Analysis of Transmission Errors and Load Sharing of Compound Stepped Planetary Gear Drives Considering Mesh Phasing

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Abstract. The compound stepped planetary gear (CSPG) drive has a higher speed ratio than simple planetary gear drives, but the selection of gear teeth for CSPG drives is more stringent due to their complex structure. Based on the involute geometry, a gear mesh analysis model is developed. From this model, the essential conditions for correct assembly of CSPG set and design guidelines are proposed. Under consideration of eccentric of carrier, the general equations of contact positions and clearance of CSPG set are derived. Based on the relations of loaded deformation and displacement, the influence of mesh phases on loaded transmission error and load sharing among planets is discussed. Four study cases are chosen for influence analysis of the meshing phases on transmission errors and load sharing.

Keywords: Compound Stepped Planetary Gear Set · Eccentric Error of Carrier · Transmission Errors · Load Sharing among Planets

1 Introduction

Planetary gear drives are widely used in various transmission application due to their advantageous features such as coaxial input and output shaft arrangement, high power density, and high-speed ratio. Compound stepped planetary gear drives (CSPG), in particular, have gained widespread use due to their superior performance under space constraints. CSPG employs two compound planet gears of different sizes on the planet shaft, as shown in Fig. 1, forming a stepped structure, which enables it to attain a higher speed ratio compared to simple planetary gear sets. However, this design requires careful consideration of gear teeth number selection and planet timing during assembly to achieve proper gear mesh.

To ensure that the CSPG gear teeth meet the assembly requirements, Myers [1] proposed a calculation method, which AGMA 6123 [2] uses as a reference to establish several calculation examples. Despite its usefulness, the equations in this method do not reveal the geometric relationship between the number of teeth and the assembly timing, making it unsuitable for establishing design guidelines for deigning CSPG.

Apart from fulfilling assembly requirements, the numbers of teeth in CSPG also affect mesh phasing, which in turn affects transmission performance [3, 4]. The observed

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differences in transmission errors and load distribution among the planets in a planetary gear set are the direct result of different meshing phases. Generally, simple planetary gear sets with three planets have two types of mesh phasing: in-phase and sequential phases. However, in CSPG, due to the design of compound planets, the mesh phasing between the sun gear (S) and the planet gears (PS), as well as between the planet gears (PA) and the annulus gear (A), are independent of each other. In other words, there can be a total of four possible combinations of in-phase meshing and sequential meshing.

Previous studies [5] introduced an analytical method based on involute geometry to analyze gear mesh and planet load distribution in simple planetary gear drives. Additionally, the same approach was utilized to determine the transmission error caused by eccentric errors [6, 7]. The present study aims to extend this analytical method to facilitate its application to CSPG. By utilizing the proposed analysis model, design guidelines for selecting tooth numbers of CSPG can be established. Moreover, the impact of the eccentric error of the carrier on transmission error and planet gear load distribution among four distinct meshing phase combinations of CSPG can be compared.

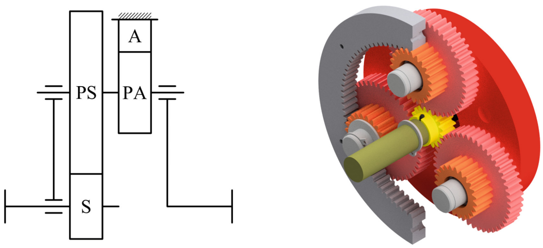

Fig. 1. Schematic diagram and section view of CSPG

2 Geometrical Relations for Gear Meshing

2.1 Basic Meshing Relations of Engaged Tooth Pairs

Contact Positions of each Engaged Tooth. The fundamental meshing relations for the contact positions of the teeth in the annulus-planet (A-PA) and sun-planet (S-PS) gear pairs can be established using the generalized geometrical relation illustrated in Fig. 2. Accordingly, the rolling angle (ξ) of each contact tooth can be determined based on the geometrical relations. The relationships among these angles do not differ from those of a simple planetary gear set, as demonstrated in [5], except for the tooth positions on the compound planet gears (PA and PS).

