




Granularity in Number and Polarity Effects

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Abstract. This paper offers a granularity-based account of the fact that round and non-round numbers may exhibit polarity effects when they are appended by even-type focus particles. The key observation is that non-round numbers appended by *mo* ‘even’ in Japanese cannot be in the scope of negation, while round numbers exhibit no restriction in scopal relation. Adopting the scope theory of *mo* and a theory of granularity ([6, 10]), we propose that an asymmetric entailment relation holds between propositions with a non-round and a round number and this entailment relation invites a proposition with a coarser granularity into the set of alternatives in computing the scalar presupposition of *mo*. Given that the scalar presupposition of *mo* with numerals is only sensitive to asymmetric entailment, we argue that the availability of asymmetric entailment from the prejacent to this additional alternative proposition is responsible for the polarity effects. We also discuss the related issues such as polarity effects observed in explicit approximators (e.g. *about*, *approximately*) and numerals with the contrastive topic marker *wa*.

Keywords: Round and non-round numbers · Granularity · Focus particles · Polarity effects

1 Introduction

The recent literature on the polarity phenomena has revealed that vagueness and granularity have an impact on the polarity effect (e.g., [14] on approximators such as *approximately* and *about*, [1] on *some* NP and minimizers). This work is yet another contribution to this trend, reporting an unnoticed contrast between round and non-round numbers when associated with focus particles in Japanese. Our analysis predicts that this phenomenon is sensitive to what granularity is assumed in the context and to whether non-round and round numbers are in competition in the relevant context.

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2 Data

It has been acknowledged that *mo* ‘even’ in Japanese invites different implications when it appears in positive or negative sentences, just like its counterpart in English. The positive version of (1a), for example, implies that Question 2 is hard, while with the negative version of (1a), Q2 is understood to be easy.¹

- (1) a. John-wa toi 2-mo {toita/toka-nakat-ta}.
 John-TOP Question 2-mo {solved/solve-NEG-PAST}
 ‘John even solved Question 2.’
 b. John solved Q2, which is hard.
 c. John didn’t solve Q2, which is easy.

When *mo* is appended to numerals, as in (2a)–(2b), the implications are about how large the interlocutors consider them to be. The positive sentence in (2a) denotes a situation where John solved five problems, and implicates that ‘5’ is considered to be large.

- (2) a. John-wa mondai-o 5-mon-mo toita
 John-TOP problem-ACC 5-CL-mo solved
 ‘John even solved five problems. (And ‘five’ is considered to be large.)’
 b. John-wa mondai-o 5-mon-mo toka-nakat-ta.
 John-NOM questions-ACC 5-CL-mo solve-NEG-PAST
 ‘John didn’t even solve five problems.’

(2b) is ambiguous: in one reading, it is true in a context where the number of problems John solved does not reach 5 (= (3a)), while in the other reading, it becomes true in a context where the number of problems John didn’t solve is five (= (3b)). These readings are associated with different implications: in Context A, ‘5’ is understood to be small, while in Context B, the same number is considered to be large. We call these two readings small and large number readings, respectively.²

¹ *Mo* has several usages as exemplified below. We will confine ourselves to the scalar usage with a similar meaning to ‘even’ in this paper. We do not make any specific assumption about the issue of whether these different usages come from a single source or not.

(i) Taro-mo, Taro-mo Jiro-mo, Dare-mo-ga ..., Dare-mo ...nai
 Taro-too, Taro-and Jiro-and, who-mo-NOM ..., who-mo ...NEG
 ‘Taro also’, ‘Taro and Jiro’, ‘Everyone...’ ‘No one ...’

² [9] notes that there is yet another reading for (2b), where truth-conditionally, John solved fewer than five problems and ‘5’ is implicated to be large. We will not consider this reading here, but our analysis can explain why this reading is legitimate both with round and non-round numbers.

- (3) a. **Context A:** John has 20 problems to solve, and he solved fewer than five, and ‘5’ is considered small. **small number reading**
- b. **Context B:** John has 20 problems to solve, and he solved 15 of them, which means that there are five problems that he didn’t solve, and ‘5’ is considered to be a large number. **large number reading**

Even in English behaves differently from its Japanese counterpart in negative sentences. It induces a small-number reading (=Context A), and the large-number reading is very hard to get, if not impossible.³

- (4) a. John didn’t even solve five problems.
John solved fewer than five problems and ‘5’ is considered to be small.
- b. Not even five people came to the party.
Fewer than five people came and ‘5’ is considered to be small.

