The Changing Bell View of Beables: A Forgotten Story

Federico Laudisa

Abstract John S. Bell is known, among other things, for the introduction of the notion of *beable*. The development of this notion inspired the so-called primitive ontology (PO) approach to the foundations of quantum mechanics, proposed for the first time by Detlef Dürr, Sheldon Goldstein and Nino Zanghì in 1992. It is not very well known, however, that the Bell theory of beables had an early formulation, in which Bell curiously adopts some Bohr-reminiscent insights to attack exactly the standard Copenhagen version of quantum mechanics. Here I reconstruct the two stages of the Bell theory of beables, showing that the first stage is in fact unable to adequately confront the foundational problem it was designed to address. Only the second stage of the Bell theory could represent a motivation for the PO approach: in this respect, it may be of some interest to compare the two-stage reconstruction of the Bell theory with recent analyses of the PO approach in terms of beables. I dedicate this paper to the dear memory of Detlef Dürr, a leading figure of the international community of the foundations of physics and a lovely man. He will be long remembered for his inspiring contributions: it was a privilege to enjoy his company and his doctrine.

1 Introduction

John Stewart Bell is unanimously recognized as one of the leading figures, if not *the* leading figure, of the foundational debate on quantum mechanics (QM) since the second half of the twentieth-century. He is also acknowledged as a fierce and relentless enemy of Copenhagenish approaches to QM: as is well known, his critical attitude toward any purely operational and instrumental understanding of quantum principles led him to encourage alternative views, ranging from Bohmian mechanics (starting from Bell 1966, his first work concerning the hidden variables' issue) to (idiosyncratic) forms of the Everett interpretation (Bell 1976), up to an explicit support to

F. Laudisa (✉)
Department of Humanities and Philosophy, University of Trento, Via Tommaso Gar 14, 38122 Trento, Italy
e-mail: federico.laudisa@unitn.it

A. Bassi et al. (eds.), *Physics and the Nature of Reality*, Fundamental Theories of Physics 215, https://doi.org/10.1007/978-3-031-45434-9_29

the so-called dynamical reduction model, or GRW version, of QM in the latest part of his career (1987, 1989, 1990). One of the most provocative proposals on the Bell part has been the introduction of the notion of *beable*, a term first introduced in 1973 with the specific aim of addressing what Bell took to be an intrinsic ambiguity in the quantum description of observation:

> This terminology, *be*-able as against *observ*able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to ordinary quantum theory." (Bell 1975, in Bell 2004[2], p. 52).

The claim that a 'theory of' beables was needed, and its connection with the issue of locality, were the focus of the seminal papers of the Seventies in which Bell started to elaborate on the notion of beable. At the time the suggestion had not been taken too seriously, but the foundational role of beables has surfaced again in more recent times, when this notion turned out to be at the source of a true research program, the *primitive ontology* (PO) approach, in the area of the foundations and interpretations of quantum mechanics.

This approach was originally proposed by Detlef Dürr, Shelly Goldstein and Nino Zanghì (Dürr et al. 1992). It emphasizes the need, for a well-founded theory, to specify in ontologically clear terms the kind of entity the theory itself is primarily supposed to account for. As to the notion of beable, it was proposed for the first time as the expression of an attitude toward the foundations of quantum mechanics inspired to (some form of) scientific realism, but at that time no PO approach was available yet. It was clear that the proposal of a notion of beable by Bell was an expression of dissatisfaction toward the standard formulation of quantum mechanics, but it is far from transparent what the anti-instrumentalistic role, assigned by Bell to that notion, should have been exactly. I wish to show that there are at least *two* different readings of the notion of beable in the development of Bell's foundational analyses, corresponding to an evolution in time of the interpretation that Bell provides for the notion itself. At an early stage, the concept of beable emerges as the consequence of a peculiar Bohrian-sounding view of the status and role of measurement in QM: within this view Bell, across several of his papers devoted to the foundations of QM, repeatedly and instrumentally exploits Bohr in different places, in order to support claims that in fact are meant to undermine the Copenhagen formulation of quantum mechanics. In this sense, Bell appears ironically to be using a Bohrian insight as a weapon *against* standard QM! I will stress that this early formulation of the notion of beable, in spite of Bell's aspirations toward a less unsatisfactory interpretation of QM, is in fact unable to improve upon the ambiguity of standard QM concerning, for instance, the description of the measurement process. Only later the Bell interpretation of the notion of beable evolves more explicitly into a second, more focused formulation. I will emphasize that it is this new formulation that is apt to intertwine with the locality/non-locality issue arising from the formulation of the 1964 Bell theorem. In retrospect, therefore, we can recognize in this *second* stage of the Bell formulation of the notion of beable one strong motivation for the PO approach to the foundations of quantum mechanics. Then the possibility arises to assess the relation between the two-stage development of the Bell notion of beable and the further evolution of the PO approach, in light of the complex relationships between the latter and the former.

