Measurement Theory Meets Mereology in Multidimensionality in Resemblance Nominalism

Satoru Suzuki(✉)

Faculty of Arts and Sciences, Komazawa University,
1-23-1, Komazawa, Setagaya-ku, Tokyo 154-8525, Japan
bxs05253@nifty.com

Abstract. The problem of particulars and universals is one of the most essential problems in the formal philosophy of language in the sense that it consists in a crossroads of ontology and semantics. According to Resemblance Nominalism, resemblance relations are primitive and the properties of a thing are defined by them. We (2020) proposed, in terms of measurement theory, a first-order modal resemblance logic MRL that can furnish solutions to the problems with which Resemblance Nominalism is confronted. Yi (2014) raises a new version of degree of resemblance problem with Resemblance Nominalism of Rodriguez-Pereyra (2002). We think this problem to be a problem of multidimensionality. When we considered this problem, we realized that the model of MRL was not able to deal appropriately with the multidimensionality of this type of problem. The aim of this paper is to revise MRL so that the revised first-order modal resemblance logic RMRL can solve Rodriguez-Pereyra-Yi Problem in terms of measurement-theoretic multidimensional representation. Measurement theory makes it possible that qualitative resemblance relations can represent quantitative (numerical) functions, whereas it is not designed to explicate the parthood between a particular and its parts referred to for determining the raking on a resemblance relation. So, in the construction of the multidimensional model of RMRL, we connect measurement-theory with mereology that can explicate the parthood between a particular and its parts referred to for determining the raking on a resemblance relation. The punch line of Resemblance Nominalism is the reducibility of universals into resemblance relations. The point of formalizing Resemblance Nominalism in RMRL is to avoid the circularity in this reduction into which it tends to slide.

Keywords: first-order logic · formal philosophy of language · measurement theory · mereology · model theory · multidimensionality · nominalism · ontology · particulars · Platonism · realism · resemblance relation · resemblance nominalism · universals

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1 Motivation

The *problem of particulars and universals* is one of the most essential problems in the *formal philosophy of language* in the sense that it consists in a cross-roads of *ontology* and *semantics*: When we translate a natural language into a first-order (modal) language, (though it is a problem which formal language we should adopt in this translation), the *semantic problem* as to which entity we should choose as the semantic value of a symbol in the *model* of first-order modal logic depends crucially on the *ontological problem* as to which ontology we should adopt. According to Rodriguez-Pereyra [5], there are at least two kinds of *Nominalism*: one that maintains that there are no *universals* and the other that maintains that there are no *abstract objects* like classes, functions, numbers and possible worlds. On the other hand, *Realism* about universals is the doctrine that there are universals, and *Platonism* about abstract objects is the doctrine that there are abstract objects. The doctrines about universals and the doctrines about abstract objects are *independent*. Nominalisms about universals can be classified into at least eight types: Trope Theory, Predicate Nominalism, Concept Nominalism, Ostrich Nominalism, Mereological Nominalism, Class Nominalism, Resemblance Nominalism, and Causal Nominalism.[1] In this paper we focus on Resemblance Nominalism. Rodriguez-Pereyra [4] is the most frequently mentioned work in the field of Resemblance Nominalism. As Rodriguez-Pereyra [5] argues, according to Resemblance Nominalism, it is not because things are scarlet that they resemble one another, but what makes them scarlet is that they resemble one another. Resemblance relations are *primitive* and the *properties* of a thing are *defined* by resemblance relations. Resemblance Nominalism reifies neither resemblance relations nor accessibility relations in themselves. Resemblance Nominalism in general is confronted with at least seven problems: Imperfect Community Problem, Companionship Problem, Mere Intersections Problem, Contingent Coextension Problem, Necessary Coextension Problem, Infinite Regress Problem, and Degree of Resemblance Problem.[2] We [8] proposed, in terms of *measurement theory*, a first-order modal resemblance logic MRL that can furnish solutions to all of these problems. Yi [10] raises a version of degree of resemblance problem. Yi [10, pp.622-625] argues as follows:

(1) Carmine resembles vermillion more than it resembles triangularity.