We observe that this ambiguity mysteriously disappears when we use a different number. The key observation here is that the small-number readings available for (2b) and (4) become mysteriously unavailable when the number included is a non-round one, such as 48, while the positive sentence does not exhibit any contrast between 50 and 48, as shown in (5a)–(5b)(see [3]).⁴

- (5) a. John-wa {50/48}-mon-mo toita.
John-TOP 50/48-CL-mo solved
‘John even solved 50/48 problems.’
- b. John-wa {50/48}-mon-mo toka-nakat-ta.
John-TOP 50/48-CL-mo solve-NEG-PAST
‘John didn’t even solve 50/48 problems.’
- c. ✓ John solved fewer than 50 problems, and 50 is a small number.
✓ There are 50 problems that John didn’t solve, and 50 is a large number.
- d. # John solved fewer than 48 problems, and 48 is a small number.
✓ There were 48 problems that John didn’t solve, and 48 is a large number.

³ Nakanishi [8, 185] notes that the large-number reading is indeed not impossible in English, as shown in (i):

(i) Al, Bill and Conan always read everything they are assigned, but this time, they each had some books that they didn’t read. Al didn’t read [one]_F book, Bill didn’t read [three]_F books and Conan ended up not even reading [five]_F.

⁴ Ijima [3] takes the sentence with 48 in (5b) is unacceptable. We found this description unsatisfactory because the 48-version of the sentence does have a legitimate interpretation with the large-number reading.

This judgment is replicated in English *even*, in which *even* with numbers in negative sentences only has a small number reading.

- (6) a. John didn't even solve {50/??48} problems.
 b. Not even {50/??48} people came to the party.

Another interesting aspect of this phenomenon is context sensitivity: in (7), '25' is judged to be weird, because in this case, the conspicuous unit of measure is 12. In other words, in this unit of measure, 25 cannot be a 'round' number.

- (7) 24 vs. 25 h
 Han'nin-no minoshirokin yokyuu-no denwa-kara mada {24/#25}
 culprit-GEN ransom demand-GEN call-from yet {24/25}
 jikan-mo tat-tei-nai.
 hours-mo pass-ASP-NEG
 (lit.) 'Even 24/25 h haven't passed yet since the culprit called to demand ransom money.'

The obvious question here is what makes 'non-round' numbers awkward in the negative context with *mo*. We propose that this contrast arises when the non-round numbers compete with round numbers in the satisfaction of the presupposition induced by *mo*.

3 The Scope Theory of *Mo* 'Even'

We follow [7] and [8] in that Japanese *mo* is best analyzed in terms of the Scope Theory of *even*-items ([4]). In this theory, *mo* introduces a scalar presupposition without contributing to the assertive content, and its scalar meaning is defined by unlikelihood.⁵

- (8) $\llbracket mo \rrbracket^{w,c} = \lambda p. p(w) = 1$, defined if $\forall q \in C [q \neq p \rightarrow q >_{\text{likely}} p]$
Scalar Presupposition

It has been argued that the hard/easy implications observed in (1a) are due to this scalar presupposition ([4]). Nakanishi [7] argues that the small and large readings also come from the scalarity of *mo*, with crucial assumptions that numeral expressions are interpreted to be one-sided, 'at least n', and that the unlikelihood is equated with asymmetric entailment, as in (9).⁶

⁵ *Even*-items including *mo* may also introduce an additive presupposition (= (i)), but we will put this component aside in this paper.

(i) $\exists q \in C [q \neq p \wedge q(w) = 1]$

Additive Presupposition .

⁶ We do not claim that the unlikelihood of *mo* is *always* based on asymmetric entailment: this simply makes a wrong prediction. In (i), for example, it has to be

- (9) Let p and q be propositions.
 p is less likely than q iff p entails q but not vice versa.

Under this setting, *John solved n problems* asymmetrically entails a proposition *John solved m problems*, where $m < n$.