Relation for the Compound Planet due to Alignment. As depicted in Fig. 2, the reference tooth L_D of planet gears PA and PS will undergo rotation by an angle φ_Y from the initial position L_A. Given that the contact position ξ_{PAi} of planet gear PA and the rotation angle φ_Y are known, the contact position of planet gear PS can be:

$$\xi_{PSi} = X_{PSi}\tau_{PS} - X_{PAi}\tau_{PA} + \psi_{bPSi} + \psi_{bPAi} - \pi + \alpha_{wPSi} + \alpha_{wPAi} - \xi_{PAi} \tag{1}$$

The integers X in Eq. (1) represent the number of intermediate teeth between the contact tooth of planet gears (PA and PS) and the reference tooth, which can be determined with integer function int from Fig. 2(b) as,

$$X_{\mathrm{PA}i} = \mathrm{int}\left(\frac{\varphi_{\mathrm{Y}}}{\tau_{\mathrm{PA}}}\right); \quad X_{\mathrm{PS}i} = \mathrm{int}\left(\frac{\pi - \varphi_{\mathrm{Y}}}{\tau_{\mathrm{PA}}}\right) \tag{2}$$

Tooth Clearances. If the contact positions of the engaged teeth of the sun gear, determined according to the two distinct relations in Fig. 2, are dissimilar, a clearance between the engaged teeth of the sun S and planet gear PS exists. The difference in the rolling angle can be expressed as follows:

$$\delta_{\mathrm{PS}i} = \xi_{\mathrm{S}i\text{ - I}} - \xi_{\mathrm{S}i\text{ - II}} = \Delta\Theta + \Delta\Phi \tag{3}$$

In the given equation, $\Delta\Theta$ represents the angular difference associated with the positional deviation of each gear and the tooth thickness deviation of the planet gear. Its value will not be zero in the presence of errors, and can be expressed with involute function inv as,

$$\begin{aligned}\Delta\Theta = {} & (\frac{z_{\mathrm{PS}}}{z_{\mathrm{S}}}+1)(\mathrm{inv}\alpha_{\mathrm{wPS}i} - \mathrm{inv}\alpha_{\mathrm{wPS1}}) - \frac{z_{\mathrm{PS}}}{z_{\mathrm{S}}}\left(\frac{|z_{\mathrm{A}}|}{z_{\mathrm{PA}}} - 1\right)(\mathrm{inv}\alpha_{\mathrm{wPA}i} - \mathrm{inv}\alpha_{\mathrm{wPA1}}) + \\ & -\frac{z_{\mathrm{PS}}}{z_{\mathrm{S}}}[\Delta\psi_{\mathrm{bPS}} + \Delta\psi_{\mathrm{bPA}}] + (\frac{z_{\mathrm{PS}}}{z_{\mathrm{S}}}\frac{|z_{\mathrm{A}}|}{z_{\mathrm{PA}}}+1)\Delta\gamma_{\mathrm{AS}i}\end{aligned} \tag{4}$$

On the contrary, $\Delta\Phi$ denotes the angular difference that are the angle between the engaged teeth of every gear. Provided that the planets are appropriately assembled, $\Delta\Phi$ holds a value of zero; otherwise, it retains a non-zero value as:

$$\begin{aligned}\Delta\Phi = \frac{2\pi}{z_{\mathrm{S}}z_{\mathrm{PA}}}\Bigg\{ & \frac{z_{\mathrm{PS}}|z_{\mathrm{A}}|+z_{\mathrm{S}}z_{\mathrm{PA}}}{n_{\mathrm{P}}} - \left[\mathrm{int}(\frac{z_{\mathrm{S}}}{n_{\mathrm{P}}})+(X_{\mathrm{PS}i} - X_{\mathrm{PS1}})\right]z_{\mathrm{PA}} \\ & -\left[\mathrm{int}(\frac{|z_{\mathrm{A}}|}{n_{\mathrm{P}}})(X_{\mathrm{PA}i} - X_{\mathrm{PA1}})\right]z_{\mathrm{PS}}\Bigg\}\end{aligned} \tag{5}$$

