Selection of an Optimum Axially Compressed Closed Cross-Section Thin-Walled Built-Up Column

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Abstract. This article focuses on the analysis of axially compressed thin-walled built-up closed cross-section columns. The built-up nature of the cross-section is carefully considered in the calculation procedures and optimization constraints, accounting for the connection of profiles through the flanges within the cross-section. The calculation procedures are based on Eurocode 3 [1], which encompasses local, distortional, and global buckling forms. Five types of closed column cross-sections, ranging in height from 2 to 4 m, and subjected to axial loads of 50, 100, 200, and 500 kN, were thoroughly analyzed and optimized. The first type of cross-section examined was a square cross-section without stiffeners, while the remaining types featured four web stiffeners with varying geometrical constraints. The constrained nonlinear optimization solver, fmincon in Matlab software, was selected for the optimization problem. The obtained optimal cross-section results indicate that the square cross-section consistently exhibited the lowest effectiveness across all column heights and axial load combinations. In contrast, the most effective cross-section type transformed to a form, similar to a circle. Moreover, cross-section types with deeper stiffeners were associated with thinner profile thickness. A comprehensive summary of the results demonstrates that the effective area ratio within the cross-section was higher for smaller column heights and larger axial loads across all cross-section types. In conclusion, this study provides valuable insights into the analysis and optimization of axially compressed thin-walled built-up closed cross-section columns. The findings underscore the superiority of cross-sections that adopt a circular form and feature deeper stiffeners, resulting in enhanced structural effectiveness.

Keywords: Cold-formed Structures · Eurocode · Stability of cross-sections · Members and frames · Slender members · Local buckling · Distortional buckling

1 Introduction

The selection of an optimum axially compressed cross-section for a thin-walled column is a critical aspect in structural engineering. The thin-walled profiles have been started to be used for their lightness and good mass to load bearing strength ratio. The built-up nature of the cross-section provides a versatile platform for exploring various cross-section

possibilities while considering the producibility of the member. Many studies have been done analyzing built-up cross-sections [2–7]. Open single profile cross-sections were optimized when calculating according to the Eurocode 3 procedures [8] and when calculating according to the Direct Strength Method [9]. Closed thin-walled cross-sections, devoid of lateral restraints, have proven to be more effective compared to open thin-walled cross-sections due to their significantly higher torsional stiffness. This article focuses on the analysis of simply supported axially compressed columns with closed cross-sections.

Closed cold-formed cross-sections are usually produced by connecting two or more. Prior experimental studies have addressed the determination of the effective thickness of the overlapping part of the cross-section in calculations [10, 11]. In these studies, closed cross-sections were analyzed, wherein the entire portion of the profiles' cross-sections overlapped, connected by a single row of fasteners. The effective thickness of the overlapping part, assumed to be equal to the profile thickness, was deemed reliable and conservative, allowing the cross-section to be treated as a continuous single unit.

Figure 1 provides visual representations of three distinct cross-section types with their respective assemblies, which are analyzed in this article. The cross-sections are connected at the flanges using self-drilling screws, ensuring complete overlap of the flange parts.
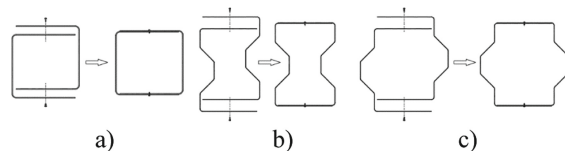

Fig. 1. The assembly of the thin-walled column cross-sections: a) – square cross-section b) – closed cross-section, web stiffeners bent in, c) – closed cross-section, web stiffeners bent out.

To determine the optimum solution for the axially compressed thin-walled built-up column, an optimization algorithm is employed. The optimization process follows the calculations outlined in Eurocode 3, as design standards typically impose bounds for the geometry of the cross-section. By adhering to these design standards, the resulting cross-sections are less complicated, more practical to produce, and easier to implement compared to calculations based on finite element methods.

In this article, a comprehensive analysis of the selection process for an optimum axially compressed closed cross-section thin-walled built-up column is presented. By considering the manufacturability, torsional stiffness, and practicality of the cross-sections, the aim is to provide valuable insights for structural engineers and designers in the field.

2 Calculation Procedure of an Axially Compressed Thin-Walled Column According to the Eurocode 3

Various methods exist for calculating thin-walled columns, each with its own approach and optimization objectives. Different column shapes using the Direct Strength Method calculations have been investigated [12], and five different types of Direct Strength

Method calculations have been analyzed [13]. This study is focused on optimizing axially compressed thin-walled columns using the Eurocode 3 calculation procedure, which is essential for identifying the most suitable cross-section solution that meets standard requirements while being practical to produce.

