Why the Book of Nature is Written in the Language of Mathematics

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Abstract The essay traces the following idea from the presocratic philosopher Heraclitus, to the Pythagoreans, to Newton's *Principia*: Laws of nature are laws of proportion for matter in motion. Proportions are expressed by numbers or, as the essay proposes, even identical to real numbers. It is argued that this view is still relevant to modern physics and helps us understand why physical laws are mathematical.

1 The "Unreasonable" Effectiveness of Mathematics

Why is mathematics so successful in describing the natural world? More profoundly, why are the fundamental laws of nature–as far as we know them today–expressed in mathematical language?

The puzzle can present itself in different ways, depending on what one takes mathematics to be. If one believes that abstract mathematical objects or structures exist in some Platonic heaven, one may wonder why they should have anything to do with the physical world and how we, as material beings in space and time, are able to acquire knowledge of them. With such questions in mind, some authors have gone as far as to suggest that the universe we live in is itself mathematical (Tegmark (2014); see also Tumulka (2017)).

If one believes that mathematics is a human invention, one must marvel at the confluence of human genius and nature's kindness that makes it so successful. One may try to deflate the "unreasonable effectiveness of mathematics" (Wigner 1960) by attributing some of it to selection bias (Wenmackers 2016), pointing to pieces of mathematics that, so far, have no use in natural science. One may also argue that our cognitive apparatus, which allowed us to invent mathematics, is the product of natural evolution and therefore well-adapted to the world (as if the traits that prevented our ancestors from being eaten by a tiger would naturally lead to the invention of complex analysis). But none of these arguments explain why the language we have been successful with is precisely that of mathematics rather than, say, biblical Hebrew or

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instructions for a Turing machine. And at the end of the day, they do little to address Wigner's sentiment that "[t]he miracle of the appropriateness of the language of mathematics for the formulation of the laws of physics is a wonderful gift which we neither understand nor deserve" (1960, p.14).

2 On the Rationality of the Cosmos in Presocratic Philosophy

To understand what mathematics has to do with natural laws–not just in practice, but in principle–it helps to go back to a time when the idea of a lawful cosmos awakened; the time of the Presocratic natural philosophers around the 6th and 5th century BCE. We must imagine an intellectual period marked by a profound insight: that we are living in a *cosmos* (lit. *order*), that the world is organized according to rational principles, and that the human intellect has, at least in principle, access to them. In short, it is a period animated by the idea that the world is comprehensible.

2.1 Parmenides

We have to start with Parmenides, the great ontologist, because much of the philosophy of the following centuries unfolds in the dialectic that he begins. Parmenides teaches, nay, proves, that What Is (*to eon*) must be uncreated, unchanging, and indivisible–one eternal whole:

> One path only is left for us to speak of, namely, that It is. In this path are very many tokens that what is is uncreated and indestructible; for it is complete, immovable, and without end. Nor was it ever, nor will it be; for now it is, all at once, a continuous one. For what kind of origin for it wilt thou look for? In what way and from what source could it have drawn its increase? ... I shall not let thee say nor think that it came from what is not; for it can neither be thought nor uttered that anything is not. (Poem of Parmenides; fr. 28 B8.1-13 DK)[1]

Recognizing What Is is the Way of Truth (*alêtheia*). It is not the world presented to us by our senses but something accessible by rational thought. Indeed, "it is the same thing that can be thought and that can be" (fr. 28 B3.1 DK).

Parmenides was also a great natural philosopher. "A whole series of important astronomical discoveries is credited to him: that the morning star and the evening star are one and the same; that the earth has the shape of a sphere ... that the phases of the moon are due to the changing way in which the illuminated half-sphere of the moon is seen from the earth" (Popper, 1992, p. 14). But as Popper argues, these discoveries–in particular, that the moon merely *appears* to be changing–only contribute to his mistrust of the senses. They pertain to the Way of *doxa*, of human beliefs or seemings, not true knowledge of What Is.