2 The Early History of Beables: Bell and Bohr

The first occurence of the term *beable* can be found in a short, programmatic Bell paper entitled "Subject and object" and published in 1973 (Bell 1973, in Bell 2004^2, pp. 40–44). First of all, the paper has a telling title. Bell decides to address the central role assigned to measurement in the standard formulation of quantum mechanics in terms of a distinction—that between 'subject' and 'object'—that has a *philosophical* flavor.[1] By pairing an *object* with a *measured system* and a *subject* with a *measurer*, Bell charges the standard formulation of quantum mechanics with a kind of subjectivism, according to which the theory is bound to retain a fundamental vagueness and ambiguity on where the boundary between subject and object is supposed to be located, no matter how good for practical use the theory is:

> The subject-object distinction is indeed at the root of the unease that may people still feel in connection with quantum mechanics. […] In extremis the subject-object division can be put somewhere at the 'macroscopic' level, where the practical adequacy of classical notions makes the precise location quantitatively unimportant. But although quantum mechanics can account for these classical features of the macroscopic world as very (very) good approximations, it cannot do more than that. The snake cannot completely swallow itself by the tail. This awkward fact remains: the theory is only *approximately* unambiguous, only *approximately* self-consistent. (Bell 1973, in Bell 2004^2, pp. 40–41, emphasis in the original).

It is in expressing his hope in a less-and-less ambiguous formulation that Bell introduces for the first time the term *beable*:

> […] it should again become possible to say of a system not that such and such may be *observed* but that such and such *be* so. The theory would not be about '*observ*ables' but about '*be*ables'." (Bell 1973, in Bell 2004^2, p. 41).

Here in "Subject and object", Bell does not elaborate as precisely as one could wish a *theory* of beables, but we can interpret his wording as suggesting at least two conditions that such a theory should satisfy:

(i) although the use of the notion of beable cannot simply amount to make quantum mechanics a classical theory in any sense, a theory of beables should account for "an image of the everyday classical world", namely they should enable us—as middle-size natural systems—to recover our subjective experience;

(ii) at the same time, a theory of beables should justify the idea that beables somehow *ground*, or, even better, *constitute* observables: as Bell says with a sort of

[1] According to a Bell biographer, the very title was a choice of the organizers of the conference in which the paper was first presented (Whitaker 2016, p. 290), but Bell employs the distinction with a conscious purpose.

'metaphysical' tone, "the idea that quantum mechanics is primarily about 'observables' is only tenable when such beables are taken for granted. Observables are *made out of* beables." (Bell 1973, in Bell 2004[2], p. 41).

Surprisingly, in order to support the plausibility of beables Bell appears to rely on a well-known passage of Niels Bohr, taken from the Bohr contribution to the 1949 celebrated volume *Albert Einstein Philosopher-Scientist*:

> […] it is decisive to recognize that, *however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.*" (Bohr 1949, p. 209, emphasis in the original).

Bell suggests not only that his notion of beable does justice to the Bohr plea for an account of evidence in classical terms, but also that—if formulated in terms of the beables' theory—such plea can be put to work in order to solve the above mentioned problem generated by the inherent ambiguity and approximation of standard quantum mechanics. The Bell suggestion is ironical, since it uses a major claim of the patriarch of the Copenhagen interpretation as a weapon *against* the Copenhagen interpretation itself: the theory of beables is introduced here clearly as an 'antidote' to the tendency to adopt an axiomatic formulation of quantum mechanics that relies essentially on an ill-defined (according to Bell) notion of measurement.

