Extension-Based Semantics for Incomplete Argumentation Frameworks: Grounded Semantics and Principles

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Abstract. Incomplete Argumentation Frameworks (IAFs) enrich classical abstract argumentation with arguments and attacks whose actual existence is questionable. The usual reasoning approaches rely on the notion of completion, *i.e.* standard AFs representing "possible worlds" compatible with the uncertain information encoded in the IAF. Recently, extension-based semantics for IAFs that do not rely on the notion of completion have been defined, using instead new versions of conflict-freeness and defense that take into account the (certain or uncertain) nature of arguments and attacks. In this paper, we give new insights on this reasoning approach, by adapting the well-known grounded semantics to this framework in two different versions. After determining the computational complexity of our new semantics, we provide a principle-based analysis of these semantics, as well as the ones previously defined in the literature, namely the complete, preferred and stable semantics.

1 Introduction

Abstract argumentation has received much attention since the seminal paper by Dung [12]. An *Argumentation Framework* (AF) is generally defined as a directed graph where nodes represent arguments, and edges represent attacks between these arguments. Since then, many generalizations of Dung's framework have been proposed, introducing the notion of support between arguments [2], weighted attacks [13] or weighted arguments [19], preferences between arguments [1], and so on.

In this paper, we focus on one such generalization of abstract argumentation, namely *Incomplete Argumentation Frameworks* (IAFs) [5,7,8] in which arguments and attacks can be defined as uncertain, meaning that the agent reasoning with such an IAF is not sure whether these arguments or attacks actually exist (*e.g.* whether they will actually be used at some step of the debate). This is particularly meaningful when modelling an agent's knowledge about her opponent in a debate [10,11], since it is a reasonable assumption that agents are not

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always able to assess precisely the uncertainty degree of a piece of information (*e.g.* meaningful probabilities may not be available). We push further a recent study of semantics defined for reasoning with IAFs, based on the idea that basic principles of argumentation semantics (namely conflict-freeness and defense) can be adapted to take into account the nature of the pieces of information in the IAF (certain or uncertain) [7,16,18]. While the initial work on this topic focuses on *Partial* AFs (which are IAFs without uncertain arguments) and the preferred semantics [7], the general IAF model and other semantics (namely complete and stable) have also been studied in [16,18]. Now we focus on the adaptation of the last classical semantics initially defined by Dung, namely the grounded semantics. For all the semantics defined in the literature and in the present paper, we also investigate the principles they satisfy, following the principle-based approach for analysing argumentation semantics [3,4,20]. Proofs are omitted for space reasons.

2 Background

Definition 1. *An* Argumentation Framework *(AF) [12] is a directed graph* $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$ *where* $\mathcal{A}$ *represents the arguments and* $\mathcal{R} \subseteq \mathcal{A} \times \mathcal{A}$ *represents the attacks between arguments.*

In this paper we assume that AFs are always finite, *i.e.* $\mathcal{A}$ is a finite set of arguments. We say that an argument $a \in \mathcal{A}$ (resp. a set $S \subseteq \mathcal{A}$) attacks an argument $b \in \mathcal{A}$ if $(a, b) \in \mathcal{R}$ (resp. $\exists a \in S$ such that $(a, b) \in \mathcal{R}$). Then, $S \subseteq \mathcal{A}$ *defends* $a \in \mathcal{A}$ if $\forall b \in \mathcal{A}$ such that $(b, a) \in \mathcal{R}$, S attacks b. A set of arguments $S \subseteq \mathcal{A}$ is called *conflict-free* when $\forall a, b \in S$, $(a, b) \notin \mathcal{R}$. In this case we write $S \in \mathsf{cf}(\mathcal{F})$. [12] defined several semantics for evaluating the acceptability of arguments, based on the characteristic function $\Gamma_{\mathcal{F}}$ of an AF:

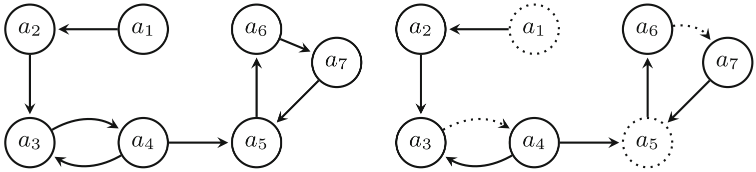
Definition 2. *Given an AF* $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$*, the* characteristic function *of* $\mathcal{F}$ *is* $\Gamma_{\mathcal{F}} : 2^{\mathcal{A}} \rightarrow 2^{\mathcal{A}}$ *defined by*

$$\Gamma_{\mathcal{F}}(S) = \{a \mid S \textit{ defends } a\}$$

Now, given $S \subseteq \mathsf{cf}(\mathcal{F})$ *a conflict-free set of arguments, S is*

- admissible *iff* $S \subseteq \Gamma_{\mathcal{F}}(S)$,
- *a* complete extension *iff* $S = \Gamma_{\mathcal{F}}(S)$,
- *a* preferred extension *iff it is a* $\subseteq$*-maximal admissible set,*
- *the unique* grounded extension *iff it is the* $\subseteq$*-minimal complete extension.*

These sets of extensions are denoted (resp.) by $\mathsf{ad}(\mathcal{F})$, $\mathsf{co}(\mathcal{F})$, $\mathsf{pr}(\mathcal{F})$ and $\mathsf{gr}(\mathcal{F})$. Finally, a last classical semantics is not based on the characteristic function: $S \in \mathsf{cf}(\mathcal{F})$ is a *stable extension* iff S attacks all the arguments in $\mathcal{A} \setminus S$. The stable extensions are denoted $\mathsf{st}(\mathcal{F})$. We sometimes write $\sigma(\mathcal{F})$ for the set of extensions of $\mathcal{F}$ under an arbitrary semantics $\sigma \in \{\mathsf{cf}, \mathsf{ad}, \mathsf{co}, \mathsf{pr}, \mathsf{gr}, \mathsf{st}\}$.



(a) $\mathcal{F}$ (b) The IAF $\mathcal{I}$

Fig. 1. Examples of AF (left) and IAF (right)

Table 1. Extensions of the AF $\mathcal{F}$

Semantics σ	Extensions $\sigma(\mathcal{F})$
co	$\{\{a_1\}, \{a_1, a_3\}, \{a_1, a_4, a_6\}\}$
pr	$\{\{a_1, a_3\}, \{a_1, a_4, a_6\}\}$
st	$\{\{a_1, a_4, a_6\}\}$
gr	$\{\{a_1\}\}$

Example 1. Figure 1a describes $\mathcal{F} = \langle \mathcal{A}, \mathcal{R} \rangle$, where the nodes represent $\mathcal{A}$ and the edges represent $\mathcal{R}$. Its extensions for the co, pr, st and gr semantics are given in Table 1.

Various decision problems can be interesting: σ-Ver is the verification that a given set of arguments is a σ extension of a given AF, σ-Cred and σ-Skep consist (resp.) in checking whether a given argument belongs to some or each σ-extension of a given AF. Finally, σ-Exist (resp. σ-NE) is the check whether there is at least one (resp. one non-empty) σ-extension for a given AF.

Incomplete Argumentation Frameworks (IAFs) generalize AFs by adding a notion of uncertainty on the presence of arguments and attacks, *i.e.* an IAF is a tuple $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ where $\mathcal{A}, \mathcal{A}^?$ are disjoint sets of arguments, and $\mathcal{R}, \mathcal{R}^?$ are disjoint sets of attacks over $\mathcal{A} \cup \mathcal{A}^?$. The arguments and attacks in $\mathcal{A}$ and $\mathcal{R}$ certainly exist, while those in $\mathcal{A}^?$ and $\mathcal{R}^?$ are uncertain. See [17] for a recent overview of IAFs. In this paper, we focus on the IAF semantics from [7,16,18]. The intuition behind this approach consists in adapting the notions of conflict-freeness and defense to IAFs, in order to define well-suited notions of admissibility and the corresponding semantics.