The scope theory of *mo* espouses that *mo* moves to a propositional level at LF, even if it is appended to a numeral or a DP. This produces the following LFs for the sentences in (2a)–(2b):

- (10) a. (2a): [mo [John solved five_F problems]] large number
 b. (2b): [mo [\neg [John solved five_F problems]]] small number
 c. (2b): [mo [five_F problems [\neg John solved t]]] large number

The LF in (10a) satisfies the presupposition of *mo* when the set of alternative propositions, C , consists of the propositions that are entailed by the prejacent. This means that the numerals included in the alternative propositions (other than the prejacent) are lower than 5. Since the prejacent proposition includes the largest number among the propositions in C , the large-number reading results.

- (11) a. [(10a)]^{w,c} is defined if $\forall q \in C$. [$q \neq$ [John solved five problems]]
 \rightarrow [John solved five problems] <_{likely} q
 b. $C = \{ \text{John solved } n \text{ problems} \mid n \leq 5 \}$

In (10b), *mo* scopes over negation, which in turn takes scope over the numeral. To satisfy the presupposition of *mo*, the alternatives in C should be the ones that have a smaller number than 5, since the negation flips the entailment. This leads to the small number reading.

- (12) a. [(10b)]^{w,c} is defined if $\forall q \in C$ [$q \neq$ [\neg John solved five problems]]
 \rightarrow [\neg John solved five problems] <_{likely} q
 b. $C = \{ \neg \text{John solved } n \text{ problems} \mid n \geq 5 \}$

The configuration in (10c) again results in a large number reading. Since the negation takes a narrower scope than the numeral, the following entailment

the case that *Taro came to the party* is less expected than, say, *Mary came to the party*, which is not in entailment relation with the former.

(i) Taro-mo paatii-ni kita.

Taro-mo party-DAT came.

‘Even Taro came to the party.’

What seems to be the case is that in the case of *mo* appended to numerals, the unlikelihood based on other than asymmetric entailment is not available.

relation holds: if there are five problems that John didn't solve, it is true that there are four problems that John didn't solve. This leads to the large-number reading.

- (13) a. $\llbracket (10c) \rrbracket^{w,c}$ is defined if ...
 $\forall q \in C [q \neq \llbracket \text{there are five problems that John didn't solve} \rrbracket$
 $\rightarrow \llbracket \text{there are five problems that John didn't solve} \rrbracket <_{\text{likely}} q]$
 b. $C = \{ \text{there are } n \text{ problems that John didn't solve} \mid n \leq 5 \}$

It should be noted here that in the scope theory of *even*-items, these items have to take scope over negation to yield appropriate interpretations. In Japanese, this is independently motivated by the general property of focus particles, which take a wider scope than negation.⁷ Take *dake* 'only', for example. In (14) below, the only interpretation possible is an interpretation where *only* takes a wider scope than negation.⁸

- (14) Taro-wa toi 2-dake toka-nakat-ta.
 Taro-TOP question 2-only solve-NEG-PAST
 (lit.) 'Taro didn't solve only question 2.'
 'It is not the case that Taro solved only question 2.' * $\neg >$ only
 'It is only question 2 that Taro didn't solve.' \checkmark only $> \neg$

The data that concerns us here is now understood in the following way: the contrast between the round and non-round numbers arises when *mo* takes a proposition in which negation takes scope over numerals (= (15b)).

- (15) a. LF: [mo [...50/48...]]
 b. LF: [mo [\neg [...50/#48...]]]
 c. LF: [mo [50/48 [\neg ...]]]

⁷ In footnote 2 we pointed out that (2b) has the third reading. The scopal relation involved in this reading should be [$\neg >$ mo $>$ n]. This apparent inconsistency to what we claim here is resolved if we consider this reading to be actually a case of external negation. A piece of evidence for this view comes from the fact that this use requires a preceding discourse that refers to the number, as in (i).

(i) A: How many students are enrolled in your class this semester? 50 students have enrolled in mine.

B: 50-nin-mo tooroku si-tei-mas-en.

50-CL-mo register do-ASP-POLITE-NEG

(lit.) 'Even 50 students hasn't enrolled (in my class).'

⁸ See [12] for morpho-syntactic reasoning of this obligatory wide-scope reading of focus particles.

