed in its assignment. It is also necessary to consider the comprehensive effect of multiple angle geometric error on the spatial error, and it is required to convert a multi-objective optimisation problem with multiple error, into a single objective optimisation problem. The objective function is therefore constructed as follows:

$$\min f\left(t_{aj}\right) = \sum_{j=1}^{17} \frac{\lambda_j}{t_{aj}} \tag{16}$$

(c) Design constraints

Two types of constraints are included in the accuracy design process, namely spatial accuracy constraints and design variable constraints.

Due to the existence of spatial errors in the machine tool, the ideal point of the machine tool does not coincide with the actual point. The spatial error can be expressed as a sphere with the theoretical point as the center and the radius as the spatial accuracy constraint requirement of the machine. That is, the machine tool spatial accuracy constraint can be expressed as:

$$\Delta r = \sqrt{\Delta P_x^2 + \Delta P_y^2 + \Delta P_z^2} \le R = 15 \tag{17}$$

where ΔP_x, ΔP_y and ΔP_z represent the spatial linear error components in the x, y and z directions, respectively.

According to the design by Chance Constrained Programming [14], in the machine tool design phase, it is necessary to allow the geometric error design result to not satisfy the constraint to some extent, but to guarantee that the probability is less than a certain confidence level. The confidence level can be adjusted according to the specific requirements during the accuracy design process. The final spatial accuracy constraint for the machine is set as:

$$P(\Delta r \le R) = 0.9 \tag{18}$$

The design value of the angle error is set too small, it will cause the actual manufacturing difficult, lost the meaning of the accuracy design, need to be bound to its design lower limit. While the design value cannot be taken excessive, it can be determined according to the tolerance levels in the regulations of GB/T1184–1996 for shape and position tolerances without note tolerances. The constraint on the design variables is thus obtained.

$$e_{ai,l} \le t_{aj} \le e_{ai,h} \tag{19}$$

where $e_{aj,l}$ is the lower limit value, set to 2 μm/m, and $e_{aj,h}$ is the upper limit value, set to 10 μm/m.

3.2 Accuracy Design Optimisation Algorithm

The Monte Carlo simulation method is used to solve the statistical constraints in the design model by generating a large number of random samples in order to approximate the estimated results and probabilities of the machine tool spatial error. In the case of problems such as single/multi-objective optimisation, the Particle Swarm Optimization (PSO) has the advantages of fast convergence and fewer optimisation parameters required. The accuracy design model is solved using an optimisation algorithm that combines Monte Carlo simulation with the PSO. The flowchart for optimal solution of accuracy design is shown in Fig. 2.

The population size is set to 100 according to the number of design variables in the accuracy design model. N ($N = 10000$) sets of data are randomly generated using the Monte Carlo algorithm within the constraints of the design variables, and for each location point in the machine tool workspace the spatial error is calculated, keeping the particles that satisfy the spatial accuracy constraints and calculating the fitness value, $n(n > 100)$ representing all points in the machine tool workspace. After 12 iterations, the optimal design results for the angle geometric errors are obtained. The fitness evolution curve is shown in Fig. 3. The results of accuracy design range for motion geometric error are shown in Table 2.

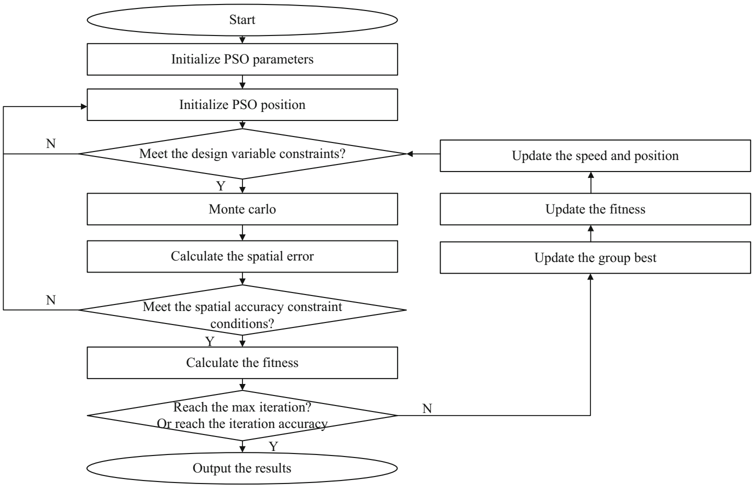

Fig. 2. Flowchart for optimal solution of accuracy design.

4 Simulation Validation

In order to verify whether the results of the motion geometric error accuracy design meet the spatial accuracy design requirements, the Monte Carlo simulation method can still be used. Two cases of machine tool workspace random simulation points and special location simulation points are selected for verification.

4.1 Validity Verification

In the workspace of the machine tool, 10000 sets of simulation points are randomly given. At each simulation point a set of randomly generated motion geometric errors to

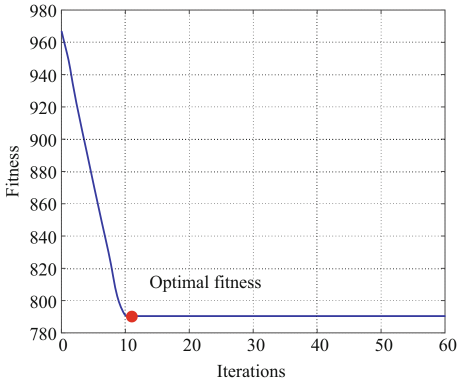

Fig. 3. Fitness evolution curve.