Eurocode 3-1-3 [14] not only provides the calculation procedure for thin-walled columns but also offers guidance on geometrical restrictions to ensure the cross-section is easily producible. These restrictions include limitations on member and sheeting thicknesses, ranging from 0.45 to 15 mm. Furthermore, the code specifies constraints on cross-section width-to-thickness ratios and stiffener sizes.

Local buckling, evaluated using the effective width theorem in Eurocode 3-1-5 [8], refers to the loss of stability that plane plate parts experience under compression before reaching the yielding stress. The effective width of cross-section parts is reduced based on a reduction factor ρ, which depends on the slenderness of the part. Calculation of the slenderness of the cross-section plate part is given in Eq. 1.

$$\lambda = \frac{b/t}{28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} \tag{1}$$

where b is the width of the cross-section plate part, t is the thickness of the cross-section plate part, k_σ is the buckling factor corresponding to the stress ratio in the cross-section plate part.

In thin-walled column cross-sections, certain parts, such as corners, are typically stiff and unaffected by local buckling. However, corners formed by stiffeners are prone to distortional buckling. To assess the stiffness and critical stress of the stiffener, the cross-sectional properties are calculated. Calculation of the critical stress of the stiffener is given in Eq. 2.

$$\sigma_{cr,s} = \frac{2 \cdot \sqrt{K \cdot E \cdot I_s}}{A_s} \tag{2}$$

where K is the spring stiffness of the stiffener per unit length, E is the elasticity modulus, I_s is the effective second moment of area of the stiffener, A_s is the cross-section area of the stiffener. In cases where stiffeners are susceptible to distortional buckling, their thickness is reduced.

Element global buckling occurs when the cross-section of a member displaces in space. The buckling strength of the column is determined by evaluating the slenderness of the member and considering the appropriate buckling curve. The calculation accounts for the effects of different buckling modes. Calculation of the buckling strength of the column is given in Eq. 3

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} \tag{3}$$

where χ is the reduction factor for buckling mode, A_{eff} is the effective area of the cross-section, f_y is the yield strength of the material, γ_{M1} is the partial factor for resistance of members to instability.

Challenge regarding the treatment of stiffeners in the thin-walled cross-sections depicted in Fig. 1b and 1c was encountered before initiating the calculations. In this article, web to consist of four separate stiffeners was considered, providing more flexibility

in cross-section geometry compared to cases where the web is assumed to have only one or two stiffeners. EN 1993-1-3 provides calculations for the effective area of the stiffener, which comprises the width of the stiffener and the effective widths adjacent to it. For this study, stiffener is considered to be a node with adjacent parts, so a width of 0 was assigned to the stiffener part itself and considered the effective area to be the sum of effective parts adjacent to the stiffener node. The stiffener closer to the flange of the cross-section was assumed to support the flange, with its spring support direction aligned with the cross-section's z-z axis. The stiffener closer to the center of the cross-section was considered to support the flange stiffener, with its spring support direction aligned with the cross-section's y-y axis. The cross-section geometry, including the stiffener cross-sections and spring support directions, is shown in Fig. 2.
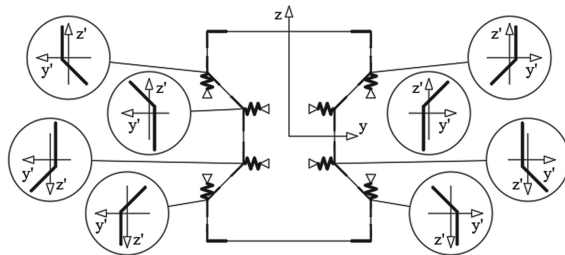

Fig. 2. The effective area of the cross-section and the directions of the spring supports of the stiffeners.

To validate the calculation algorithm, it's calculation results were compared to experimental results from a similar type of axially compressed built-up thin-walled column. Experiments on a closed built-up cross-section with external stiffeners was conducted [10], which resembles the type of cross-section given in Fig. 1c in this article. The Eurocode 3 calculation algorithm for the tested cross-sections of 300 mm, 2000 mm, and 3200 mm with a profile thickness of 0.48 mm and 1.0 mm was applied. Calculated buckling strength of the columns was 13.5% to 18.9% higher than the experimental strength of the columns. The results are conservative and reliable, conclusion was made, that the calculation algorithm written according to the Eurocode 3 methodology can be used optimizing the selected axially compressed built-up thin-walled cross-section types.