[1] Unless stated otherwise, Presocratic fragments are quoted in the translation by Burnet (1920).

It remains unclear how the two relate to one another. Parmenides' rationalism goes so far that, on his Way of Truth, little attempt is made to save the phenomena. Something about the holistic *Being* has to give if it is supposed to explain the cosmos we experience.

2.2 Anaxagoras

In response to Parmenides, Anaxagoras separates mind and matter, leaving a cosmic intelligence–the *Nous*–as a moving principle to act upon the material world. The *Nous* causes change and diversification by creating a cosmic vortex through which matter begins to separate into its constituent elements. *Nous* is also in *us*, as our minds that control our bodies. The implication is that we can understand the world because we share in the cosmic intelligence that shapes it. The testimony of the senses is not entirely dismissed, but its tentative character is expressed in the doctrine that "appearances are a sight of the unseen" (fr. 59 B21a DK). True knowledge requires the refinement of sense experience by rational thought.

It remains unclear how to understand the *Nous* when it comes to the subjective or individual aspects of mind, what we might call consciousness or, less anachronistically, soul (*psyche*). While the Presocratics don't always get a fair shake from Aristotle, his criticism of Anaxagoras as conflating mind and soul (*De anima* 1.2) seems pertinent.

2.3 Heraclitus

Heraclitus, "the Dark One," is very clear on one point, that he speaks about something which is *common to all* (frr. 22 B2, B80, B89, B113, B114 DK). For example:

> The waking have one common world, but the sleeping turn aside each into a world of his own. (B89 DK)

Erwin Schrödinger sees therein the idea of an external reality emerging "from the fact that part of our sensations and experiences overlap" (2014, p. 73). We can put it in a different way. While Anaxagoras separates the all-encompassing BEING of Parmenides into matter and mind, Heraclitus splits off the cognizing subject, leaving an external world as the object of cognition (cf. Dürr and Lazarovici (2012)).

Common to all is also the *logos*, the ordering and unifying principle of the world. Since recovering from the influence of Hegel, it has become widely accepted that cosmology, not logic or dialectic, is the right starting point for understanding this central concept of Heraclitean philosophy (Kurtz 1971). *Logos* does not rule some abstract realm of thought; it rules the universe we all inhabit. We may start with fragments like the following:

> This world order (*kosmos*), the same for all, none of the gods or humans made it, but it always was and is and will be: fire ever-living, kindled in measures (*métra*) and extinguished in measures. (B30 DK; translated by Laks and Most (2016))

Heraclitus is sometimes presented as the great antagonist of Parmenides because the reality he describes appears like the opposite of static being. It is a world in flux, an endless process of *becoming*, opposites united in a ceaseless cycle of transmutation. And yet, in this flow of change, Heraclitus recognizes something constant, something that manifests order and reflects the underlying *logos*. Fire, which Heraclitus takes to be the most fundamental element, transforms *in measures*, that is, in certain regular *proportions*:

> Turnings of fire: first sea; then half of the sea, earth; and the other half, lightning storm. [...] It spreads out as sea and its measure reaches the same *logos* as it was before it became earth. (B31 DK; translated by Laks and Most (2016))

If one wants to settle on a translation for "logos," the best fit here is indeed *proportion* (Kurtz 1971). Compared to the *Nous* of Anaxagoras, the Heraclitean *logos* is a more abstract and impersonal concept, coming closer to that of *natural law*.

The last quote is one of the notoriously obscure fragments of Heraclitus, whose precise meaning is hard to reconstruct. The meaning of "lightning storm" (*prêstêr*) is disputed–is it a form of fire, or a fourth element, viz. air? Also ambiguous is the subject of the second sentence and hence what transformation it describes (maybe of water back into fire; almost certainly, Heraclitus describes a kind of cycle process).[2] These issues notwithstanding, it seems clear enough that the fragment expresses a law of the form *water : earth = water : storm*, and presumably also *fire : water = water : earth*.