2.2 Essential Conditions for Correct Assembly

Fundamental prerequisite for the assembly of the compound planets is the absence of clearance, signified by $\Delta\xi_i = 0$. In the ideal scenario where $\Delta\Theta = 0$, the angular difference $\Delta\Phi$ for tooth-number relations must also be equal to zero. Equation (5) can be utilized to derive the assembly relation with $\mathrm{X}_{\mathrm{PA1}}$ set to 0, i.e.,

$$\frac{z_{\mathrm{S}}z_{\mathrm{PA}} + |z_{\mathrm{A}}|z_{\mathrm{PS}}}{n_{\mathrm{P}}} = \left[\mathrm{int}(\frac{z_{\mathrm{S}}}{n_{\mathrm{P}}}) + (X_{\mathrm{PS}i} - X_{\mathrm{PS1}})\right]\cdot z_{\mathrm{PA}} + \left[\mathrm{int}(\frac{|z_{\mathrm{A}}|}{n_{\mathrm{P}}}) - X_{\mathrm{PA}i}\right]\cdot z_{\mathrm{PS}}, \tag{6}$$

where all the variables X are integer. Equation (6) is valid only when the item on the left side of the equation must be also an integer, i.e.,

$$z_{\mathrm{PR}}z_{\mathrm{S}} + |z_{\mathrm{R}}|z_{\mathrm{PS}} = n_{\mathrm{P}} \cdot X. \tag{7}$$

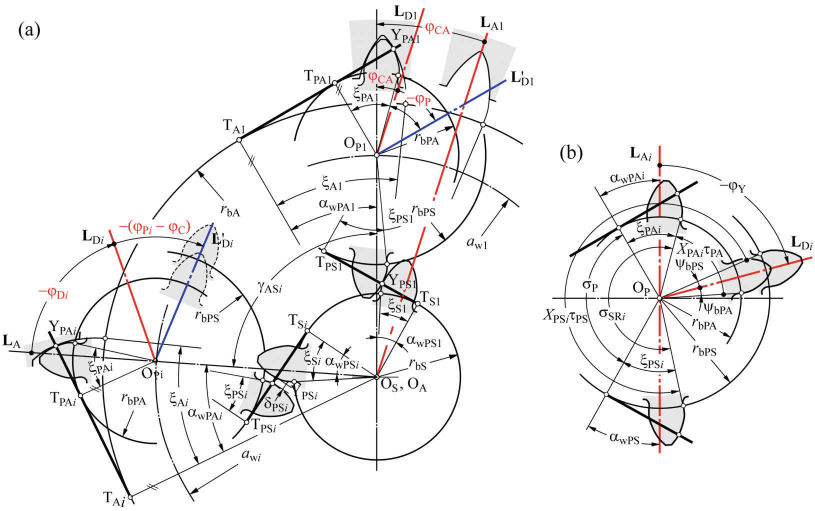

Fig. 2. Generalized geometric relation of gear mesh in CSPG

Equation (7) is the basic requirement to be fulfilled for assembly of the equally spaced compound planets.

Design Rules and Timing Sequence. Except for essential requirement of Eqs. (6), (7) must be also valid, where X_{PAi} is the aligning number of teeth of planet i at assembly, and the variable difference X_{PSi}–X_{PS1} must be also an integer, i.e.,

$$X_{PSi} - X_{PS1} = \left[\frac{z_S}{n_P} - \mathrm{int}(\frac{z_S}{n_P})\right] + \left[\frac{|z_A|}{n_P} - \mathrm{int}(\frac{|z_A|}{n_P}) + X_{PAi}\right] \cdot \frac{z_{PS}}{z_{PA}} \in \text{integer} \quad (8)$$

The tooth numbers of the gears can be determined according to the classified six rules listed in Table 1 which are based on the equality requirement of Eq. (8). This classification is more systematic and understandable than the existing literature [1, 2].

2.3 Meshing Relations due to the Presence of Eccentric Error of the Carrier

Considering the presence of the eccentric error e_S of the carrier, the positions of the compound planets relative to the sun and the annulus gear will be deviated and varied with the rotation angle φ_C, Fig. 3. This change can be characterized by the deviation angles ϑ_{AS1} and ϑ_{ASi}, which are respectively the function of φ_C and affect the relative rotation angle φ_{CA}, the separation angle γ_{ASi}, the working pressure angles $\alpha_{wPA1,i}$, $\alpha_{wPS1,i}$ and the center distances $a_{w1,i}$. The positions of each contact tooth are varied by these deviated parameters, and a clearance δ_i between engaged teeth of the planet PSi and the sun gear S is caused accordingly. The relevant relations are derived as follows.