Definition 3. *Let $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ be an IAF, and $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ a set of arguments. S is* weakly *(resp.* strongly*)* conflict-free *iff $\forall a, b \in S \cap \mathcal{A}$ (resp. $a, b \in S$), $(a, b) \notin \mathcal{R}$ (resp. $(a, b) \notin \mathcal{R} \cup \mathcal{R}^?$).*

Definition 4. *Let $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ be an IAF, $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ a set of arguments, and $a \in \mathcal{A} \cup \mathcal{A}^?$ an argument. S* weakly *(resp.* strongly*)* defends *a iff $\forall b \in \mathcal{A}$ (resp. $b \in \mathcal{A} \cup \mathcal{A}^?$) s.t. $(b, a) \in \mathcal{R}$ (resp. $(b, a) \in \mathcal{R} \cup \mathcal{R}^?$), $\exists c \in S \cap \mathcal{A}$ s.t. $(c, b) \in \mathcal{R}$.*

The weak (resp. strong) conflict-free and admissible sets of an IAF $\mathcal{I}$ are denoted by $cf_w(\mathcal{I})$ and $\mathsf{ad}_w(\mathcal{I})$ (resp. $\mathsf{cf}_s(\mathcal{I})$ and $\mathsf{ad}_s(\mathcal{I})$). Combining weak (resp. strong) conflict-freeness with weak (resp. strong) defense yields a notion of weak (resp. strong) admissibility, and the corresponding preferred and complete semantics.

Definition 5. *Let $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ be an IAF, and $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ a set of arguments. S is a*

- *weak (resp. strong) preferred extension of $\mathcal{I}$ if S is a $\subseteq$-maximal weak (resp. strong) admissible set,*
- *weak (resp. strong) complete extension of $\mathcal{I}$ if S is a weak (resp. strong) admissible set which does not weakly (resp. strongly) defend any argument outside of S.*

These semantics are denoted by $\mathsf{pr}_x(\mathcal{I})$ and $\mathsf{co}_x(\mathcal{I})$, with $x \in \{w, s\}$. The stable semantics has been adapted as well.

Definition 6. *Let $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ be an IAF, and $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ a set of arguments. S is a weak (resp. strong) stable extension iff it is a weak (resp. strong) conflict-free set s.t. $\forall a \in \mathcal{A} \setminus S$ (resp. $a \in (\mathcal{A} \cup \mathcal{A}^?) \setminus S$), $\exists b \in S \cap \mathcal{A}$ s.t. $(b, a) \in \mathcal{R}$.*

We use $\mathsf{st}_x(\mathcal{I})$ with $x \in \{w, s\}$ to denote the weak and strong stable extensions of an IAF.

Example 2. Figure 1b describes an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ where the dotted nodes (resp. edges) represent the uncertain arguments $\mathcal{A}^?$ (resp. attacks $\mathcal{R}^?$). Certain arguments and attacks are represented as previously. Its extensions are given in Table 2.

Table 2. Extensions of the IAF $\mathcal{I}$

Semantics σ	Extensions $\sigma(\mathcal{F})$
co_w	$\{\{a_1, a_2, a_4, a_6, a_7\}\}$
pr_w	$\{\{a_1, a_2, a_4, a_6, a_7\}\}$
st_w	$\{\{a_2, a_4, a_6, a_7\}, \{a_2, a_4, a_5, a_6, a_7\},$ $\{a_1, a_2, a_4, a_6, a_7\}, \{a_1, a_2, a_4, a_5, a_6, a_7\}\}$
co_s	$\{\{a_1\}, \{a_1, a_6\}\}$
pr_s	$\{\{a_1, a_6\}\}$
st_s	$\emptyset$

The complexity of reasoning with these semantics has been established in [16,18], the results are summarized in Table 3.

3 Grounded Semantics

Now we fulfill the landscape of extension-based semantics for IAFs by defining weak and strong variants of the grounded semantics. Following Dung's original approach, we define characteristic functions of an IAF, corresponding to the notions of weak and strong defense from Definition 4.

Definition 7 (Characteristic Functions). *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, the x-characteristic function of $\mathcal{I}$ (where $x \in \{w, s\}$) is defined by*

$$\Gamma_{x,\mathcal{I}}(S) = \{a \in \mathcal{A} \cup \mathcal{A}^? \mid S \text{ } x\text{-defends } a\}$$

We show that the results by Dung regarding the characteristic function of an AF [12, Section 2.2] can be adapted to our framework. The following lemmas are easy to prove. First, the x-characteristic function preserves the x-conflict-freeness.

Lemma 1. *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, $x \in \{w, s\}$ and $S \subseteq \mathcal{A} \cup \mathcal{A}^?$, if $S \in \mathsf{cf}_x(\mathcal{I})$ then $\Gamma_{x,\mathcal{I}}(S) \in \mathsf{cf}_x(\mathcal{I})$.*

The following lemma also shows that the usual relation between admissibility and the characteristic function(s) also works for the strong and weak admissible sets defined in [16, 18].