(2) is a resemblance-nominalistic formulation that expresses what makes (1) true:

(2) Some carmine particular resembles some vermillion particular more closely than any carmine particular resembles any triangular particular.

Rodriguez-Pereyra [4, p.65] defines the *degree of resemblance* as follows :

Definition 1 (Degree of Resemblance). *The particulars resemble to the degree n iff they share n properties.*

[1] Refer to Rodriguez-Pereyra [5] for details of these eight types.
[2] Refer to Rodriguez-Pereyra [4] for details of these seven problems.

By "properties", Rodriguez-Pereyra means *sparse* properties. Rodriguez-Pereyra [4, p.20,pp.50-52] adopts the following Lewis [2]'s distinction between abundant and sparse properties:

> [The *abundant* properties] pay no heed to the qualitative joints, but carve things up every which way. Sharing them has nothing to do with *similarity* [(*resemblance*)] ... There is one of them for any condition we could write down, even if we could write at infinite length and even if we could name all those things that must remain nameless because they fall outside our acquaintance. [They] are as abundant as the sets themselves, because for any whatever, there is the property of belonging to that set ... The *sparse* properties are another story. Sharing of them makes for qualitative *similarity* [(*resemblance*)], they carve at the joints, they are intrinsic, they are highly specific, the sets of their instances are *ipso facto* not miscellaneous, they are only just enough of them to characterise things completely sand without redundancy.[2, pp. 59-60]

In this paper, we use "properties" in this sense of sparse properties as well as Rodriguez-Pereyra. Under Definition 1, (2) compares the *maximum* degrees of resemblance. But (2) is false because a possible carmine particular completely resembles a possible triangular particular. For the same particular might be both carmine and triangular. Rodriguez-Pereyra [6] responses to Yi by replacing (2) by (3):

> (3) Some carmine particular resembles some triangular particular less closely than any carmine particular resembles any vermillion particular.

Again under Definition 1, (3) compares the *minimum* degrees of resemblance. Rodriguez-Pereyra [6, p.225] argues that (3) is true because the minimum degree to which a carmine particular can resemble a triangular particular (degree 0) is smaller than the minimum degree to which a carmine particular can resemble a vermillion particular (a degree greater than 0). Yi [11, p.796] criticizes this Rodriguez-Pereyra's response by arguing that it rests on a false assumption: the minimum degree to which a carmine particular can resemble a vermillion particular is greater than 0. For, on Rodriguez-Pereyra's notion of resemblance, a carmine particular cannot resemble a vermillion particular unless they share a sparse property, but they might not share any such property. A carmine particular and a vermillion particular might share no non-color sparse property, and two such particulars share also no color sparse property because they have different determinate color properties (i.e., carminity and vermillionity). Although they share *determinable* color properties (e.g., red), this does not help because, in Rodriguez-Pereyra's view, determinable properties are not sparse properties. So the minimum degree to which a carmine particular can resemble a vermillion particular might be 0. No doubt this argument by Yi needs examining in detail, but we can safely say that the main culprit of this Rodriguez-Pereyra-Yi Problem is Definition 1 on which both (2) and (3) are based. We consider this problem to be a problem of multidimensionality (such three dimensionality as carminity, vermillionity and triangularity) that requires *quantitative (numerical)* representations because we cannot have computational method of *aggregation* only in terms of qualitative resemblance relations. When we considered this problem, we realized that the model of MRL was not able to deal appropriately with the multidimensionality of this type of problem. The *aim* of this paper is to revise MRL so that the *revised first-order modal resemblance logic* RMRL can solve Rodriguez-Pereyra-Yi Problem in terms of measurement-theoretic multidimensional representation.[3] Measurement theory makes it possible that qualitative resemblance relations can represent quantitative (numerical) functions, whereas it is not designed to explicate the *parthood between a particular and its parts* (referred to for determining the raking on a resemblance relation). So, in the construction of the *multidimensional model* of RMRL, we would like to connect measurement-theory with *mereology*[4] that can explicate the *parthood between a particular and its parts* referred to for determining the raking on a resemblance relation. The punch line of Resemblance Nominalism is the *reducibility* of universals into resemblance relations. The point of *formalizing* Resemblance Nominalism in RMRL is to avoid the *circularity* in this reduction into which it tends to slide. In this paper, we try to give a solution to Rodriguez-Pereyra-Yi Problem by *defining* in RMRL the *degree of unresemblance* (*Definition 20*), instead of using Definition 1 (on which both (2) and (3) are based) that is the main culprit of this problem so that, in the multidimensional comparison of unresemblance of (1),