3   Optimized Cross-Sections

To optimize the axially compressed thin-walled built-up cross-section, a calculation algorithm for its strength was incorporated into the optimization solver. By considering the y and z axes of the cross-section as axes of symmetry, the calculation of cross-section properties can be simplified, reducing the complexity and computation time of the procedure. This is crucial since the calculation procedure is executed multiple times during the optimization algorithm.

Various optimization algorithms available in the Matlab software [15] were tested for optimizing the axially compressed thin-walled cross-section, but ultimately the fmincon solver was used for the problem, because it was the fastest and was providing good results. Fmincon is a constrained nonlinear optimization solver designed to find the optimum of a function with multiple variables. It should be noted that fmincon is a local minimum optimization algorithm, so determining whether the obtained optimization result is the global or local minimum is not straightforward. In this paper, the fmincon optimization solver was utilized to optimize the axially compressed thin-walled column cross-section. Different optimization starting points and geometric constraints were tested to assess if the optimized cross-section represents a global solution. The goal of the optimization problem was to find the smallest cross-section area for each cross-section type of the column under each column height and axial load combination.

This section is focused on selecting and optimizing simply supported axially compressed columns with the same effective length along both cross-section axes. The lengths of the columns considered were 2, 3, and 4 m, while the compressive loads applied were 50, 100, 200, and 500 kN.

Five types of cross-sections were analyzed in this study. The A Type represented a square cross-section without stiffeners. The basic starting geometry of the A Type cross-section is given in Fig. 3a. Although the A Type cross-section had a single constraint in terms of buckling strength, its absence of cross-section nodes only allowed for transformation into a different-sized square or rectangle with varying cross-section thickness (Fig. 3).
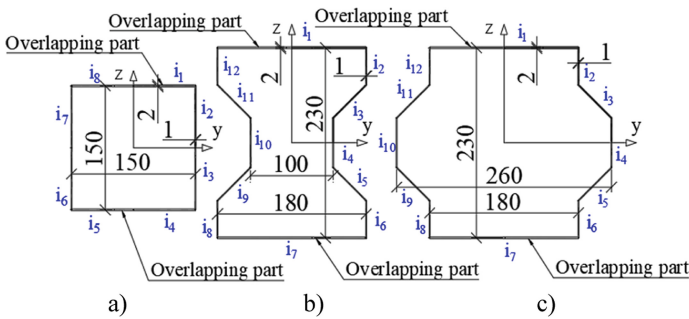

Fig. 3. The basic starting geometry of the optimized cross-sections: a) – A Type cross-section, b) – B and C Type cross-sections, c) – D and E Type cross-sections.

The B Type and C Type specimens were similar, both featuring closed cross-sections designed with four intermediate web stiffeners bent inwards. The basic starting geometry of the B and C Type cross-section is given in Fig. 3b. The goal of differentiating between these types was to assess the influence of the stiffener angle on the effectiveness of the cross-section. The B Type cross-sections had no restrictions on stiffener geometry, while the C Type cross-sections had a constraint on the angle between the stiffener and the web plate, set to 45°.

The D Type and E Type specimens had the same initial geometry for optimization, comprising closed cross-sections with four intermediate web stiffeners bent outwards.

The basic starting geometry of the D and E Type cross-section is given in Fig. 3c. The D Type cross-sections introduced two geometric restrictions: first, the width of the middle part of the web should not exceed the flange width, and second, the maximum distance of the farthest web part from the y-y axis was limited, following a similar restriction as specified for edge stiffeners in Eurocode 3. On the other hand, the E Type specimens had complete freedom in selecting the geometry of the stiffeners.

The optimized specimens exhibited clear trends: as the column height and applied load increased, the cross-sections became wider. The geometry of the optimized specimens subjected to axial compressive loads of 50 kN and 100 kN is illustrated in Fig. 4, while the specimens subjected to loads of 200 kN and 500 kN are shown in Fig. 5.

The optimization results demonstrated that the A Type cross-section, which was a square shape, consistently emerged as the optimal choice across all combinations of column height and load. This cross-section type exhibited the largest optimal cross-section area compared to the other types at each axial load and column height combination.

The optimal B Type cross-section shared a similar geometry with the initial design, but as the axial load increased, the B Type cross-section transformed into a configuration with three web stiffeners instead of four. The angle of the web stiffener ranged from 46 to 86°.

In most cases, the optimal C Type cross-section had three stiffeners instead of four. It was slightly smaller in size for a 50 kN load and larger for 100 kN, 200 kN, and 500 kN loads at each column height compared to the optimal B Type cross-sections.