The use of the name of Bohr in the 1973 paper is not new to Bell, though. It occurs in the very first section of the first article devoted by Bell to the issue of hidden variables, namely the paper *On the problem of hidden variables in quantum mechanics*, written in 1963 but published in 1966. It is the path-breaking article in which Bell reviews the existing impossibility proofs for a hidden variable re-interpretation of quantum mechanics—from von Neumann 1932 to Jauch-Piron 1963, through the work of Gleason in 1957—only to find them all wanting. As is well known, according to Bell all these proofs—no matter what the internal variants were—shared a common drawback, that of requiring assumptions that it was not reasonable to require from *any* possible, hypothetical hidden variable completion of quantum theory.[2] It is in the context of anticipating, in the first section, the core of the article that Bell exploits the name of Bohr, in order to support his claim and make the unreasonableness of the existing impossibility proofs even more apparent:

> It will be urged that these analyses [i.e. the above mentioned proofs] leave the real question untouched. In fact it will be seen that these demonstrations require from the hypothetical dispersion free states, not only that appropriate ensembles thereof should have all measurable properties of quantum mechanical states, *but certain other properties as well*. These additional demands appear reasonable when results of measurement are loosely identified with properties of isolated systems. They are seen to be quite unreasonable when one remembers with Bohr 'the impossibility of any sharp distinction between the behaviour of atomic objects and the interaction with the measuring instruments which serve to define the conditions under which the phenomena appear'. (Bell 1966, in Bell 2004[2], pp. 1–2, my emphasis).

The Bohr view, referred to by Bell, is that in a quantum measurement process a peculiar, non-classical form of non-separability emerges between object system and

[2] For recent re-assessments of the Bell arguments against von Neumann-Gleason and Jauch-Piron see, respectively, Acuna 2021, and (Laudisa 2023).

apparatus. In his 1966 paper Bell appears to exploit this Bohrian non-separability in support of his critical attitude toward the no-hidden variable theorems by von Neumann, Gleason and Jauch-Piron. In other words, Bell presents the Bohr view as an early instance of would have been called 'contextuality', suggesting at the same time that this should have long taught von Neumann, Gleason, Jauch and Piron that any serious hypothetical hidden-variable completion of quantum mechanics was bound to incorporate a form of context-dependence in the first place.[3]

In the same vein, the name of Bohr emerges in the 1971 Bell paper *Introduction to the hidden-variable question*, where Bell first introduces the family of stochastic hidden variable theories. In discussing "the very essential role of apparatus" in the quantum–mechanical description of the measurement process, Bell argues that.

> The result of the measurement does not actually tell us about some property previously possessed by the system, but about something which has come into being in the combination of system and apparatus. *Of course, the vital role of the complete physical set-up we learned long ago, especially from Bohr*." (Bell 1971, in Bell Bell 2004^2, p. 35).

Bell returns to the same point in his later 1982 article "The impossible pilot wave". In recalling once again the lack of generality of the early no-hidden variable theorems, Bell writes about what he calls 'the Gleason-Jauch argument':

> For a given operator P_1 it is possible (when the dimension N of the spin space exceeds 2) to find more than one set of other orthogonal projection operators to complete it:

$$\begin{aligned} 1 &= P_1 + P_2 + P_3 \ldots \\ &= P_1 + P_2' + P_3' \ldots \end{aligned}$$

> where P'_2 … commute with P_1 and with one another, but not with P_2 …. And the extra assumption is this: the result of 'measuring' is independent of which complementary set… or… is 'measured' at the same time. The de Broglie-Bohm picture does not respect this. […] In denying the Gleason-Jauch independence hypothesis, the de Broglie-Bohm picture illustrates rather the importance of the experimental set-up as a whole, *as insisted on by Bohr*. *The Gleason-Jauch axiom is a denial of Bohr's insight.* (Bell 1982, in Bell 2004^2, p. 165).

We have evidence, then, that Bohr has a place in the Bell line of thought about the foundations of quantum mechanics already in the early Sixties, as a forerunner of the idea of contextuality.