Basic Relations due to the Error. As the relations in Fig. 3 show, the deviation angles ϑ_{AS1} and ϑ_{ASi} can be derived based on the law of sines as

$$\tan\vartheta_{AS1} = -\frac{e_C \sin\varphi_C}{a_{C1} + e_C\cos\varphi_C}, \tan\vartheta_{ASi} = -\frac{e_C \sin(\gamma_{Ci} + \varphi_C)}{a_{Ci} + e_C\cos(\gamma_{Ci} + \varphi_C)}. \quad (9)$$

Table 1. Assembly conditions and corresponding meshing phasing

Rule	Tooth number				Installation type	Mesh phasing	
	z_{S}	$\lvert z_{\mathrm{A}}\rvert$	z_{PS}	z_{PA}		S-PS	A-PA
A1	$n_{\mathrm{P}}X_1$	$n_{\mathrm{P}}X_2$	mutually prime		Specific	In-phase	In-phase
A2	$n_{\mathrm{P}}X_1$	$n_{\mathrm{P}}X_2$	$C_{\mathrm{P}}\ X_3$	$C_{\mathrm{P}}\ X_4$	Interval	In-phase	In-phase
A3	$n_{\mathrm{P}}X_1$	$n_{\mathrm{P}}X_2$	$X_{\mathrm{N}}\ z_{\mathrm{PA}}$	z_{PA}	Random	In-phase	In-phase
B1	$n_{\mathrm{P}}X_1 - \lvert z_{\mathrm{A}}\rvert X_{\mathrm{N}}$	$\neq n_{\mathrm{P}}X_2$	$X_{\mathrm{N}}\cdot z_{\mathrm{PA}}$	z_{PA}, $X_{\mathrm{N}} \neq n_{\mathrm{P}}X_3$	Random	Seq.-phase	Seq.-phase
B2	$n_{\mathrm{P}}X_1$	$\neq n_{\mathrm{P}}X_2$	$X_{\mathrm{N}}\cdot z_{\mathrm{PA}}$	z_{PA}, $X_{\mathrm{N}} = n_{\mathrm{P}}X_3$	Random	In-phase	Seq.-phase
C	not in the above relations, but acc. to Eq. (8)				Depending on the factorizing		

The rotation angle φ_{CA} and the separation angle γ_{ASi}, can be calculated accordingly,

$$\varphi_{\mathrm{CA}} = \varphi_{\mathrm{C}} + \vartheta_{\mathrm{AS1}},\ \gamma_{\mathrm{AS}i} = \gamma_{\mathrm{C}i} - \vartheta_{\mathrm{AS1}} + \vartheta_{\mathrm{AS}i},\ \text{or}\ \Delta\gamma_{\mathrm{AS}i} = \gamma_{\mathrm{AS}i} - \gamma_{\mathrm{C}i} = \vartheta_{\mathrm{AS}i} - \vartheta_{\mathrm{AS1}}. \tag{10}$$

The center distances a_{w1}. And $a_{\mathrm{w}i}$.are calculated according to the law of cosines, i.e.,

$$a_{\mathrm{w1}} = \sqrt{a_{\mathrm{C1}}^2 + e_{\mathrm{C}}^2 + 2a_{\mathrm{C1}}e_{\mathrm{C}}\cos\varphi_{\mathrm{C}}},\ a_{\mathrm{w}i} = \sqrt{a_{\mathrm{C}i}^2 + e_{\mathrm{C}}^2 + 2a_{\mathrm{C}i}e_{\mathrm{C}}\cos(\varphi_{\mathrm{C}} + \gamma_{\mathrm{C}i})}. \tag{11}$$

The pressure angle $\alpha_{\mathrm{wPA1},i}$, $\alpha_{\mathrm{wPS1},i}$ are equal to

$$\cos\alpha_{\mathrm{wPA1},i} = \frac{m_{\mathrm{A}}\cdot(\lvert z_{\mathrm{A}}\rvert + z_{\mathrm{PA}})\cdot\cos\alpha_0}{2\cdot a_{\mathrm{w1},i}},\ \cos\alpha_{\mathrm{wPS1},i} = \frac{m_{\mathrm{S}}\cdot(z_{\mathrm{S}} + z_{\mathrm{PS}})\cdot\cos\alpha_0}{2\cdot a_{\mathrm{w1},i}}. \tag{12}$$