Lemma 2. *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, $x \in \{w, s\}$, and $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ such that $S \in \mathsf{cf}_x(\mathcal{I})$, $S \in \mathsf{ad}_x(\mathcal{I})$ if and only if $S \subseteq \Gamma_{x,\mathcal{I}}(S)$.*

Also, the correspondence between fixed-points of the characteristic functions and the strong and weak complete extensions holds in our framework as well.

Lemma 3. *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, $x \in \{w, s\}$, and $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ such that $S \in \mathsf{cf}_x(\mathcal{I})$, $S \in \mathsf{co}_x(\mathcal{I})$ if and only if $S = \Gamma_{x,\mathcal{I}}(S)$.*

Now, we prove that the $\Gamma_{x,\mathcal{I}}$ functions are monotonic.

Lemma 4. *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, $x \in \{w, s\}$, and two sets of arguments $S, S' \subseteq \mathcal{A} \cup \mathcal{A}^?$ such that S, S' are x-conflict-free, if $S \subseteq S'$ then $\Gamma_{x,\mathcal{I}}(S) \subseteq \Gamma_{x,\mathcal{I}}(S')$.*

Finally we define the grounded semantics of IAFs:

Definition 8. *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ and $x \in \{w, s\}$, the unique x-grounded extension of $\mathcal{I}$ is the fixed point obtained by iteratively applying the x-characteristic function of $\mathcal{I}$ using $\emptyset$ as the starting point.*

This means that we can compute the x-grounded extension with Algorithm 1, which follows the usual approach for computing the grounded extension of an argumentation framework: take the arguments which do not need to be defended (*i.e.* compute $\Gamma_{x,\mathcal{I}}(\emptyset)$, in the case where $x = w$, these are the arguments which are not certainly attacked by certain arguments; in the case where $x = s$ it means that they are not attacked at all). Then, while it is possible, we add to the extension arguments that are defended by the arguments already member of the extension. The process stops when nothing can be added anymore.

Algorithm 1. Computation of the x-grounded extension

Require: $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, $x \in \{w, s\}$

1: result $= \Gamma_{x,\mathcal{I}}(\emptyset)$
2: **while** result $\neq \Gamma_{x,\mathcal{I}}$(result) **do**
3:     result $= \Gamma_{x,\mathcal{I}}$(result)
4: **end while**
5: **return** result

Example 3. Continuing the previous example, we have $\mathsf{gr}_w(\mathcal{I}) = \{\{a_1, a_2, a_4, a_6, a_7\}\}$ and $\mathsf{gr}_s(\mathcal{I}) = \{\{a_1\}\}$.

From Lemma 4, we deduce that the iterations of the loop (line 2 in Algorithm 1) only add arguments to the result being constructed. So the number of iterations of this loop is bounded by the number of arguments, which means that this process is polynomial, as well as all the classical decision problems for these semantics. The P-hardness comes from the known results for standard AFs [14].

Proposition 1. *For* $x \in \{w, s\}$*, the problems* gr_x-Ver*,* gr_x-Cred *and* gr_x-Skep *are* P*-complete,* gr_x-Exist *is trivial, and* gr_x-NE *is in* L*.*

From Lemma 3, it is obvious that the x-grounded extension of an IAF is also a x-complete extension. It is also the case that any complete extension must contain the arguments which do not need to be x-defended, and then it must contain all the arguments from the x-grounded extension. So the x-grounded extension can be characterized as the (unique) $\subseteq$-minimal x-complete extension, similarly to the "classical" grounded extension. This implies that the coNP upper bound for co_x-Skep [16] can be made more precise, since co_x-Skep $=$ gr_x-Skep.

Corollary 1. *For* $x \in \{w, s\}$*,* co_x-Skep *is* P*-complete.*

Table 3 summarizes the known complexity results for reasoning with the semantics of IAFs. Grey cells correspond to new results provided in this paper, while the other cells correspond to results from [16] (for σ_x-Ver, σ_x-Cred and σ_x-Skep) and [18] for (σ_x-Exist and σ_x-NE).