But let us return to what we called the Bell theory of beables, as expressed by the conditions (i) and (ii). These conditions appear far from uncontroversial, when

---

[3] For the meaning and role of contextuality in the Bohr philosophy of quantum mechanics is a relevant issue in the Bohrian scholarship: see for instance (Dieks 2017). Given that Bohr was standardly conceived as the major representative of an approach to the foundations of quantum mechanics that could not be more alien to Bell in many respects, Abner Shimony has playfully described Bell's use of the Bohr claim: "Bell, *by a judo-like manoeuvre*, cited Bohr in order to vindicate a family of hidden variables theories in which the values of observables depend not only upon the state of the system but also upon the context." (Shimony 1984, in Shimony 1993, p. 121, my emphasis).

referred to the early, Bohrian characterization of beables by Bell. If condition (i) sounds milder, since it seems to require just compatibility with common sense, condition (ii) is more puzzling. What sort of 'constitution' property is supposed to be involved in the claim that observables are 'made out of' beables? What are beables supposed to be in order to 'make up' observables? And what is the exact relation of such intuition of 'constitution' with the Bohrian view of quantum measurements? The use of Bohr *against* Copenhagen quantum mechanics would do no harm as such, but the Bell strategy is dubious by a *conceptual* point of view. I wish to argue that the Bohrian requirement to express experimental evidence in 'classical' terms, in order for linguistic communications among scientists to be consistenly preserved, can hardly be put usefully to work to provide the unambiguous description of the quantum measurement process that Bell was searching for.

The extent to which the reference to Bohr may really play the role of dissolving the ambiguity deplored by Bell is a matter of dispute, since it concerns the status of an issue that is still debated in the reconstructions of the Bohr attitude toward quantum mechanics: the issue of whether, according to Bohr, quantum mechanics should be taken as universal—i.e. applicable to *all* physical systems, including measuring instruments—or not. The problem of the universality of quantum mechanics in principle emerged since the very origins of quantum theory, due to the increasing divergence from all preceding classical physics that was apparent in the experimental development of the theory already in the first decades of the twentieth century. In the early days of the debate on the foundations of quantum mechanics, it was far from clear what the relation between the classical and the quantum regimes was supposed to be, until the mathematical treatment of the theory in the 1932 von Neumann treatise allowed physicists to put the problem in a clearer light in terms of the notorious 'measurement problem', raising for the first time the universality issue for quantum mechanics. The von Neumann treatment, and the place occupied by this problem in his first formally rigorous formulation of quantum theory, already revealed how controversial the status of measurement in quantum mechanics would have been, to the extent that the very notion of measurement would turn out to be the *locus classicus* for emphasizing the lack of consensus on the interpretation of the theory: von Neumann explicitly confronts the implications of the assumption that—in the context of a measurement of a physical quantity on a quantum system S with an apparatus A—the laws of QM govern *both* S *and* A. This view has acquired with time the status of a commonplace: 'quantum fundamentalism'—this is how, for instance, Zinkernagel (2015) calls it—is the claim that "Everything in the universe (if not the universe as a whole) is fundamentally of a quantum nature and ultimately describable in quantum–mechanical terms." In Zinkernagel's words:

> In this formulation, quantum fundamentalism contains both an ontological and an epistemological thesis: that everything is of a quantum nature is an ontological claim, whereas the idea that everything can (at least in principle) be *described* in quantum terms is epistemological. The ontological component of quantum fundamentalism can also be expressed as the idea that we live in a quantum world. (Zinkernagel 2015, p. 419, emphasis in the original).

In fact Bohr never discussed explicitly the measurement problem in the von Neumann formal context. A wide consensus was established among most Bohr scholars, however, according to which his overall philosophical outlook legitimates a *non*-universalistic reading of quantum mechanics, mainly due to the special role attributed to classical categories in accounting for the experimental evidence in quantum measurements. For instance in a recent, qualified defense of this consensus, Zinkernagel (2015) refers to a 1938 paper in which Bohr argues that.

> […] in each case some ultimate measuring instruments, like the scales and clocks which determine the frame of space-time coordination – on which, in the last resort, even the definitions of momentum and energy quantities rest – must always be described *entirely* on classical lines, and consequently kept outside the system subject to quantum mechanical treatment." (Bohr 1938, p. 104, emphasis in the original).