This generalization is supported by the fact that the contrast is not observed when *mo* takes a narrower scope than other ‘negative’ operators. In Japanese, *mo* does not move across a clause boundary ([8]), which is evidenced by the lack of the ‘easy’ reading in (16). If *mo* takes a proposition within which a ‘negative’ operator does not take scope over a numeral, it is predicted that there will be no contrast between 50 and 48 observed. This prediction seems to be borne out, as shown in (17).

- (16) [[Taro-ga toi 2-mo toita]-towa] odoroki-da.
 Taro-NOM question 2-mo solved-COMP surprise-COPULA
 ‘It’s surprising that Taro even solved question 2.’
 Question 2 is hard/*easy
- (17) a. [[Taro-ga {50/48}-mon-mo toita]-towa] odoroki-da.
 Taro-NOM {50/48}-CL-mo solved-COMP surprise-COPULA
 ‘It is surprising that Taro even solved 50/48 problems.’
 50/48 is a large number.
- b. [Moshi Taro-ga {50/48}-mon-mo toita-ra] kurasu-de
 if Taro-NOM {50/48}-CL-mo solved-CONDITIONAL class-in
 ichiban-ni nar-eru-daroo.
 no.1-DAT become-can-will
 ‘If Taro even solved 50/48 problems, he will be the best student in
 the class.’ 50/48 is a large number.

In the next section, we explain why the generalization in (15) holds, based on granularity.

4 Proposal

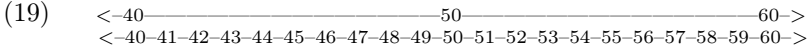
4.1 Granularity in Number

Before moving to our proposal, we first lay out how round and non-round numbers are treated in this paper.

We have described ‘50’ can be a round number, while ‘48’ is not. In other words, we understand ‘48’ as a precise number when we say ‘Taro solved 48 problems,’ while ‘50’ in ‘Taro solved 50 problems.’ can be understood to denote an exact number Taro solved or an approximate number he solved. Krifka [6] formulates this in terms of the Coarsest Scale Principle in (18): That ‘50’ is on the coarser and finer scales in (19) makes it possible to have an approximate interpretation.

(18) The Coarsest Scale Principle

If a measure expression α occurs on scales that differ in granularity, then uttering α implicates that the most coarse-grained scale on which α occurs is used. ([6, 119-120])



We follow [10] and [14] in that granularity is formulated as a contextual parameter of interpretation. A granularity function, \mathbf{gran}_i is a function that maps a number, n to the interval $[n - 1/2 \times i \leq n \leq n + 1/2 \times i]$, where i represents the granularity level. Under this formulation, for example, ‘50’ on the coarser scale in (19) denotes the interval from [45-55], and ‘48’ on the finer scale in (19) denotes the interval from [47.5-48.5].

- (20) a. $\llbracket 50 \rrbracket^g = \mathbf{gran}_{10}(50) = [45-55]$
- b. $\llbracket 48 \rrbracket^g = \mathbf{gran}_1(48) = [47.5-48.5]$

We can now define the relative coarseness of granularity functions, as in (21). \mathbf{gran}_1 is finer than \mathbf{gran}_{10} since the former returns a narrower interval when it is applied to a number than when the latter is applied to the same number.

- (21) Relative coarseness of granularity functions
 \mathbf{gran} is finer than \mathbf{gran}' iff for any number n ,
 $\max(\mathbf{gran}(n)) - \min(\mathbf{gran}(n)) < \max(\mathbf{gran}'(n)) - \min(\mathbf{gran}'(n))$

Under this interpretation of granularity, if there is a context where you can truthfully say (22a), then there should be a context where you can truthfully say (22b). This relation holds when the interval denoted by $\mathbf{gran}_1(48)$ falls within the one denoted by $\mathbf{gran}_{10}(50)$.

- (22) a. $\llbracket \text{John solved 48 problems.} \rrbracket^{w,c} = 1$, iff
 Taro solved (at least) $\mathbf{gran}_1(48) =$ Taro solved at least [47.5-48.5] problems.
- b. $\llbracket \text{John solved 50 problems.} \rrbracket^{w,c} = 1$, iff
 Taro solved (at least) $\mathbf{gran}_{10}(50) =$ Taro solved at least [45-55] problems.