4 Principle-Based Analysis of IAF Semantics

Now we study the properties of the extension-based semantics of IAFs. More precisely, we focus on some principles already mentioned in the literature [3, 20]. However, we do not mention some principles which are not relevant here, like *admissibility* or *reinstatement*, which do not make sense if they are directly applied to IAFs. Since our semantics have been defined to satisfy weak or strong counterparts of admissibility (except weak stable semantics), there is nothing to prove regarding these principles adapted to IAFs. We adapt to IAFs several principles from the literature, and show which ones are satisfied by our semantics.

Table 3. Complexity of σ_x-Ver, σ_x-Cred, σ_x-Skep, σ_x-Exist and σ_x-NE for $\sigma \in \{\mathsf{cf}, \mathsf{ad}, \mathsf{gr}, \mathsf{st}, \mathsf{co}, \mathsf{pr}\}$ and $x \in \{w, s\}$. $\mathcal{C}$-c means $\mathcal{C}$-complete.

Semantics σ_x	σ_x-Ver	σ_x-Cred	σ_x-Skep	σ_x-Exist	σ_x-NE
cf_x	in L	in L	trivial	trivial	in L
ad_x	in L	NP-c	trivial	trivial	NP-c
gr_x	P-c	P-c	P-c	trivial	in L
st_x	in L	NP-c	coNP-c	NP-c	NP-c
co_x	in L	NP-c	P-c	trivial	NP-c
pr_x	coNP-c	NP-c	Π_2^P-c	trivial	NP-c

The I-maximality principle states that no extension should be a proper subset of another extension.

Principle 1. *An extension-based semantics σ satisfies the* I-maximality *principle if, for any AF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, $\forall S, S' \in \sigma(\mathcal{I})$, if $S \subseteq S'$ then $S = S'$.*

Proposition 2. *I-maximality is satisfied by st_s as well as pr_x and gr_x for $x \in \{w, s\}$. It is not satisfied by co_x for $x \in \{w, s\}$, nor by st_w.*

Roughly speaking, the next principle states that if an argument belongs to an extension, and is attacked by another extension, then there should be a third one which abstains to give a status to this argument (*i.e.* this argument does not belong to the third extension, and is not attacked by it).

Given $S \subseteq \mathcal{A} \cup \mathcal{A}^?$, $S^+ = \{a \in \mathcal{A} \cup \mathcal{A}^? \mid \exists b \in S \text{ s.t. } (b, a) \in \mathcal{R} \cup \mathcal{R}^?\}$ is the set of arguments attacked by S.

Principle 2. *An extension-based semantics σ satisfies the* allowing abstention *principle if, for any IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, and any $a \in \mathcal{A} \cup \mathcal{A}^?$, if there are two extensions $S_1, S_2 \in \sigma(\mathcal{I})$ such that $a \in S_1$ and $a \in S_2^+$, then there is a third extension $S_3 \in \sigma(\mathcal{I})$ such that $a \notin S_3 \cup S_3^+$.*

Proposition 3. *For $x \in \{w, s\}$, gr_x satisfies allowing abstention. For $\sigma \in \{\mathsf{pr}, \mathsf{st}\}$ and $x \in \{w, s\}$, σ_x does not satisfy allowing abstention. Finally, co_s satisfies it, and co_w does not satisfy it.*

Notice that allowing abstention can be considered either as trivially satisfied (as in [20]) or non-applicable (as in [3]) for single-status semantics like the grounded semantics. Here we use the first option for presenting the results.

The next principle is based on the notion of contaminating framework. To define it, we need to introduce $\mathcal{I}_1 \sqcup \mathcal{I}_2 = \langle \mathcal{A}_1 \cup \mathcal{A}_2, \mathcal{A}_1^? \cup \mathcal{A}_2^?, \mathcal{R}_1 \cup \mathcal{R}_2, \mathcal{R}_1^? \cup \mathcal{R}_2^? \rangle$.