The optimal D Type cross-section exhibited a geometry resembling a plus sign, but with some exceptions, resulting in a shape close to a circle. This cross-section type had the largest cross-section height and width for the majority of axial load and column height combinations.

The optimal E Type cross-section closely resembled a circle in every axial load and column height combination. In many cases, the stiffener closer to the flange merged with the flange, eliminating the vertical stiffener cross-section part and deleting the web part adjacent to the flange. This type of cross-section had the smallest cross-section area for each axial load and column height combination. Figure 7 presents the optimized cross-section areas for each type at different column heights and axial loads, including the average values presented with the "AVG" lines for each cross-section type.

To assess the effectiveness of the optimized cross-section types, the optimal cross-section area of each type was compared to the optimal A Type cross-section area at the same axial load and column height combination. The B and C Type cross-section areas in between each other showed a difference of approximately 0.15%. On average, these cross-sections were 11%, 14%, and 16% more effective than the A Type cross-sections at column heights of 2, 3, and 4 m, respectively. The D Type cross-section areas were, on average, 32%, 34%, and 35% more effective than the A Type cross-sections at the corresponding column heights. Similarly, the E Type cross-section areas were, on average, 36%, 39%, and 40% more effective than the A Type cross-sections at the respective column heights. The effectiveness of using cross-sections with web stiffeners increased with greater column height, with slightly greater effectiveness observed for smaller axial loads.
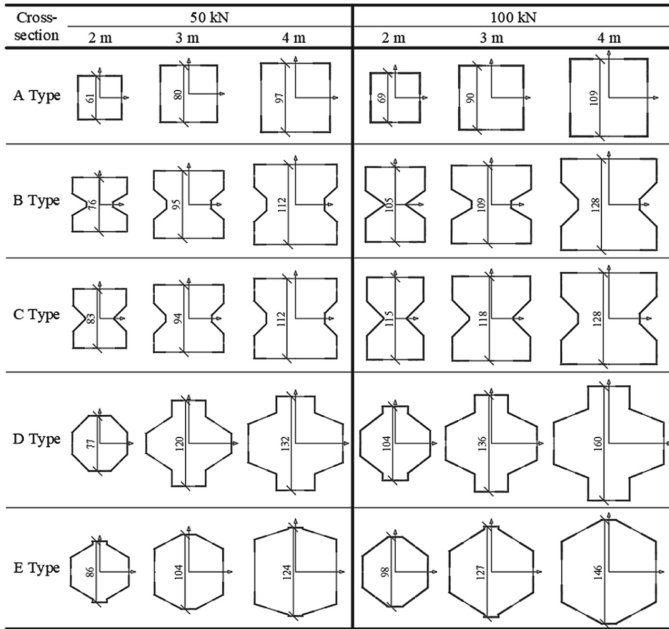
Fig. 4. The optimized cross-section types of 2, 3, and 4 m column height loaded with 50 or 100 kN axial compressive load.

The optimization results indicated that a cross-section geometry for an axially compressed thin-walled column can be selected such that the reduced thickness of the stiffeners closely matches or equals the thickness of the profile. The optimization algorithm identifies an optimal cross-section with increased dimensions and thickness to accommodate higher loads. This approach minimizes the inefficient parts of the cross-section while ensuring the stiffeners maintain the maximum possible reduced thickness.

The analysis of the effective area of the optimized cross-sections revealed that the ratio of the optimized effective cross-section area to the optimized gross cross-section area was higher for shorter columns and increased with higher axial loads. Refer to Fig. 7 for the detailed results as well as average values which are presented with the "AVG" lines for each cross-section type.