> the *weighted sum* of the degrees of unresemblance of carmine particulars to triangular particulars may be greater than that of carmine particulars to vermillion particulars.

In so doing, RMRL obtains the capacity to deal with multidimensionality in general beyond Rodriguez-Pereyra-Yi Problem. In the semantics of RMRL, a resemblance relation is *primitive* and the degree of unresemblance is *defined* in Definition 20 by it via Representation Theorem (Theorem 3) and Uniqueness Theorem (Theorem 4).

The structure of this paper is as follows. In Subsect. 2.1, we define the language $\mathscr{L}$ of RMRL. In Subsubsect. 2.2.1, we define three measurement-theoretic concepts. In Subsubsect. 2.2.2, we prepare the seven steps to a mereological additive difference factorial proximity structured model $\mathfrak{M}$ of RMRL. In Subsubsect. 2.2.3, we provide RMRL with a satisfaction definition relative to $\mathfrak{M}$, define the truth at $w \in W$ in $\mathfrak{M}$, define validity. In Subsubsect. 2.2.4, we show the representation and uniqueness theorems for (multidimensional) resemblance predicates. In Sect. 3, we conclude by giving a solution to Rodriguez-Pereyra-Yi Problem by RMRL.

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[3] About measurement-theoretic multidimensional representation, refer to Suppes et al. [7].

[4] About mereology, refer to Varzi [9].

2 Measurement Theory Meets Meleology in RMRL

2.1 Language

In this paper, we focus only on the ontology of*properties* that are the sematic values of one-place predicate symbols. So we do not introduce n-place predicate symbols ($n \geq 2$) in general into the language of RMRL the semantic values of which are n-ary relations, though we introduce *four-place resemblance predicate symbols* indexed by one-place predicate symbols. We define the language $\mathscr{L}$ of revised first-order modal resemblance logic RMRL:

Definition 2 (Language).

- *Let* $\mathscr{V}$ *denote a class of individual variables,* $\mathscr{C}$ *a class of individual constants, and* $\mathscr{P}$ *a class of* one-place *predicate symbols.*
- *Let* $\leqslant_F$ *denote a* four-place resemblance predicate symbol *indexed by* F*.*
- *When* $n \geq 2$*, let* $\leqslant_{F_1 \times \cdots \times F_n}$ *denote a four-place resemblance predicate symbol indexed by* $F_1, \ldots, F_n$*.*
- *The language* $\mathscr{L}$ *of* RMRL *is given by the following BNF grammar:*

$$
\begin{aligned}
&t ::= x \mid a \\
&\varphi ::= F(t) \mid t_1 = t_2 \mid \bot \mid \neg\varphi \mid \varphi \wedge \psi \mid \\
&(t_1, t_2) \leqslant_F (t_3, t_4) \mid (t_1, t_2) \leqslant_{F_1 \times \cdots \times F_n} (t_3, t_4) \mid \Box\varphi \mid \forall x\varphi,
\end{aligned}
$$