Definition 9. *Two IAFs $\mathcal{I}_1 = \langle \mathcal{A}_1, \mathcal{A}_1^?, \mathcal{R}_1, \mathcal{R}_1^? \rangle$ and $\mathcal{I}_2 = \langle \mathcal{A}_2, \mathcal{A}_2^?, \mathcal{R}_2, \mathcal{R}_2^? \rangle$ are disjoint if $(\mathcal{A}_1 \cup \mathcal{A}_1^?) \cap (\mathcal{A}_2 \cup \mathcal{A}_2^?) = \emptyset$.*

An IAF $\mathcal{I}^*$ is contaminating for a semantics σ if and only if for any $\mathcal{I}$ disjoint from $\mathcal{I}^*$, $\sigma(\mathcal{I}^*) = \sigma(\mathcal{I}^* \sqcup \mathcal{I})$.

The existence of such a contaminating IAF $\mathcal{I}^*$ can be seen as a weakness of the semantics, because adding $\mathcal{I}^*$ to another IAF $\mathcal{I}$ somehow causes a crash of the reasoning in $\mathcal{I}$.

Principle 3. *An extension-based semantics σ satisfies the* crash resistance *principle iff there is no contaminating IAF for σ.*

Proposition 4. *For $\sigma \in \{\mathsf{co}, \mathsf{pr}, \mathsf{gr}\}$ and $x \in \{w, s\}$, σ_x satisfies crash resistance. For $x \in \{w, s\}$, st_x does not satisfy crash resistance.*

A set of arguments is called isolated if none of its elements attacks or is attacked by an argument outside the set.

Definition 10. *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, a set of arguments $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ is called* isolated *in $\mathcal{I}$ if*

$$((S \times ((\mathcal{A} \cup \mathcal{A}^?) \setminus S)) \cup (((\mathcal{A} \cup \mathcal{A}^?) \setminus S) \times S)) \cap (\mathcal{R} \cup \mathcal{R}^?) = \emptyset$$

Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ and $S \subseteq \mathcal{A} \cup \mathcal{A}^?$, $\mathcal{I}_{\downarrow S}$ is the IAF defined by $\mathcal{I}_{\downarrow S} = \langle \mathcal{A} \cap S, \mathcal{A}^? \cap S, \mathcal{R} \cap (S \times S), \mathcal{R}^? \cap (S \times S) \rangle$.

Principle 4. *An extension-based semantics σ satisfies the* non-interference *principle iff for any IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, and for any $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ isolated in $\mathcal{I}$, $\sigma(\mathcal{I}_{\downarrow S}) = \{E \cap S \mid E \in \sigma(\mathcal{I})\}$.*

Proposition 5. *For $\sigma \in \{\mathsf{co}, \mathsf{pr}, \mathsf{gr}\}$ and $x \in \{w, s\}$, σ_x satisfies non-interference. For $x \in \{w, s\}$, st_x does not satisfy non-interference.*

Finally, the three last principles are based on the notion of unattacked sets of arguments, *i.e.* sets that can attack arguments from outside, but which are not attacked by arguments from the outside (notice that these sets do not have to be conflict-free).

Definition 11. *Given an IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$, the set of arguments $S \subseteq \mathcal{A} \cup \mathcal{A}^?$ is called* unattacked *in $\mathcal{I}$ if and only if $\forall a \in (\mathcal{A} \cup \mathcal{A}^?) \setminus S$, $\forall b \in S$, $(a, b) \notin \mathcal{R} \cup \mathcal{R}^?$.*

The set of unattacked sets of $\mathcal{I}$ is denoted by $\mathcal{US}(\mathcal{I})$.

Principle 5. *An extension-based semantics σ satisfies the* directionality *principle iff for any IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ and any $S \in \mathcal{US}(\mathcal{I})$, $\sigma(\mathcal{I}_{\downarrow S}) = \{E \cap S \mid E \in \sigma(\mathcal{I})\}$.*

As in Dung's framework, directionality implies non-interference, which implies crash resistance.

The next principles are weaker versions of directionality, where there is only an inclusion relation between $\sigma(\mathcal{I}_{\downarrow S})$ and $\{E \cap S \mid E \in \sigma(\mathcal{I})\}$ instead of an equality. This means that a semantics which satisfies directionality obviously satisfies both of them, but a semantics which does not satisfy directionality may satisfy one of them (but not both).