The thickness of the cross-sections was similar at each column height but increased with higher axial loads. The A Type optimal cross-sections exhibited the greatest thickness, ranging from 1.2 to 4.0 mm thickness. Then the E Type optimal cross-sections had their thickness ranging from 0.8 to 3.0 mm. B, C and D Type optimal cross-sections had similar cross-section thickness which varied from 0.7 to 2.7 mm.
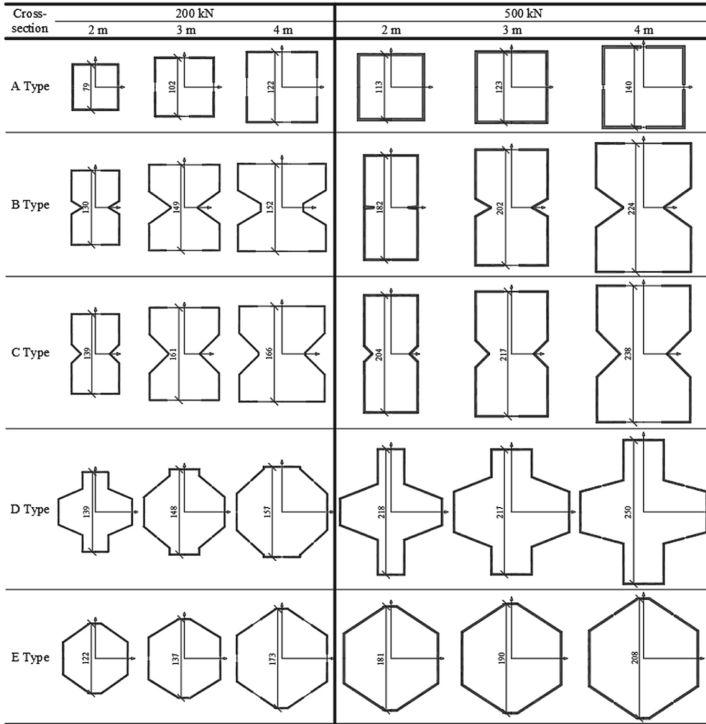
Fig. 5. The optimized cross-section types of 2, 3, and 4 m column height loaded with 200 or 500 kN axial compressive load.
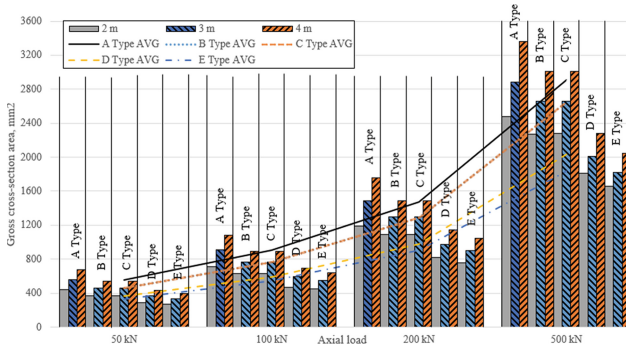

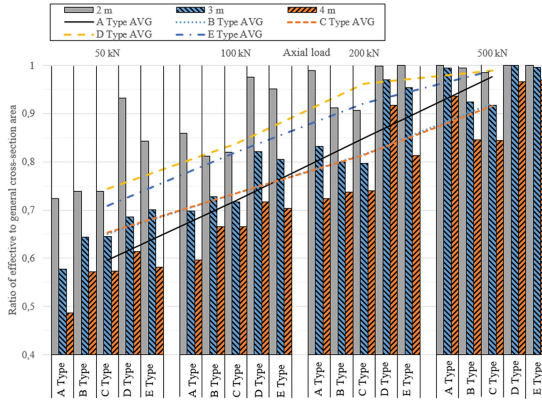**Fig. 6.** Gross cross-section area of the column types.

Fig. 7. Ratio of effective to general cross-section area of the column types.

4 Conclusions

In this study, the optimization of five different built-up thin-walled column closed cross-sections was carried out. These cross-sections were analyzed at varying column heights of 2, 3, and 4 m, and under different axial load combinations of 50, 100, 200, and 500 kN. The cross-section types included a square cross-section without stiffeners, as well as four cross-section types with four web stiffeners. Eurocode 3 calculation algorithms for axially compressed thin-walled columns, both with and without intermediate web stiffeners, were utilized. The column cross-sections consisted of two thin-walled profiles connected at the flanges.

The results of the optimization, based on calculation algorithms according to the Eurocode 3, highlighted that the E Type cross-section, which exhibited a circular-like form, consistently demonstrated the highest effectiveness in terms of cross-section area at every column height and axial load combination. On the other hand, the square cross-section without web stiffeners consistently exhibited the lowest effectiveness across all column heights and axial load combinations.

Remarkably, the optimized results for axially compressed thin-walled columns proved to be able to accommodate the constraints for the geometry of the cross-section. The inclusion of certain geometry restrictions, such as the angle of the stiffeners or specific geometrical proportions, led to alternative cross-section configurations that were nearly as effective as the optimal results. These restrictions can be implemented to facilitate the design of cross-sections for easier fabrication of profiles based on specific production technologies.

This study successfully demonstrated the selection of the cross-section and optimization for an axially compressed thin-walled column according to the Eurocode 3 calculation algorithm. The findings emphasize the best performance of the E Type cross-section, resembling a circular shape, when compared to other cross-section types and provide valuable insights into the adaptive nature of the calculation algorithm. The results offer practical implications for the design and fabrication of cross-sections, enhancing the efficiency of the built-up thin-walled column structures.

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