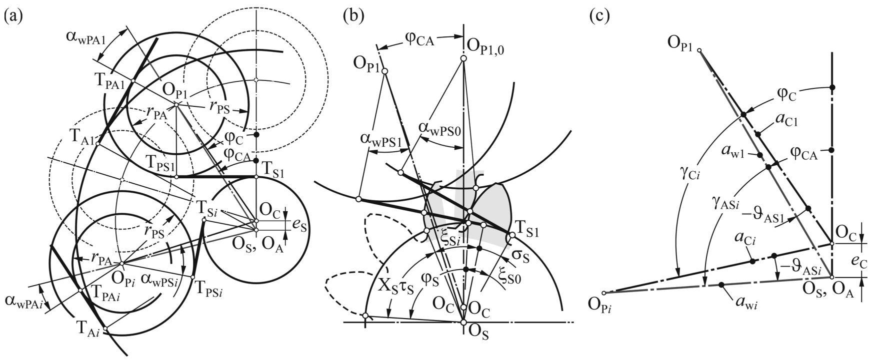

Fig. 3. Deviated positions of the planets relative to the sun and annulus gear

Rotation Angle of the Sun Gear $\varphi_{\mathrm{S\text{-}P1}}$ is determined according to the geometric relation in Fig. 3, i.e., $\varphi_S = \sigma_S + \xi_{S1} - \xi_{S0} + X_S\tau_S$. With ξ_{S1} and ξ_{S0}, $\varphi_{\mathrm{S\text{-}P1}}$ can be calculated based on the gear mesh of sun and planet 1,

$$\varphi_{\text{S - P1}} = (u_{ST} + 1)\varphi_{\text{C}} + (u_{\text{ST}} + 1)\vartheta_{\text{AS1}} + X_{\text{S}}\tau_{\text{S}} - \frac{z_{\text{PS}}}{z_{\text{S}}}(X_{\text{PS1}} - X_{\text{PS0}})\tau_{\text{PS}} + \\ + \left[\frac{z_{\text{PS}}}{z_{\text{S}}} + 1\right](\text{inv}\alpha_{\text{wPS1}} - \text{inv}\alpha_{\text{wPS0}}) - \frac{z_{\text{PS}}}{z_{\text{S}}}\left(\frac{|z_{\text{A}}|}{z_{\text{PA}}} - 1\right)(\text{inv}\alpha_{\text{wPA1}} - \text{inv}\alpha_{\text{wPA0}}) \tag{13}$$

Non-loaded Transmission Error (*NLTE*) is determined as the difference of the actual and the ideal value of the rotation angle φ_{S} of the sun gear under a given rotation angle φ_{C} of the carrier. Because of multiple compound planets, the actual angle φ_{S} must be involved all the contact conditions of the planet-sun gear pairs. The eccentricity error of the carrier may also cause flank interference in sun-planet gear pairs other than sun-planet 1. The sun gear must rotate reversely to avoid the interference. Therefor the result transmission error NLTE is equal to

$$NLTE = \varphi_{\text{S}} - (u_{\text{ST}} + 1)\varphi_{\text{C}} + \frac{\min(\delta_i)}{r_{\text{bS}}}, \quad i = 2, \cdots, n, \text{ and only if} \delta_i < 0. \tag{14}$$

3 Analysis Model for Shared Loads on Contact Tooth Pairs

The shared loads among the planets in the planetary gear sets are calculated based on the relations of loaded deformation and displacement, as well as the load equilibrium of the planets and the sun gear. The loaded deformation w of the engaged tooth pair is equal to the product of the compliance f of the tooth and the acting load F according to Hook's law, i.e., $w = F{\cdot}f$. The compliance f involves the influences of Hertz contact and tooth bending deformation and is varied with the contact position of the tooth, more details can be found in [1, 2].