3 Mathematical Interlude

It may not be obvious to us today that the term "measures" already points to something mathematical. Perhaps we need a definition:

Definition 1 Two magnitudes A and B of the same kind, are *commensurable* if there exists a third magnitude ϵ and natural numbers n, m such that $A = n \cdot \epsilon$ and $B = m \cdot \epsilon$. In this case, ϵ is a *measure* of A and B and the *ratio* $A : B$ corresponds to $n : m$.

It must be emphasized that magnitudes are not numbers, but physical or geometrical quantities (lengths, areas, masses, etc.). Only the *ratio* of two commensurable magnitudes corresponds to the ratio of two numbers–or what we now recognize as a (rational) number in its own right. It is important to keep this in mind, especially when we talk about the Pythagoreans, because the above definition anticipates a fundamental motive of their science and philosophy.

[2] On these questions, see, e.g., Kurtz (1971); Jones (1972); Schadewaldt (1978); Kirk et al. (1983).

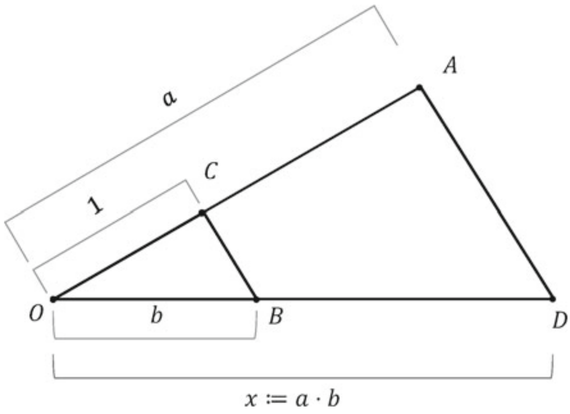

Fig. 1 Descartes' construction of the product of $a = \overline{OA}$ and $b = \overline{OB}$. The segment $\overline{OC}$ is (arbitrarily) chosen as unity. By the intercept theorem, the constructed $x = \overline{OD}$ satisfies $a : 1 = x : b$

Magnitudes by themselves have only some of the structure of numbers (see Maudlin (2014, pp. 9–25)). Two magnitudes of the same kind can be added and subtracted but they cannot be multiplied or divided (to yield a third magnitude of the same kind). A workaround, at least for line segments, is introduced much later in Descartes' *La Géométrie* (1637) and requires some arbitrary length to be designated as unity (see Fig. 1). This allowed for the very powerful algebraization of geometrical problems that paid off immediately with a precise characterization of (im)possible constructions with compass and straightedge.

But the Cartesian solution is very non-Pythagorean and indeed nonsensical from a strictly geometric point of view. It corresponds to defining the product of 2m and 3m as 6m, when we would could have just as well chosen a different unit, say cm, and multiplied the same two lengths to 60000.

It is rarely noticed that we are committing the same sin when we represent numbers as points on the number line. There is nothing numerical about a linear continuum *per se*. An arbitrary segment must be designated as a unit length, say between two points marked "0" and "1". Only relative to the scale thus introduced can we say that points (or their distances from 0) correspond to numbers.

4 Pythagoreanism and Platonism

4.1 Plato

We saw that Heraclitus, in his cosmological fragments, describes the *logos* as a law of proportion for the transformations of elements. It is this *logos* that unifies the different elements in cycles of change. While the context differs, we find the same kind of calculation in the creationist cosmogony of Plato's *Timaeus*:

> God in the beginning of creation made the body of the universe to consist of fire and earth. But two things cannot be rightly put together without a third; there must be some bond of union between them. And the fairest bond is that which makes the most complete fusion of itself and the things which it combines; and proportion is best adapted to effect such a union. [...] God placed water and air in the mean between fire and earth, and made them to have the same proportion so far as was possible (as fire is to air so is air to water, and as air is to water so is water to earth); and thus he bound and put together a visible and tangible heaven. And for these reasons, and out of such elements which are in number four, the body of the world was created, and it was harmonized by proportion [...]. (Tim. 31b–32c; translated by Jowett (1892))