The relation between these two is one of entailment: if (22a) is true in a context, then (22b) has to be true in another context. This notion of entailment is thus formulated as follows:

- (23) Let **gran** be finer than **gran'** and n and m be variables for numbers. For any number n , if there is a context c and a number m such that if $\mathbf{gran}(n) \subset \mathbf{gran}'(m)$, then $\llbracket \phi(n) \rrbracket^{c, \mathbf{gran}} = 1 \Rightarrow \llbracket \phi[n/m] \rrbracket^{c, \mathbf{gran}'} = 1$, where ϕ does not contain \neg .

(23) is, in effect, the condition for rounding numbers. Thus '48' can be rounded to '50', but not vice versa.

4.2 Polarity Effects Explained

In the previous section, we made a crucial assumption that the unlikelihood of *mo*, when appended to numerals, is equated with asymmetric entailment. In other words, *mo* appended to numerals is sensitive to entailment relation between its possible alternatives. We propose that an alternative set C contains a proposition with a coarser granularity when it satisfies the relation in (23) with the prejacent. In simpler terms, when the numeral in the prejacent can be rounded to another numeral, then C has to include the proposition with the round number as one of the alternative propositions.

Let us now proceed to how this proposal accounts for our data. Consider first the affirmative cases. Since *mo* does not contribute to the assertive content, we will only consider whether the scalar presupposition is satisfied. (24a) has the prejacent proposition that may be truthfully denoted by a proposition with a round number. Thus the set of alternative propositions in (25b) has to include that proposition (=the underlined one), in addition to the propositions with the same granularity level. Since the prejacent 'John solved $48_{\mathbf{gran}1}$ problems.' entails all the other propositions in C , the scalar presupposition of *mo* is satisfied.

- (24) a. John-wa 48-mon-mo toita.
 John-TOP 48-CL-mo solved
 'John even solved 48 problems.'
- b. John-wa 50-mon-mo toita.
 John-TOP 50-CL-mo solved
 'John even solved 50 problems.'

- (25) a. $\llbracket (24a) \rrbracket^{w,c}$ is defined, if
 $\forall q \in C [q \neq \llbracket \text{John solved 48 problems} \rrbracket^{w,c}$
 $\qquad \qquad \qquad \rightarrow \llbracket \text{John solved 48 problems} \rrbracket^{w,c} <_{\text{likely}} q]$
- b. $C = \{ \text{John solved } 48_{\mathbf{gran}1} \text{ problems, John solved } 47_{\mathbf{gran}1} \text{ problems,}$
 $\text{John solved } 46_{\mathbf{gran}1} \text{ problems, } \dots \underline{\text{John solved } 50_{\mathbf{gran}10} \text{ problems}} \}$
- c. 'John solved $48_{\mathbf{gran}1}$ problems' entails 'John solved $50_{\mathbf{gran}10}$ problems.'

the scalar presupposition satisfied

(24b), in turn, does not include alternatives with different granularity levels, since the prejacent does not entail, say, ‘John solved 48_{gran1} problems.’ The computation of the scalar presupposition goes through as usual, with either of C₁ or C₂.

- (26) a. $\llbracket (24b) \rrbracket^{w,c}$ is defined, if
 $\forall q \in C \llbracket \text{John solved 50 problems} \rrbracket^{w,c} <_{\text{likely}} q$
- b. C₁ = { John solved 50_{gran1} problems, John solved 49_{gran1} problems, John solved 48_{gran1} problems, ... }
the scalar presupposition satisfied
- c. C₂ = { John solved 50_{gran10} problems, John solved 40_{gran10} problems, John solved 30_{gran10} problems, ... }
the scalar presupposition satisfied

In the case of negative sentences, two LF are possible, (27b) and (27c).

- (27) a. (=5b)
 John-wa 50/48-mon-mo toka-nakat-ta.
 John-TOP 50/48-CL-mo solve-NEG-PAST
 ‘John didn’t even solve 50/48 problems.’
- b. LF₁: [mo [¬ [John solved 48 problems]]]
- c. LF₂: [mo [48 problems [¬ John solved t]]]

Let us first consider the wider scope negation reading with ‘48’. Since ‘48’ is a number that conforms to the relation in (23), C has to include a proposition with a different granularity as its member (=the underlined one in (28b)). Since we espouse the ‘at least’ semantics of numerals, the prejacent does not entail ‘¬ John solved 50_{gran10} problems.’ (see (29)). This leads to the unsatisfied presupposition, and thus the unacceptability results.