The load relationship of a compound planetary gear mechanism is no different from that of a simple planetary gear set, except for the differences in the load equilibrium of the compound planets, i.e., for the i^{th} planet gear the equation is valid,

$$r_{\text{bPA}} \cdot \sum_{j=1}^{t_{\text{PA}i}} F_{\text{PA}i,j} = r_{\text{bPS}} \cdot \sum_{j=1}^{t_{\text{PS}i}} F_{\text{PS}i,j} \tag{15}$$

The complete equations for the CSPG. With n_{P} planets, n_{PA} planet-annulus tooth pairs and n_{PS} planet-sun tooth pairs in contact can be expressed in a matrix [1, 2],

$$\begin{bmatrix} \mathbf{f}_{PA} & \mathbf{0} & -r_{bPA}\mathbf{I} & \mathbf{0} \\ \mathbf{0} & f_{PS} & r_{bPS}I & -r_{bS}J \\ \mathbf{r}_{bPA}\mathbf{I} & -r_{bPS}\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & r_{bS}\mathbf{J}^T & \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{F}_{PA} \\ \mathbf{F}_{PS} \\ \mathbf{\Lambda}_P \\ \lambda_S \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \Delta_{PS} \\ \mathbf{0} \\ T_{in} \end{bmatrix} \tag{16}$$

In the equation, $\mathbf{f}_{\text{PA}}$ is $n_{\text{PA}} \times n_{\text{PA}}$ matrix of compliance, $\mathbf{f}_{\text{PS}}$ $n_{\text{PS}} \times n_{\text{PS}}$ matrix of compliance, $\mathbf{I}$ $n_{\text{P}} \times n_{\text{P}}$ unit matrix and $\mathbf{J}$ $n_{\text{P}} \times 1$ unit vector. The tooth clearances $\delta_{\text{PS}i}$ calculated from Eq. (3) due to errors are integrated in vector $\mathbf{\Delta}_{\text{PS}}$. The unknown variables to be solved, including the acting force on the contact tooth pairs, the rotational deformation angles of planets and sun, are all integrated separately into vectors $\mathbf{F}_{PA}$, $\mathbf{F}_{PA}$, $\mathbf{\Lambda}_P$, and λ_S.

Loaded Transmission Error (*LTE*) is the sum of *NLTE* and deformation angle λ_S calculated from Eq. (3),

$$LTE = NLTE + \lambda_S \tag{17}$$

4 Numerical examples

Four study cases are chosen for influence analysis of the meshing phases on transmission errors and load sharing, see Tables 2, 3, and 4. All possible combinations of mesh phasing of the gear pairs S-PS and PA-A are included in the cases, Table 3. The eccentric error of the carrier is 2.5 µm, which is the limit for precision machining.

Table 2. Common gearing data used in study cases

	Unit	Sun S	Planet PS	Planet PR	Annulus A
Normal module m_n	mm	1	1	1	1
Pressure angle α_P	deg	20	20	20	20
Face width b	Mm	10	10	10	10
Planet number n_P	–	3			
E-modulus E	MPa	207,000	207,000	207,000	207,000
Poison's ratio ν	–	0.3	0.3	0.3	0.3
Input Torque T	Nm	80	–	–	–
Input Speed	Rpm	1,400			--

Table 3. Essential data of study cases, tooth numbers and mesh characteristics

Study case	Tooth number				Design rule	Installation type	Mesh phasing	
	z_S	z_{PS}	z_{PR}	z_R			S-PS	PA-A
Case 1	18	43	20	−81	A1	Specific	In-phase	In-phase
Case 2	17	44	22	−83	B1	Random	Seq.-phase	Seq.-phase
Case 3	18	45	22	−85	C	Specific	In-phase	Seq.-phase
Case 4	17	43	21	−81	C	Specific	Seq.-phase	In-phase

Table 4. Gear pair data of study cases

Study case	Total ratio	Center distance a	Profile-shifting factor			
			x_S	x_{PS}	x_{PR}	x_R
Case 1	10.675	30.5	0.3049	−0.3049	0.517	− 0.517
Case 2	10.765	30.5	0.3327	−0.3327	0.4946	− 0.4946
Case 3	10.659	31.5	0.3134	−0.3134	0.4930	−0.4930
Case 4	10.756	30.0	0.3285	−0.3285	0.5062	−0.5062

5 Analysis Results

5.1 Loaded Transmission Error

Using the calculation in Eq. (17), when the planetary gear set is loaded, transmission errors can be caused due to the tooth deformation. Since the loaded transmission error analysis result with error-free condition does not have long wave variation, but only short wave variation under the mesh of tooth pairs, the results of four cases are shown in the smaller range of carrier angle, see the left side of Fig. 4.