Plato also makes explicit what we can only surmise for Heraclitus, that numbers (expressing proportions) are a reflection of the eternal in a world in motion:

> Now the nature of the ideal being was everlasting, but to bestow this attribute in its fullness upon a creature was impossible. Wherefore he [the creator] resolved to have a moving image of eternity, and when he set in order the heaven, he made this image eternal but moving according to number, while eternity itself rests in unity; and this image we call time. (37d)

True knowledge is knowledge of the eternal forms. In the world of change, we can only deal in likelihood. This epistemological principle is itself expressed as a law of proportion: "As being is to becoming, so is truth to belief." (*Tim.* 29c; cf. *Rep.* VII 534a). The genesis of the soul explains the possibility of knowledge. It was created out of the divisible and material on the one hand and the indivisible and unchangeable on the other, and therefore partakes of the nature of both. It is noteworthy that soul and number are ascribed a similar status as intermediates between the physical world and the realm of the eternal (cf. Plato's analogy of the divided line in *Rep.* VI 509d–511e).

4.2 The Pythagoreans

Between Heraclitus and Plato, we have the Pythagoreans, and among them a group known as the *mathēmatikoi*.[3] They developed four sciences or *mathemata*, which would come to form the classical *quadrivium* of education: arithmetic, geometry, astronomy, and music (or harmonics).

The study of musical harmony began with the observation that the simultaneous striking of different chords produces consonance when the cord lengths stand in certain ratios: 2:1 (the octave), 3:2 (the perfect fifth), 4:3 (the perfect fourth), etc. It later turned into a more axiomatic science of harmonic proportions and the musical scale. The Pythagorean astronomers recognized the same harmonic proportions in

[3] The Pythagorean influence on Plato is undeniable (the Platonic character Timaeus is commonly identified as a Pythagorean). Placing Heraclitus in the same lineage is more contentious. Plato criticizes Heraclitus on the basis that if everything were in flux, truth and knowledge would not be possible (*Cratylus* 402a ff.). Heraclitus calls Pythagoras an "imposter" (fr. B129 DK), someone who has studied many things but lacks understanding (B40 DK). Heraclitus was not an easy fellow. Nonetheless, a reconciliation of these great thinkers is not only possible but plausible, and I set forth the connections as they seem correct to me.

the motions of celestial bodies, postulating that the sun, the moon, and the planets (including Earth) move uniformly in circular orbits around a "central fire". The idea of a "music of the spheres" would culminate 2000 years later in Kepler's *Harmonice Mundi* (1619). Geometry was the study of proportions in their purest form, the discovery of mathematical laws in the relations of lengths, areas, and angles. The Pythagorean theorem is just the most obvious example.

The idea of Pythagoreanism as holding that all things are literally made out of numbers is a caricature based on the school's mystical currents. Undoubtedly, though, Number was considered divine, the universal principle behind harmony, rationality, and beauty in the skies and on Earth. Only through Number is it possible to understand the cosmos:

> Indeed, it is the nature of Number which teaches us all things which would otherwise remain impenetrable and unknown to every man. For there is nobody who could get a clear notion about things in themselves, nor in their relations, if there was no Number or Number-essence. By means of sensation, Number instills a certain proportion, and thereby establishes among all things harmonic relations [...]; it incorporates intelligible reasons of things, separates them, individualizes them, both in limited and unlimited things. (Philolaus, fr. B11 DK, cited by Guthrie and Fideler (1987))

4.3 The Discovery of Incommensurability

If one can still sense how sublime and fulfilling the Pythagorean worldview must have seemed to believers, it helps to understand the shock caused by the discovery of *incommensurability*. The Pythagoreans had an algorithm–today, we call it the Euclidean algorithm–to find the greatest common measure of two like magnitudes. Subtract the smaller magnitude as often as possible from the greater, and then the remainder from the smaller, and so on. *Hippasus* is usually credited with the discovery that, for certain line segments–such as the diagonal and side of a square or a regular pentagon–this algorithm never terminates (see Fig. 2 below). The discovery of incommensurability thus also marks the beginning of the mathematical struggle with infinity. Legend has it that Hippasus was drowned at sea as punishment for his blasphemy (fr. 18 A4 DK).