- (28) a. $\llbracket (27b) \rrbracket^{w,c}$ is defined, if
 $\forall q \in C [q \neq \llbracket \neg \text{John solved 48 problems} \rrbracket^{w,c}$
 $\rightarrow \llbracket \neg \text{John solved 48 problems} \rrbracket^{w,c} <_{\text{likely}} q]$
- b. C = { ¬ John solved 48_{gran1} problems, ¬ John solved 49_{gran1} problems, ¬ John solved 50_{gran1} problems, ...
¬ John solved 50_{gran10} problems }
- c. ‘¬ John solved 48_{gran1} problems.’ does not entail ‘¬ John solved 50_{gran10} problems.’
the scalar presupposition unsatisfied

$$(29) \quad \underbrace{\text{John didn't solve } 50_{\text{gran}10}} \quad \underbrace{-48}_{[45-55]}$$

With the numeral taking wider scope, as in (27c), the proposition ‘there are $50_{\text{gran}10}$ problems that John didn’t solve.’ has to be added to the set of alternatives, but this time it does not do any harm: Just like the affirmative case, ‘there are $48_{\text{gran}1}$ problems that John didn’t solve’ entails ‘there are $50_{\text{gran}10}$ problems that John didn’t solve’. The scalar presupposition of *mo* is satisfied in this reading.

Our proposal that a proposition with a different granularity level is added to the set of alternatives when we compute the scalar presupposition of *mo* thus predicts the contrast between the round and non-round numbers we observed in Sect. 2.

4.3 Some Predictions

The current proposal is based on the idea that a numeral + *mo* sounds awkward when it can be rounded without making the sentence false. This reasoning leads to the prediction that if a numeral is not rounded to another one, then it does not exhibit awkwardness. This prediction is actually borne out, as shown in (30b). According to the definition in (23), ‘3’ cannot be rounded to any number with, say, gran_{10} :

- (30) a. **Context:** John had 20 problems to solve, and he only solved two of them.
- b. John-wa mondai-o 3-mon-mo toka-nakat-ta.
 John-wa problems-ACC 3-CL-mo solve-NEG-PAST
 ‘John didn’t even solve three problems.’ ✓ $\neg > 3$

Another consequence of the proposal is that if the context in question makes it easier to access a particular measure of the unit, the numerals that would not show a contrast in other contexts may exhibit a difference in acceptability. (7), repeated here as (31) below, is just the case: In (31), the conspicuous measure of the unit is 12, and thus 25 can be replaced by 24, without making the (affirmative) sentence false.

- (31) Han’nin-no minoshirokin yokyuu-no denwa-kara mada {24/#25}
 culprit-GEN ransom demand-GEN call-from yet {24/25}
 jikan-mo tat-tei-nai.
 hours-mo pass-ASP-NEG
 ‘Not even 24/25 h have passed yet since the culprit called to demand ransom money.’

Putting a proposition with a different granularity level into a set of alternatives when the number can be rounded to another one thus explains apparently mysterious contrasts between round and non-round numbers in negative sentences.

5 Discussion

This section presents some issues that arise from our analysis.

5.1 Approximately N

Solt [14] observes that numerals modified by ‘approximators’ such as *approximately*, *about*, *roughly* avert negative contexts:

- (32) Lisa { has/*doesn’t } have {about/roughly/approximately} 50 sheep. [14,91]

Japanese behaves in the same way, in that the reading where the modified numeral has a narrower scope than negation is not available in (33).

- (33) Lisa-wa mondai-o oyoso 50-mon toka-nakat-ta.
 Lisa-TOP problems-ACC about 50-CL solve-NEG-PAST
 ‘There are 50 problems that Lisa didn’t solve.’
 $*\neg > \text{about } 50, \checkmark \text{about } 50 > \neg$

The possible scopal relation is confined to the one where negation takes a narrower scope when modified numerals appended by *mo*:

- (34) John-wa mondai-o oyoso 50-mon-mo toka-nakat-ta.
 John-TOP problems-ACC about 50-CL-mo solve-NEG-PAST
 ‘John didn’t even solve about 50 problems.’
 $*mo > \neg > \text{about } 50, \checkmark mo > \text{about } 50 > \neg$

Our analysis does not seem to predict this distribution. Let us assume that the approximators restrict possible granularity functions to the coarsest possible in the given context (cf. [10, 13]). Since this does not require a proposition with a different granularity in its alternatives, the scalar presupposition of *mo* should be satisfied.