One can make sense of this argument, according to Zinkernagel, only under the assumption that quantum mechanics actually *fails* to be universal:

> A way to understand Bohr's requirement is that we need a reference frame to make sense of, say, the position of an electron (in order to establish with respect to what an electron has a position). And, by definition, a reference frame has a well-defined position and state of motion (momentum). Thus the reference frame is not subject to any Heisenberg uncertainty, and it is in this sense (and in this context) classical. This does not exclude that any given reference system could itself be treated quantum mechanically, but we would then need another – classically described – reference system e.g. to ascribe position (or uncertainty in position) to the former. (Zinkernagel 2015, p. 430).[4]

This view has been challenged. Already (Landsman 2007), for instance, had argued that the Bohr texts would not justify an interpretation of his thought to the effect that there exists an independent natural realm of an intrinsic classical character. Let us consider the following passage, contained in a famous Bohr paper entitled "On the notions of causality and complementarity", published in 1948 on the philosophical journal *Dialectica*:

> The construction and the functioning of all apparatus like diaphragms and shutters, serving to define geometry and timing of the experimental arrangements, or photographic plates used for recording the localization of atomic objects, will depend on properties of materials which are themselves essentially determined by the quantum of action. (Bohr 1948, p. 145).

On the basis of texts like this, Landsman claims that the division system/apparatus, in which the former is described quantum-mechanically whereas the latter is described classically, has no *ontological* import:

> there is no doubt that both Bohr and Heisenberg believed in the fundamental and universal nature of quantum mechanics, and saw the classical description of the apparatus as a purely *epistemological* move, which expressed the fact that a given quantum system is being used as a measuring device" (Landsman 2007, p. 437, emphasis added).

In a recent contribution Dieks reinforces this challenge, defending an exclusive *epistemic* reading of the role of the classical notions in the Bohr view of the quantum measurement process, denying any *ontological* quantum non-universalism by Bohr (Dieks 2017).

---

[4] A more sustained defense of this view is contained in Zinkernagel (2016).

This dispute on the ontological or epistemological flavor of the quantum/classical divide, however, leaves the ambiguity point that concerns us here untouched. We do not need to take a stance on whether the boundary between the classical and the quantum world concerns our knowledge or the ultimate structure of Nature to see that we are forced anyway, within the Heisenberg-Bohr Copenhagen framework, to acknowledge that, on one hand, we cannot but locate somewhere the infamous 'cut' between system and apparatus, and on the other hand there is no rigorous recipe even on a pragmatic level about where *exactly* we should put it. As Dieks himself remarks, in the very first section of the seminal complementarity paper published in 1927, Bohr emphasizes that.

> The circumstance [...] that in interpreting observations use has always to be made of theoretical notions entails that for every particular case *it is a question of convenience* at which point the concept of observation involving the quantum postulate with its inherent "irrationality" is brought in" (Bohr 1934, p. 54, emphasis added),

Wolfgang Pauli echoed the same point in a 1949 paper, entitled "The Philosophical Significance of the Idea of Complementarity":

> [...] modern physics generalizes the old placing in opposition of apprehending subject on one hand and object apprehended on the other to the idea of the cut between the observer or instrument of observation and the system observed. While the existence of such a cut is a necessary condition of human cognition, modern physics regards the position of the cut as to a certain extent arbitrary, and as the result of a choice partly determined by considerations of expediency, and therefore partly free. (Pauli 1950, p. 41, emphasis added).

As a consequence, the 'ambiguity' and 'approximation' of the standard formulation of quantum mechanics cannot be removed by the use of the Bohrian framework, and Bell needed to say (and later did say) more to characterize the kind of solution he envisioned. In particular, the Bohrian model of the quantum measurement may at most satisfy the Bell condition (i), namely, the 'functionalistic' recovery of subjective experience, but fails to satisfy unambiguously the 'constitutive' Bell condition (ii), since the concrete individuation of the relevant beables depends on *arbitrary* criteria: with the resources allowed by the Bohr framework, quantum observables simply *cannot* be 'made out' of beables.