Principle 6. *An extension-based semantics σ satisfies the* weak directionality *principle iff for any IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ and any $S \in \mathcal{US}(\mathcal{I})$, $\sigma(\mathcal{I}_{\downarrow S}) \supseteq \{E \cap S \mid E \in \sigma(\mathcal{I})\}$.*

Principle 7. *An extension-based semantics σ satisfies the* semi-directionality *principle iff for any IAF $\mathcal{I} = \langle \mathcal{A}, \mathcal{A}^?, \mathcal{R}, \mathcal{R}^? \rangle$ and any $S \in \mathcal{US}(\mathcal{I})$, $\sigma(\mathcal{I}_{\downarrow S}) \subseteq \{E \cap S \mid E \in \sigma(\mathcal{I})\}$.*

Proposition 6. *For $\sigma \in \{\mathsf{co}, \mathsf{pr}, \mathsf{gr}\}$ and $x \in \{w, s\}$, σ_x satisfies directionality. For $x \in \{w, s\}$, st_x does not satisfy directionality.*

Table 4. Satisfaction (✓) or non-satisfaction (✗) of the principles

Principles	co	gr	pr	st	co_s	gr_s	pr_s	st_s	co_w	gr_w	pr_w	st_w
I-max	✗	✓	✓	✓	✗	✓	✓	✓	✗	✓	✓	✗
Allow. abst	✓	✓	✗	✗	✓	✓	✗	✗	✗	✓	✗	✗
Crash resist	✓	✓	✓	✗	✓	✓	✓	✗	✓	✓	✓	✗
Non inter	✓	✓	✓	✗	✓	✓	✓	✗	✓	✓	✓	✗
Direct	✓	✓	✓	✗	✓	✓	✓	✗	✓	✓	✓	✗
Weak Direct	✓	✓	✓	✓	✓	✓	✓	??	✓	✓	✓	??
Semi-Direct	✓	✓	✓	✗	✓	✓	✓	??	✓	✓	✓	??

Let us discuss the results of our principle-based analysis, summarized in Table 4. In most of the cases, the semantics of IAFs have the same properties as their counterpart for standard AFs. We notice few exceptions, and some open questions. First, while strong complete semantics has the same properties as the complete semantics of AFs, it is not the case of the weak complete semantics which does not satisfy allowing abstention. Also, while classical stable semantics of AFs and strong stable semantics of IAFs satisfy I-maximality, it is not the case for the weak stable semantics of IAFs. Then, while it is known that the stable semantics of AFs satisfy weak directionality (and thus does not satisfy semi-directionality), the status of strong and weak stable semantics regarding these properties is still open.

5 Related Work

While our approach for defining semantics for IAFs is, in a way, the original one (since it was initially proposed for *Partial* AFs in [7]), most of the work on reasoning with IAFs is based on the notion of completions [5,6,17], *i.e.* standard AFs that correspond to one possible way to "solve" the uncertainty in the IAF. Using completions, all classical decision problems can be adapted in two versions: the possible view (the property of interest is satisfied in some completions) and the necessary view (the property of interest is satisfied in all the completions). This reasoning approach captures the intuition that the agent reasoning with the IAF uses it to represent a set of possible scenarios and must accept arguments if they are acceptable in some/all scenarios. On the contrary, the approach from [7,18] which is also followed in the current paper considers that the agent uses directly the structure of the IAF for reasoning, instead of using the structure of the (exponentially many) completions of the IAF. Studying whether there are relations between the "completion-based" and the "direct" semantics of IAFs is an interesting question for future work.

6 Conclusion

This paper describes new results on a new family of reasoning approaches for Incomplete Argumentation Frameworks (IAFs), inspired by the original semantics for Partial AFs, a subclass of IAFs. We have shown that Dung's grounded semantics can be adapted to IAFs in two variants, namely weak and strong grounded semantics. As it is usually the case, reasoning with such semantics is doable in polynomial time. Then, we have established which principles from the literature are satisfied by our new semantics, as well as the extension-based semantics for IAFs defined in previous work.

Among possible interesting tracks for future research, of course we plan to fill the gaps regarding the stable semantics in the principle-based analysis, *i.e.* removing the question marks in Table 4. Also, it would be interesting to study whether there are connections between the acceptability of argument with respect to our semantics and their status with respect to completion-based reasoning methods. Then, we wish to apply our semantics in a context of controllability [9,15] and automated negotiation [11]. Also, it would be interesting to parameterize the weak semantics by the number of uncertain conflicts that can be contained in a weak extension, in a way in the same spirit as weighted argumentation frameworks [13].

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