- (35) $[[\text{about } 50]]^g = \mathbf{gran}(50)$, where **gran** is the coarsest functions available in the context.

- (36) a. $\llbracket \neg \text{John solved about 50 problems-mo} \rrbracket^{w,c}$ is defined if
 $\forall q [q \neq \llbracket \neg \text{John solved about 50 problems} \rrbracket^{w,c}$
 $\rightarrow \llbracket \neg \text{John solved about 50 problems} \rrbracket^{w,c} <_{\text{likely}} q]$
- b. $C = \{ \neg \text{John solved about 50 problems, } \neg \text{John solved about 60}$
 $\text{problems, } \neg \text{John solved 70 problems, } \dots \}$

Solt [14] argues that the PPI-hood of numerals modified by approximators comes from the conversational principle in (37) ([5]) when they are in competition with bare numerals in their structural terms. Under her denotations of modified and bare numerals, these two may not be better than the other from the informational perspective, but the latter is definitely simpler in form and thus better in this respect. So if the speaker uses a proposition with a modified numeral, she/he implicates that an alternative proposition with a bare numeral cannot be asserted.

- (37) Conversational principle: Do not use ϕ if there is another sentence $\phi' \in \text{ALT}(\phi)$ such that both (i) ϕ' is better than ϕ and (ii) ϕ' is weakly assertable.

Suppose we understand that *about 50 problems* denotes the interval around (precisely) '50'. In that case, the negative sentence with the modified numeral causes a contradiction, while the affirmative does not cause any trouble: asserting that John didn't solve [50-k, 50+k] problems implicates the speaker cannot assert that John didn't solve 50 problems, which in turn means that John solved 50 problems.

We might thus explain the oddness of (34) with wide-scope negation reading by resorting to the PPI-hood of modified numerals.

5.2 Contrastive Topic Marker *Wa* and Numerals

Ijima ([3]) has made another observation that when a non-round number is appended by the contrast topic marker *wa*, the sentence becomes odd whether it is in an affirmative or negative context:

- (38) a. John-wa mondaio-o {50/??48}-mon-wa toita.
 John-TOP questions-ACC {50/??48}-CL-CT solved.
 'John solved (at least) {50/48} problems.
- b. John-wa mondaio-o {50/??48}-mon-wa toka-nakat-ta.
 John-TOP questions-ACC {50/??48}-CL-CT solve-NEG-PAST.
 'John didn't solve (at least) {50/48} problems.

Let us first adopt a scalar analysis of *wa* ([11]). Sawada [11] claims that *wa* works as a mirror image of *mo* 'even', proposing the following semantics.

- (39) $\llbracket wa_{CT} \rrbracket^{w,c} = \lambda p. p(w) = 1$, defined if
- $\exists q [q \in C \wedge q \neq p \wedge \neg q(w)=1]$ Anti-Additive Presupposition
 - $\forall q [q \in C \wedge q \neq p \rightarrow q <_{\text{likely}} p]$ Scalar Presupposition

The scalar presupposition in (39b) requires that the prejacent p should be the most likely among the set of alternative propositions C (i.e., p is entailed by all the other alternatives in C .)

If we apply this denotation of wa to (38a), we will get the following results:

- (40) a. $\llbracket \text{John solved 48-wa problems} \rrbracket^{w,c}$ is defined if
- $\exists q [q \in C \wedge q \neq \llbracket \text{John solved 48 problems} \rrbracket^{w,c} \wedge \neg q(w)=1]$
 - $\forall q [q \in C \wedge q \neq \llbracket \text{John solved 48 problems} \rrbracket^{w,c} \rightarrow q <_{\text{likely}} \llbracket \text{John solved 48-mo problems} \rrbracket^{w,c}]$
 - $C = \{ \text{John solved 48}_{\text{gran1}} \text{ problems, John solved 49}_{\text{gran1}} \text{ problems, John solved 50}_{\text{gran1}} \text{ problems, } \dots \underline{\text{John solved 50}_{\text{gran10}} \text{ problems.}} \}$

Just like the other examples with ‘48’, the alternative set includes a proposition with a different granularity (=the underlined one in (40d)), which is entailed by the prejacent ‘John solved 48-wa problems’. The scalar presupposition is not thus satisfied and the infelicity is predicted as desired.