Euclid (Book X, Def. 1.3) and before him Plato (*Rep.* VII 534d, VIII 546c) already refer to incommensurable line segments as *irrational*,[4] Plato in a way that suggests the term had been established before, maybe by the Pythagoreans themselves. It is still a big conceptual leap from here to understanding the proportions of incommensurable magnitudes as (irrational) *numbers*, but the step seems almost inevitable.

The Pythagoreans had more immediate concerns. They had to save their sciences, in particular geometry, whose arithmetic foundation crumbled with Hippasus' discovery of incommensurability. A fundamental question that arose is what it means for different magnitudes to stand in the same proportion if this proportion no longer

[4] *árritos*, which translates more literally to *ineffable* or *inexpressible*; also *alogon*.

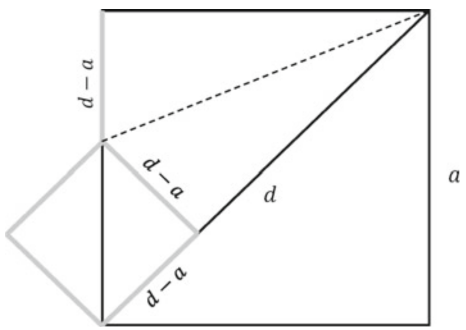

Fig. 2 Euclidean algorithm for the side and diagonal of a square. Trying to find a common measure leads to an infinite regress. In the next step, we have to subtract the side of the new square ($a_2 = d - a$) from its diagonal ($d_2 = 2a - d$)

corresponds to a ratio of natural numbers. In other words, one needs an identity criterion for proportions that applies also in the incommensurable case. The following solution is attributed to Eudoxus of Cnidus, a student of Plato. It provides the basis for the theory of proportions presented in Book V of Euclid's *Elements*.

Definition 2 Let A, B and C, D be magnitudes of the same kind. The ratio $A : B$ is equal to $C : D$ if for all natural numbers n, m one of the following three cases holds:

$$\begin{aligned} m \cdot A < n \cdot B &\text{ and } m \cdot C < n \cdot D \\ m \cdot A = n \cdot B &\text{ and } m \cdot C = n \cdot D \\ m \cdot A > n \cdot B &\text{ and } m \cdot C > n \cdot D \end{aligned}$$

The second case can only occur for commensurable magnitudes (for which then $A : B = C : D = n : m$). But if we take the step of recognizing rational numbers (and thus license writing $\frac{n}{m}$), we can see from Eudoxus' definition that any proportion *partitions* the rationals such that either $\frac{n}{m} < A : B$ or $\frac{n}{m} \geq A : B$. This is precisely the idea behind Richard Dedekind's construction of the real numbers, though the fact that it took 2000 years (and the invention of set theory) to carry it out shows the magnitude of the achievement. While Dedekind (1872) makes a point of looking for arithmetic as opposed to geometric principles for the continuum, it is straightforward to translate his account into a completion of Eudoxus' theory of proportions with all the structure of the real numbers. For instance, multiplication: Given two proportions $A : B$ and $C : D$, their product is the smallest proportion $E : F$ such that, for all $k, l, m, n \in \mathbb{N}$, $nB < mA$ and $kD < lC$ implies $(lm)E \leq (kn)F$. Non-positive numbers can be included by admitting magnitudes of positive or negative orientation.

With the discovery of incommensurability, we lose the crutch of saying that the ratio of two magnitudes *is like* the ratio of two (natural) numbers. Instead, we are led to recognize proportions as numbers in their own right–those forming the continuum of reals.