- In Case 1, the planets mesh with the sun gear and the annulus gear both in-phase simultaneously, resulting in the highest amplitude of the overall transmission error with a peak-to-peak value of about 5 μm.
- In Case 2, the sun-planet gear pairs and the annulus-planet gear pairs are in sequential-phase simultaneously, causing the smallest peak-to-peak value of LTE among all cases, about 1 μm.
- In Case 3, the sun-planet gear pairs are in-phase, and the annulus-planet gear pairs in sequential-phase, which causes the LTE trend to be closer to Case 2 but with a larger peak-to-peak value of about 2 μm.
- In Case 4, the sun-planet gear pairs are in sequential-phase, and the annulus-planet gear pair in-phase, resulting in an LTE trend closer to Case 1, but with a smaller peak-to-peak value of about 4 μm.

If an eccentric error of the carrier is present, additional periodic variations in transmission errors will occur. The results in the right side of Fig. 4 demonstrates the effect. In comparison to the error-free LTE, the high-frequency wave remains unchanged, while the low-frequency wave is affected by the carrier's eccentricity.

5.2 Load Sharing Among Planets—Error-Free

Figure 5 demonstrate that the mesh phasing of the sun-planet gear pairs and the annulus-planet gear pairs affect the load sharing among planets under error-free condition. Because of in-phase mesh of all the gear pairs in Case 1, each engaged tooth pair has the same mesh position and meshing stiffness at any given time, resulting in an equally load distribution among the planets. In contrast, the occurrence of sequential phase mesh between sun-planet pairs or annulus-planet pairs, or both, in Case 2, 3, and 4, results in

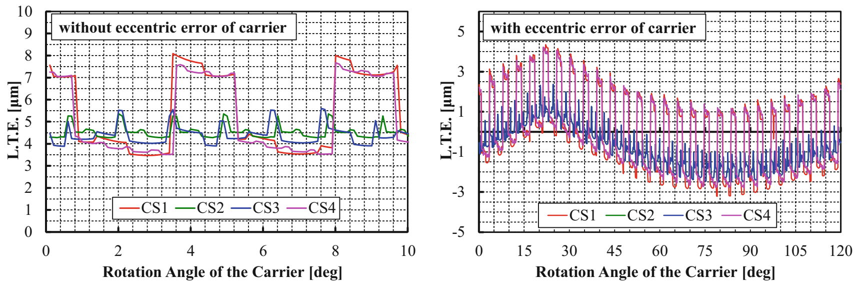

Fig. 4. Loaded transmission errors

uneven distribution of planetary loads. In Case 2, both the sun-planet pairs and annulus-plant pairs mesh sequential-phase, causing significant fluctuations in load sharing with clear sequential variation. The maximum load sharing factor K_γ is about 1.54. Similar results can be also found in Case 3 with $K_\gamma = 1.50$, where the mesh of the sun-planet pair is in-phase, and annulus-planet pairs sequential-phase. Comparatively, the load sharing in Case 4 is relatively smaller than that in Case 3. The significant difference between Cases 3 and 4 is attributed to the waveform period. In Case 4, the planets mesh with the sun gear in sequential phase, resulting in a reduced period of the load sharing waveform.

5.3 Load Sharing Among Planets—With Eccentric Error of Carrier

Taking the eccentric error of the carrier into consideration, the load sharing curve can be regarded as a combination of a short-period wave and a long-period wave, as illustrated in Fig. 6. The long-period wave is caused mainly by the eccentric error, and the short-period wave is dependent on the mesh phasing.

The short-period load sharing in Case 1 varies minor fluctuations due to eccentric error which cause deviations in the contact position, instead of remaining constant as expected. The load sharing variation in short period is similar to the variation under the error-free condition in the other cases, and all are influenced by the mesh phasing of the gear pairs.