3 Beyond Bohr: The New Life of Beables

In the first appearance of the notion of beable, the early Bell move—use Bohr *against* Copenhagen quantum mechanics—looks then rather unfortunate. But the role that we have analyzed in the previous section starts to be replaced in the subsequent development of the notion itself. For Bell returns to beables in a 1975 paper, whose title ("The theory of local beables") this time mentions explicitly the need for a *theory* of these 'objects', whatever they are meant to be. At first sight, the very opening of the paper is in line with the Bohrian attitude we have alluded to:

> This is a pretentious name for a theory which hardly exists otherwise, but which ought to exist. The name is deliberately modelled on 'the algebra of local observables'. This terminology, *be*-able as against *observ*able, is not designed to frighten with metaphysic those dedicated to realphysic. It is chosen rather to help in making explicit some notions already implicit in, and basic to ordinary quantum theory. For, in the words of Bohr, ' it is decisive to recognize that, however far the phenomena transcend the scope of classical physical explanation, the account of all evidence must be expressed in classical terms.' It is the ambition of the theory of local beables to bring these 'classical terms' into the equations, and not relegate them entirely to the surrounding talk. (Bell 1975, in Bell 2004[2], p. 52).

In clarifying what beables are supposed, or meant, to be, Bell refers again to macroscopic pieces of experimental settings in a broad sense—and this is, once again, entirely Bohrian in spirit—but, *this time*, he expresses explicitly the need for a clear *theory* of them, in terms of a more robust sense of physical reality:

> The beables must include the settings of switches and knobs on experimental equipments, the current in coils, and the readings of instruments. 'Observables' must be *made*, somehow, out of beables. The theory of local beables should contain, and give precise physical meaning to, the algebra of local observables. (Bell 1975, in Bell 2004[2], p. 52).

This appears to be a turning point in the Bell characterization of beables. Not only Bell refers to the difference in electromagnetism between 'physical' entities (like the electric and magnetic fields) and 'unphysical' entities (like potentials), in order to set up a distinction according to which beables should be clearly located on the 'physical' side. He also points here to what we have called above a condition of 'constitution', a more fundamental status that beables should be endowed with: it is this status that in principle justifies the observables being *made out* of beables. This conjunction of realism—beables *are out there*—and constitution—beables are what *make up* observables and all that gravitates around observation—characterizes the new Bell theory of beables, and his later paper "Beables for quantum field theory" (1984) testifies it:

> There is nothing in the mathematics to tell what is 'system' and what is 'apparatus', nothing to tell which natural processes have the special status of 'measurements'. Discretion and good taste, born of experience, allow us to use quantum theory with marvelous success, despite the ambiguity of the concepts named above in quotation marks. But it seems clear that in a serious fundamental formulation such concepts must be excluded. In particular we will exclude the notion of 'observable' in favour of that of '*be*able'. The beables of the theory are those elements which might correspond to elements of reality, to things which exist. Their existence does not depend on 'observation'. Indeed observation and observers must be made out of beables. (Bell 1984, in Bell 1987, p. 174).

That beables should correspond "to elements of reality, to things which exist" might still sound compatible with the Bell early, Bohrian-sounding formulation that we analyzed in the previous section, but clearly this is not the case with the claim that the existence of beables *does not depend on 'observation'*: in Bohrian terms, on the contrary, it is exactly the reference to the context of observation that allows macroscopic pieces of experimental settings (namely, what Bell takes as beables in his early formulation) to be part of a scientifically meaningful experience.

In connection with this emphasis *both* on the 'reality' of beables and their 'constitutive' nature, Bell introduces for the first time a connection with an intuitive sense of *locality*, called here *local causality*[5]:

> We will be particularly concerned with *local* beables, those which (unlike the total energy) can be assigned to some bounded space-time region. [...] It is in terms of local beables that we can hope to formulate some notion of local causality. (Bell 1975, in Bell 2004[2], p. 53, emphasis in the original).