  where $x \in \mathscr{V}$*,* $a \in \mathscr{C}$*, and* $F_1, \ldots, F_n \in \mathscr{P}$*.*
- $\top$*,* $\vee$*,* $\rightarrow$*,* $\leftrightarrow$*,* $<_F$*,* $<_{F_1 \times \cdots \times F_n}$*,* $\Diamond$ *and* $\exists$ *are introduced by the standard definitions.*
- $(t_1, t_2) \leqslant_F (t_3, t_4)$ *means that* t_3 does not resemble t_4 more than t_1 resembles t_2 with respect to F-ness.
- *When* $n \geq 2$*,* $(t_1, t_2) \leqslant_{F_1 \times \cdots \times F_n} (t_3, t_4)$ *means that* t_3 does not resemble t_4 more than t_1 resembles t_2 with respect to F_1-ness and ... and F_n-ness.
- *The set of all well-formed formulae of* $\mathscr{L}$ *is denoted by* $\Phi_{\mathscr{L}}$*.*

Remark 1 (Modal Part of RMRL). *In this paper, we do not deal with Contingent Coextension and Necessary Coextension Problems above neither of which relates to multidimensionality that is the main topic of this paper, though we did in [8]. The motivation to introduce a modality* $\Box$ *into* $\mathscr{L}$ *is only to solve Contingent Coextension and Necessary Coextension Problems.*

2.2 Semantics

2.2.1 Three Measurement-Theoretic Concepts Here we would like to define such measurement-theoretic concepts as

1. *scale types*,
2. *representation* and *uniqueness theorems*, and
3. *measurement types*

on which the argument of this paper is based: *First*, according to Roberts [3, pp. 64-69], we classify *scale types* in terms of the class of *admissible transformations* φ:

Definition 3 (Scale Types).

- *A scale is a triple* $(\mathfrak{U}, \mathfrak{V}, f)$ *where* $\mathfrak{U}$ *is an observed relational structure that is qualitative,* $\mathfrak{V}$ *is a numerical relational structure that is quantitative, and* f *is a homomorphism from* $\mathfrak{U}$ *into* $\mathfrak{V}$*.*
- *Sometimes we sloppily refer to* f *alone as a scale.*
- *Suppose that* $\mathscr{D}$ *is the domain of* $\mathfrak{U}$ *and that* $\mathscr{D}'$ *is the domain of* $\mathfrak{V}$*. Suppose that* φ *is a function that maps the range of* f*, the set* $f(\mathscr{D}) := \{f(\mathfrak{d}) : \mathfrak{d} \in \mathscr{D}\}$*, into* $\mathscr{D}'$*. Then the composition* $\varphi \circ f$ *is a function from* $\mathscr{D}$ *into* $\mathscr{D}'$*. If* $\varphi \circ f$ *is a homomorphism from* $\mathfrak{U}$ *into* $\mathfrak{V}$*, we call* φ *an* admissible transformation of scale.
- *When the admissible transformations are all the functions* φ *of the form* $\varphi(x) := \alpha x; \alpha > 0$*.* φ *is called a* similarity transformation*, and a scale with the similarity transformations as its class of admissible transformations is called a* ratio scale*.*
- *When the admissible transformations are all the functions* φ *of the form* $\varphi(x) := \alpha x + \beta; \alpha > 0$*,* φ *is called a* positive affine transformation*, and a corresponding scale is called an* interval scale*.*
- *When the admissible transformations are all the functions* φ *of the form* $\varphi(x) := \alpha x + \beta; \alpha \neq 0$*,* φ *is called an* affine transformation*, and a corresponding scale is called a* quasi-interval scale*.*
- *When a scale is unique up to* order*, the admissible transformations are monotone increasing functions* $\varphi(x)$*, that is, functions* $\varphi(x)$ *satisfying the condition that* $x \precsim y$ *iff* $\varphi(x) \leq \varphi(y)$*, where* $\precsim$ *is a binary relation on* $\mathscr{D}$*. Such a scale is called an* ordinal scale*.*

Example 1 (Mass and Temperature).