4.4 The Birth of Modern Physics

The Pythagorean influence was still very present at the time of the scientific revolution, where it combined with the right amount of empirical methodology (not too little, but also not too much). Modern physics was born with the discovery of mathematical laws that are laws of proportion for the motion of matter.

Galileo performed his acceleration experiments and reported that "the spaces traversed were to each other as the squares of the times" (1638/1954, p. 179). Expressed here is not the algebraic formula $s = \frac{1}{2}at^2$, which relates dimensionful quantities on both sides, but the fact that, for pairs (s_1, t_1), (s_2, t_2) of times and corresponding distances, s_1 and s_2 have the same ratio as the squares of t_1 and t_2 (see Thm. II, Prop. II on naturally accelerated motion in the *Discorsi*). That Galileo thought geometrically is also evident in the famous passage from *Il Saggiatore* (1623) that inspired the title of this essay:

> Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth. (Quoted from Drake (1957, p. 238))

Around the same time, Kepler combined Pythagorean ideas with Copernican heliocentrism and found his *harmonic law* for planetary motion: *The square of the orbital period is proportional to the cube of the semi-major axis of its orbit.* Two generations later, Newton proved that this law follows from a centripetal force inversely proportional to the squares of the distances (Prop. XV, Thm. VII in the *Principia*). In the *Principia*, one still looks in vain for differential equations or even the famous formula $F = \frac{GmM}{r^2}$. Classical mechanics is developed geometrically, including "the method of the first and last ratios of quantities" introduced to apply results of Euclidean geometry to curve segments as they become vanishingly small.[5]

5 Why Laws of Nature are Mathematical

In the preface to the first edition of the *Principia*, Newton made explicit how he saw the relationship between mathematics and natural philosophy in the task of "reduc[ing] the phenomena of nature to mathematical laws" (Newton 1687/1999, p. 381). The practical side of mechanics involves the manual art of measuring magnitudes and carrying out geometrical constructions. "[G]eometry is founded on mechanical practice and is nothing other than that part of universal mechanics which reduces the art of measuring to exact propositions and demonstrations"

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[5] Although Newton had developed a more abstract differential calculus in his *Method of Fluxions* (completed 1671, but not published until 1736), it was not used in the *Principia* (first published 1687).

(p. 382). In essence, mechanics, as an empirical science, falls short of exact geometry only through practical limitations, particularly the inaccuracies of measurements.

It would be an overstatement to call Newton a Pythagorean. But he is part and pinnacle of a long tradition of thought that recognizes geometry–in the sense of the rational investigation of relations between magnitudes–as the nexus between physics and mathematics. The understanding we can gather from this tradition is that the appropriacy of mathematics for the formulation of the laws of nature is neither accidental nor merely a matter of convenience. There is something genuinely mathematical about the very concept of natural laws.

Why are the laws of physics mathematical? Because physics is the science of matter in motion. Regularities of motion manifest themselves in proportions of times, distances, and other geometric or perhaps kinematic quantities. Proportions are numbers. And numbers are mathematical.

I believe this answer is still relevant today, as our physics and mathematics have become so much more sophisticated. A physical theory can involve whatever kind of abstract calculus and higher-order mathematical structures we need. At the end of the day, the theory must link up to *matter in motion*, and this is where mathematics meets the physical world, both conceptually and metaphysically. This presupposes, however, two things that can no longer be taken for granted in contemporary physics: the laws must be mathematically consistent and precise. And the theory must postulate a clear ontology of matter as that to which the mathematical formalism ultimately refers.[6]

There would thus be another story to tell about how the Pythagorean understanding has been lost in more recent times; perhaps completely when Bohr declared that the formalism of quantum mechanics "represents a purely symbolic scheme" (in Schilpp (1949, p.110)). What a fall from grace for *theory*, from a vision of the divine *logos* to a meaningless manipulation of symbols that refers to nothing in the world. But I'll leave this tragedy for another time.