It is *this* focus on locality—I argue—that determines a new twist for the formulation of a theory of beables, a formulation which starts to diverge from the Bohrian-sounding notion reviewed in the previous section and receives a more distinctive 'fundamental' status in somewhat ontological terms. Bell attempts to figure out a definition of local causality that can work also in an indeterministic setting, an attempt that leads him to introduce an expression like $\{A|\Lambda\}$, that stands for the probability of a particular value A, given particular values Λ (Bell 1975, in Bell 2004[2], p. 54). An interesting point to note here is that, in introducing this expression, Bell employs the term 'beable' to denote a *value* (of a physical quantity), something very different from "settings of switches and knobs on experimental equipments", which was the original, Bohrian-sounding meaning attached to the term. On this new background Bell operates in a much more explicitly 'realistic' (and much less 'Bohrian') vein—a background in which it is perfectly sensible to conceive an observer-independent world whose unveiling is a major task for fundamental physics—and the new reading of beables in terms of values is immediately put to work in an EPR-kind of context:

> Let A be localized in a space-time region 1. Let B be a second beable localized in a second region 2 separated from 1 in a spacelike way. Now my intuitive notion of local causality is that events in 2 should not be 'causes' of events in 1, and viceversa. But this does not mean that the two sets of events should be uncorrelated, for they could have common causes in the overlap of their backward light cones. It is perfectly intelligible then that if Λ in 1 does not contain a complete record of events in that overlap, it can be usefully supplemented by information from region 2. So in general it is expected that
>
> $$\{A|\Lambda, B\} \neq \{A|\Lambda\}$$
>
> However, in the particular case that Λ contains already a *complete* specification of beables in the overlap of the two light cones, supplementary information from region 2 could reasonably be expected to be redundant. (Bell 1975, in Bell 2004[2], p. 54, emphasis in the original).

It is quite clear, then, that the above mentioned specification of beables makes sense in the Bell second, ontologically-loaded formulation of the notion of beable (and not in the old, Bohrian-sounding one). Moreover, this new formulation is immediately put to work in the investigation on whether, in the Bell language, quantum mechanics might be shown to be 'locally causal' if reformulated as a sub-theory of a 'more complete' theory:

[5] As already remarked by others, this expression is likely to be misleading in suggesting that the influence at stake *should* have a direction, which in fact is not necessarily the case.

> But could it not be that quantum mechanics is a fragment of a more complete theory, in which there are other ways of using the given beables, or in which there are additional beables – hitherto 'hidden' beables? And could it not be that this more complete theory has local causality? Quantum mechanical predictions would then apply not to given values of all the beables, but to some probability distribution over them, in which the beables recognized as relevant by quantum mechanics are held fixed. We will investigate this question, and answer it in the negative. (Bell 1975, in Bell 2004^2, p. 55).

Thus, the notion of beable (in his second, mature sense) appears to have been a major factor for motivating the development of the PO research program in the foundations of QM. After the initial proposal by Detlef Dürr, Shelly Goldstein and Nino Zanghì, in more recent times the scientific literature on the evaluation of the PO approach has been growing significantly in quantity and depth (Allori 2013, 2015). Our aim here was just to provide an attempt of reconstructing the Bell own conceptual evolution on the notion of beable over the years: an interesting, open question is to investigate how the Bell theory of beables fares with respect to some recent claims concerning the relationship between beables and the PO approach, and whether the evolution from the first to the second formulation of the Bell theory of beables can shed some light on the study of this relationship.

References

V. Allori, Primitive ontology and the structure of fundamental physical theories, in *The Wave Function: Essays on the Metaphysics of Quantum Mechanics*. ed. by D.Z. Albert, A. Ney (Oxford University Press, Oxford, 2013), pp.58–75

V. Allori, Primitive ontology in a nutshell. Int. J. Quantum Found. **1**, 107–122 (2015)

J.S. Bell, On the problem of hidden variables in quantum mechanics. Rev. Mod. Phys. **38**, 447–452 (1966), reprinted in Bell 2004^2, pp. 1–13.

J.S. Bell, Introduction to the hidden-variable question, in *Foundations of Quantum Mechanics*, ed. by B. D'Espagnat (Academic Press, New York, 1971), pp. 171–181, reprinted in Bell 2004^2, pp. 29–39.

J.S. Bell, Subject and object, in *The Physicist's Conception of Nature*, ed. by J. Mehra (Reidel, Dordrecht, 1973), pp. 697–690, reprinted in Bell 2004^2, pp. 40–44.

J.S. Bell, The theory of local beables, TH-2053-CERN (1975), reprinted in Bell 2004^2, pp. 52–62

J.S. Bell, The measurement theory of Everett and de Broglie's pilot wave, in *Quantum Mechanics, Determinism, Causality and Particles*, ed. by M. Flato et al. (Reidel, Dordrecht, 1976), pp. 11–17, reprinted in Bell 2004^2, pp. 93–99.