Unfortunately, the same analysis cannot be extended to the negative sentence in (38b): we predict that the scalar presupposition of wa is *satisfied* in this case.

- (41) a. $\llbracket \text{John didn't solve 48-wa problems} \rrbracket^{w,c}$ is defined if
- $\exists q [q \in C \wedge q \neq \llbracket \neg \text{John solved 48 problems} \rrbracket^{w,c} \wedge \neg q(w)=1]$
 - $\forall q [q \in C \wedge q \neq \llbracket \neg \text{John solved 48 problems} \rrbracket^{w,c} \rightarrow q <_{\text{likely}} \llbracket \neg \text{John solved 48-mo problems} \rrbracket^{w,c}]$
 - $C = \{ \dots, \neg \text{John solved 46}_{\text{gran1}} \text{ problems, } \neg \text{John solved 47}_{\text{gran1}} \text{ problems, } \quad \neg \text{John} \quad \text{solved} \quad 48_{\text{gran1}} \text{ problems, } \underline{\neg \text{John solved 50}_{\text{gran10}} \text{ problems.}} \}$

In the negative environment, the entailment relationship is reversed, and the underline proposition above entails the prejacent. Thus, the prejacent is the most likely in C , and the scalar presupposition is satisfied.

Let us now adopt a non-scalar analysis of wa ([2]). Hara ([2]) also proposes that wa introduces definiteness condition without contributing to the assertion, which requires the existence of at least one stronger proposition than the assertion. Furthermore, this produces an uncertainty implicature where it is possible that the stronger proposition is false.

- (42) a. $\llbracket wa \rrbracket^{w,c} = \lambda p. p(w) = 1$, defined if $\exists q \in C [q \Rightarrow p \wedge p \not\Leftarrow q]$
 b. Implicates: $\diamond \neg q$

The application of this analysis to our case does not give us what we want: since there is at least one stronger alternative in C in (42c), say, John solved 50_{gran1} problems, this should not cause any problems, even if we have an alternative with different granularity.

- (43) a. $\llbracket \text{John solved 48-wa problems} \rrbracket^{w,c}$ is defined if
 b. $\exists q [q \in C \wedge q \rightarrow \llbracket \text{John solved 48 problems} \rrbracket^{w,c} \wedge \llbracket \text{John solved 48 problems} \rrbracket^{w,c} \not\Leftarrow q]$
 c. $C = \{ \text{John solved } 48_{\text{gran1}} \text{ problems, Jon solved } 49_{\text{gran1}} \text{ problems, John solved } 50_{\text{gran1}} \text{ problems, } \dots \underline{\text{John solved } 50_{\text{gran10}} \text{ problems}} \}$

The same holds of the negation, and thus we cannot explain the distribution of ‘48’ with *wa*.

The above discussions indicate that unlike the cases with *mo*, the incompatibility of *wa* with non-round numbers is not due to the entailment relation between round and non-round numbers. Thus, we need a different, non-entailment-based analysis, and we speculate that the uncertainty implicature induced by *wa* is at odds with the fine granularity of non-round numbers. In (38a) and (38b), the speaker implicates that she or he does not have perfect knowledge about numbers greater/smaller than 48 but at the same time, she or he uses the non-round number, indicating that she or he has sufficient knowledge to choose the fine granularity scale. Given that the choice of the precise scale increases the speaker’s certainty, non-round numbers seem to be incompatible with the contrastive topic marker *wa*. However, we leave the detailed exposition of this analysis for future work.

6 Conclusion

This paper has discussed an unfamiliar polarity effect observed with non-round and round numbers appended by *mo* and proposes that a proposition with a non-round number has to include a proposition with a number with a coarser granularity. It is important to note here that the contrast reported here is not confined to Japanese *mo*: as we noted above English exhibits the same contrast. This indicates that this phenomenon could be robust across languages, which we have to leave for future work. We hope that our work will contribute to the understanding of the roles of granularity in polarity effects, which has gained a lot of attention in recent literature.

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