5.1 Numbers as Proportions

When I say that (real) numbers are proportions, I mean that they are relations between magnitudes. Magnitudes themselves are not numerical (only relative to a chosen unit of measurement) and include spatiotemporal relations as well as concrete physical properties. The metaphysical details of this proposal remain to be spelled out elsewhere.[7] Here, I want to make the point that the understanding of numbers as proportions (rather than abstract objects of set theory) narrows the gap between what we now call Platonism and nominalism.

[6] Ideally, it needs what Dürr, Goldstein, and Zanghì (1992) named *primitive ontology* (see Lazarovici and Reichert (2022) for a recent discussion) or what John Bell (2004, Chap. 7) called *local beables*.

[7] I will also not discuss the ontological status of other mathematical objects. Both a selective realism and full-blown Platonism are consistent with the view I propose in regard to numbers.

The ratio of the diagonal to the side of a square is $\sqrt{2}$, as is the ratio of the sides of two squares where the first has twice the surface area of the second. These are true identity statements, necessarily and a priori. Numbers are universals transcending their various instantiations since everything that is particular to given lengths or areas or other magnitudes quite literally cancels out when we consider their proportions. This is why the Pythagoreans insisted, as Proclus reports, that "numbers are purer and more immaterial than magnitudes" and appear "to every mind as one and not many, and as free of any extraneous figure or form" (1992, p. 78).

On the other hand, if space and time are actually continua, all real numbers are instantiated in the physical world–in space-time itself and (if this is still too abstract) in the motions of material entities. This requires less than a metric structure since we don't need absolute distances. In fact, the nominalist program of Hartry Field (*Science without Numbers*, 2016), which builds on Hilbert's axiomatization of Euclidean geometry, can be read as an exploration of how far one can get with only intrinsic structure, such as relations of congruence. I just don't think it thereby "eliminates" numbers in any metaphysically interesting sense. One can debate the question of ontological priority (if one is so inclined). But if, say, a circle exists in your universe, then the number π exists as well.

5.2 Conclusion

I am certainly not advocating a return to the mathematics of the early 18th century or dismissing the awesome progress we have made ever since. We have explored so much more of the mathematical universe, set our inquiries on solid logical foundations, and developed powerful concepts and mathematical methods without which modern science and technology would not be possible. We have gained tremendous knowledge, but we have also lost some of the wisdom of past giants.

It is easy to get lost in formalism and mathematical abstraction, to the point that it seems a great mystery how any of it could have anything to do with the natural world, let alone a *logos* that is not of our own making. This often combines with a tendency to make us humans both too small and too large at the same time: it seems inconceivable that we could have insight into either mathematical truth or the laws of nature unless we are somehow the engineers of both. Detlef Dürr strictly rejected such thinking. For him, the purpose of doing science was not only to understand the cosmos, but also to recognize our proper place in it.

I believe–and this is one of the many insights I owe to Detlef–that the understanding of numbers as proportions is at least the beginning of an answer to why the laws of nature are mathematical. The more profound mystery is why laws of nature exist in the first place; what explains the very rationality and comprehensibility of the universe. With this, I leave the final word to my teacher:

> What is the origin of physical law? We could answer: there is no origin; it is a brute fact that everything can be described by a law, and in the end, it is our human law because our senses experience regularities. And we are looking for a code to describe these experiences. And

> mathematics is a good code that we have developed in a process of trial and error. This is wrong. We are not working like that, at least not as physicists. If we did, we would pile up all kinds of mathematical garbage just as moles pile up mounds of earth. Galilei didn't do that, Newton didn't do that, and least of all Einstein. To better understand why this idea is wrong, you must understand the mathematical formulation of the law, or rather of the laws that we have discovered so far. It is not a summary of our observations that all bodies fall to the ground; it is not said that some bodies do this and others do that; it is not a bookkeeper's order that we write down. We are looking purposefully for the underlying law of everything. We are guided by ideas of beauty, simplicity, elegance that the law should satisfy, and with these categories, we are successful. There is no good explanation for our successes [...].
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> — Detlef Dürr (2007): *Was heißt und zu welchem Ende studiert man Physik?*[8]

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