J.S. Bell, On the impossible pilot wave, *Foundations of Physics*, vol. 12, pp. 989–999 (1982), reprinted in Bell 2004^2, pp. 159–168.

J.S. Bell, Beables for quantum field theory, CERN-TH 4035/84 (1984), reprinted in Bell 2004^2, pp. 173–180.

J.S. Bell, Are there quantum jumps?, in *Schrödinger: Centenary Celebration of a Polymath*, ed. by C.W. Kilmister (Cambridge University Press, Cambridge, 1987), pp. 41–52, reprinted in Bell 2004^2, pp. 201–212.

J.S. Bell, *Speakable and Unspeakable in Quantum Mechanics* (Cambridge University Press, Cambridge, 2004^2)

N. Bohr, *Atomic theory and the description of nature* (Cambridge University Press, Cambridge, 1934)

N. Bohr, The causality problem in atomic physics, in *New Theories in Physics*, International Institute of Intellectual Cooperation, Paris, pp. 11–45 (1938) (reprinted in ed. by J. Faye, H.J. Folse, *The Philosophical Writings of Niels Bohr*, *Vol. IV: Causality and Complementarity,* (Ox Bow, Woodbridge, 1998), pp. 94–121).

N. Bohr, On the notions of causality and complementarity, *Dialectica* 2, pp. 312–319 (1948) (reprinted in ed. by J. Faye, H.J. Folse, *The Philosophical Writings of Niels Bohr*, *Vol. IV: Causality and Complementarity* (Ox Bow, Woodbridge, 1998), pp. 141–148).

N. Bohr, Discussion with Einstein on the epistemological problems in atomic physics, in *Albert Einstein Philosopher-Scientist*, ed. by P.A. Schilpp (Harper & Torchbooks, New York, 1949), pp. 199–243

D. Dieks, Niels Bohr and the formalism of quantum mechanics, in 2017, *Niels Bohr and the Philosophy of Physics. Twenty-First Century Perspectives*, ed. by J. Faye, H.J. Folse (Bloomsbury, London-New York, 2017), pp. 303–334.

D. Dürr, S. Goldstein, N. Zanghì, Quantum equilibrium and the origin of absolute uncertainty. J. Stat. Phys. **67**, 843–907 (1992)

N.P. Landsman, Between classical and quantum, in *Handbook of the Philosophy of Science. Philosophy of Physics*, ed. by J. Butterfield, J. Earman (Elsevier, Amsterdam, 2007), pp. 417–553.

F. Laudisa, How and why did locality become 'local realism'? A historical and critical analysis (1963–1978). Stud. Hist. Philos. Sci. **97**, 44–57 (2023)

W. Pauli, Die philosophische Bedeutung der Idee der Komplementarität, in *Experientia*, vol. 6, p. 72–75 (1950) (Engl. ed. The philosophical significance of the idea of complementarity, in ed. by W. Pauli, *Writings on Physics and Philosophy*, ed. by C.P. Enz, K. von Meyenn) (Springer, Berlin 1994), pp. 35–42 (page references to the English edition)

A. Shimony, Contextual hidden variables theories and Bell's inequalities. *British Journal for the Philosophy of Science* 35, pp. 25–45 (reprinted in *Search for a Naturalistic World View. Natural Science and Metaphysics*, vol. II (Cambridge University Press, Cambridge, 1993, 1984), pp. 104–129

A. Whitaker, *John Bell and Twentieth-Century Physics* (Oxford University Press, Oxford, Vision and Integrity, 2016)

H. Zinkernagel, Are we living in a quantum world? Bohr and quantum fundamentalism, in *One Hundred Years of the Bohr Atom*, *Proceedings from a Conference. Scientia Danica. Series M: Mathematica et Physica*, ed. by F. Aaserud & H. Kragh, vol. 1 (Royal Danish Academy of Sciences and Letters, Copenhagen, 2015), pp. 419–434.

H. Zinkernagel, Niels Bohr on the wave function and the classical/quantum divide. Stud. Hist. Philos. Mod. Phys. **53**, 9–19 (2016)