Table 2. The results of accuracy design range for motion geometric error.

Motion geometric error	Accuracy design range/[μm/m, μm/m]	Motion geometric error	Accuracy design range/[μm/m, μm/m]
$\varepsilon_x(x)$	[− 6.42, 6.42]	$\varepsilon_x(b)$	[− 4.51, 4.51]
$\varepsilon_y(x)$	[− 7.86, 7.86]	$\varepsilon_y(b)$	[− 6.43, 6.43]
$\varepsilon_z(x)$	[− 9.47, 9.47]	$\varepsilon_z(b)$	[− 6.14, 6.14]
$\varepsilon_x(y)$	[− 2.8, 2.8]	S_{zx}	[− 5.32, 5.32]
$\varepsilon_y(y)$	[− 6.01, 6.01]	S_{zy}	[− 2.73, 2.73]
$\varepsilon_z(y)$	[− 6.25, 6.25]	S_{xy}	[− 3.59, 3.59]
$\varepsilon_x(z)$	[− 6.17, 6.17]	S_{xb}	[− 3.94, 3.94]
$\varepsilon_y(z)$	[− 7.6,7.6]	S_{zb}	[− 4.05, 4.05]
$\varepsilon_z(z)$	[− 8.13, 8.13]		

meet the accuracy design requirements are substituted into Eq. (6), and finally obtain the x, y, z-directional spatial linear error and corresponding spatial error. The estimated spatial error frequency distribution shown in Fig. 4. It can be seen that the probability is 95.64%, 94.17% and 97.94% for x-, y- and z-directional spatial linear error less than 10 μm, respectively. The probability that the spatial error of the machine meets the design specification is 95.25%, thus verifying the effectiveness of the proposed accuracy design method.

4.2 Applicability Validation

In order to ensure that the accuracy design results have applicability, it is necessary to ensure that the machine tool accuracy design results can also meet the machine tool spatial accuracy requirements at the point of maximum sensitivity. In this simulated point, 1000 sets of motion geometric errors were randomly generated to meet the accuracy

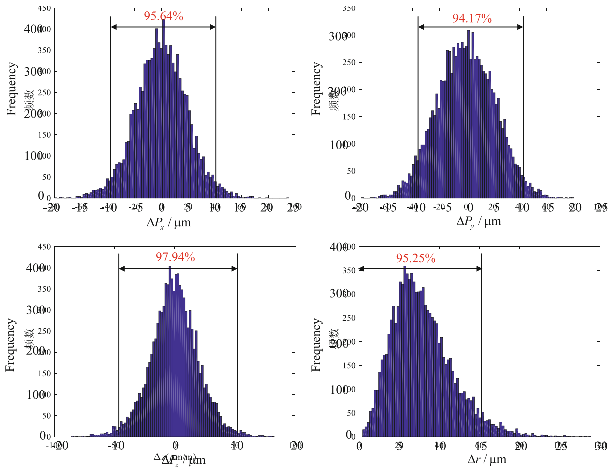

Fig. 4. Machine tool estimated spatial error frequency distribution histogram.

design requirements, calculating the spatial errors of the machine, and getting the spatial error distribution as shown in Fig. 5. There are 987 qualified points and 13 over-qualified points. With 90% of the estimated spatial error of the simulation points are mainly distributed in the range of 6.8–13.2 μm. The probability of meeting the spatial accuracy requirements is 98.7%.

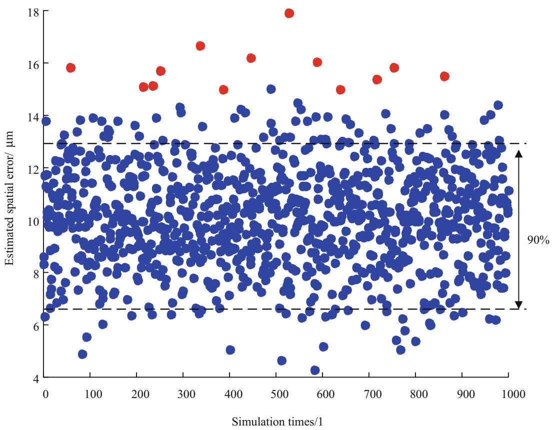

Fig. 5. Machine tool spatial error distribution.

5 Conclusions

In this paper, a geometric accuracy design method for machine tool with spatial accuracy constraints has been proposed, which is applied to a vertical machining center. The effectiveness and accuracy of the proposed method can be verified through simulation, and the following conclusions can be drawn:

1. The spatial error model reflects the importance of angle geometric error on spatial accuracy. In this regard, the proposed design method completes the motion geometric accuracy design purposefully.
2. The proposed geometric accuracy design method uses the machine tool spatial accuracy as a constraint, which is more conducive to ensure the machining accuracy. There is no need to adjust the cost function according to different machine tools, which is universal. In addition, the problem of inaccurate accuracy design results caused by empirical functions in accuracy design is avoided.
3. The combination of Monte Carlo simulation and PSO algorithms for accuracy design, aided by statistics, enables accurately and quickly obtained accuracy design results.

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