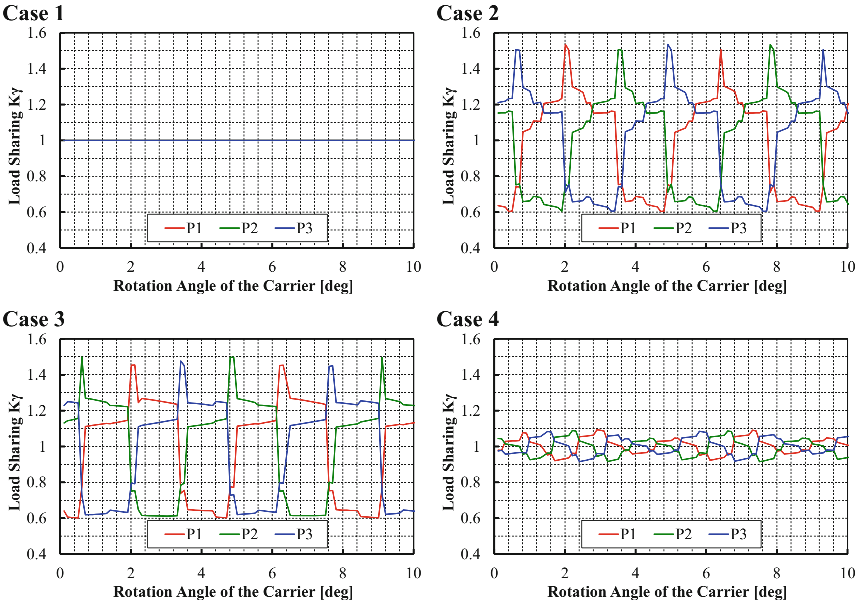

Fig. 5. Load sharing among planets without eccentric error of carrier

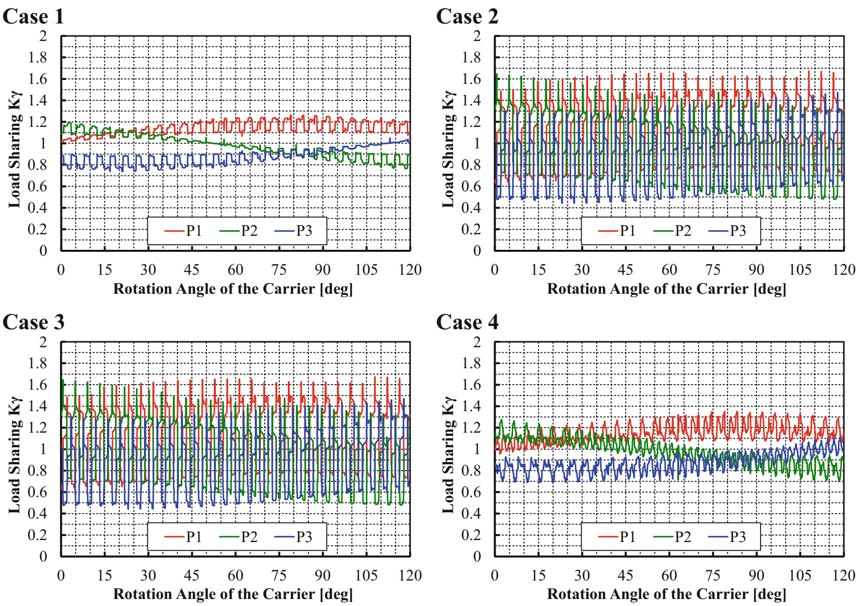

Fig. 6. Load sharing among planets with eccentric error of carrier

6 Conclusion

This paper proposes a set of general equations for determining contact positions and load-deformation relationships of CSPG sets, from which the correct number of teeth for assembly can be derived. A gear meshing and load analysis model was also modified to account for the presence of eccentric error in the carrier. To explore the effects of different combinations of mesh phasing in CSPG, transmission errors and load sharing were analyzed. Based on these findings, the following conclusions can be drawn:

- The number of gear teeth in CSPG can be easily determined using the proposed equations and design guidelines, which are more systematic and easier to understand than existing literature [1, 2].
- According to the factorizing of the sun gear and the annulus gear respectively, CSPG can have four different combinations of mesh phasing. The mesh phase of the sun gear and annulus gear significantly affects transmission error and load sharing among planets.
- If the planets mesh with the sun and annulus all in-phase (Case 1), it will result in a larger transmission error, but the load among the planets will be evenly distributed.
- If the planets mesh with the sun and annulus sequentially (Case 2), the transmission error will decrease, but load sharing among the planets will be uneven.
- The mesh phasing of planet-annulus gear pair has a greater impact on the transmission performance than that of planet-sun gear pair. The transmission performance of Case 1 is similar to that of Case 4, and the same is true for Case 2 and Case 3.
- The eccentric error of the carrier results in long-periodic fluctuations in transmission error and load sharing, while mesh phasing only affects short-periodic variations that can be influenced by profile errors or profile modifications.

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