- *The measurement of mass is the assignment of a homomorphism* f *from the observed relational structure* $(A, H, \bigcirc)$ *(where we judge* $\mathfrak{d}_1$ *to be heavier than* $\mathfrak{d}_2$ *and the binary operation satisfies* $f(\mathfrak{d}_1 \bigcirc \mathfrak{d}_2) = f(\mathfrak{d}_1) + f(\mathfrak{d}_2)$ *for any* $\mathfrak{d}_1, \mathfrak{d}_2 \in A$*) to the numerical relational structure* $(\mathbb{R}, >, +)$*. Mass is an example of a ratio scale.*
- *The measurement of temperature is the assignment of a homomorphism* f *from the observed relational structure* (A, W) *(where* A *is a set of objects and the binary relation* $\mathfrak{d}_1 W \mathfrak{d}_2$ *holds iff we judge* $\mathfrak{d}_1$ *to be warmer than* $\mathfrak{d}_2$*) to the numerical relational structure* $(\mathbb{R}, >)$*. Temperature is an example of an interval scale.*

Second, according to Roberts [3, pp. 54-56], we define *representation* and *uniqueness theorems*:

Definition 4 (Representation Theorem and Uniqueness Theorem).

- *The first basic problem of measurement theory is the* representation problem*: Given a numerical relational structure $\mathfrak{V}$, find conditions on an observed relational structure $\mathfrak{U}$ (necessary and) sufficient for the* existence *of a homomorphism f from $\mathfrak{U}$ to $\mathfrak{V}$ that preserves all the relations and operations in $\mathfrak{U}$.*
- *The theorem stating conditions on $\mathfrak{U}$ are (necessary and) sufficient for the existence of f is called a* representation theorem.
- *The second basic problem of measurement theory is the* uniqueness problem*: Find the transformation of the homomorphism f under which all the relations and operations in $\mathfrak{U}$ are preserved.*
- *The theorem stating the type of transformation up to which f is unique is called a* uniqueness theorem.

Third, according to Roberts [3, pp. 122-131, pp. 134-142] and Krantz et al., [1, pp. 136-157], we classify *measurement types*:

Definition 5 (Measurement Types).

- *Suppose $\mathscr{D}$ is a set, $\precsim'$ is a binary relation on $\mathscr{D}$, $\bigcirc$ is a binary operation on $\mathscr{D}$, $\precsim$ is a quaternary relation on $\mathscr{D}$, and f is a real-valued function.*
- *Then we call the representation $\mathfrak{d}_1 \precsim' \mathfrak{d}_2$ iff $f(\mathfrak{d}_1) \leq f(\mathfrak{d}_2)$, for any $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$, and $f(\mathfrak{d}_1 \bigcirc \mathfrak{d}_2) = f(\mathfrak{d}_1) + f(\mathfrak{d}_2)$, for any $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$, extensive measurement.*
- *We call the representation $(\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}_3, \mathfrak{d}_4)$ iff $f(\mathfrak{d}_1) - f(\mathfrak{d}_2) \leq f(\mathfrak{d}_3) - f(\mathfrak{d}_4)$, for any $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4 \in \mathscr{D}$, when the direction of differences is taken into consideration,* positive-difference measurement*, when the direction of differences is not taken into consideration,* algebraic-difference measurement.
- *We call the representation $(\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}_3, \mathfrak{d}_4)$ iff $|f(\mathfrak{d}_1) - f(\mathfrak{d}_2)| \leq |f(\mathfrak{d}_3) - f(\mathfrak{d}_4)|$ for any $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4 \in \mathscr{D}$,* absolute-difference measurement.

2.2.2 Seven Steps to Construct Model $\mathfrak{M}$ of RMRL

By using some measurement-theoretic concepts of Krantz et al. [1] and Suppes et al. [7], we prepare the following *seven steps* to construct a model $\mathfrak{M}$ of RMRL:

2.2.2.1 ***First Step***

The first step is a step to prepare an *absolute difference* structure for the semantics of $\leqslant_F$ and $\leqslant_{F_1 \times \cdots \times F_n}$. We resort to an *absolute difference structure* in order to solve the problems of Resemblance Nominalism. Krantz et al. [1, pp.172-173] define an absolute difference structure:

Definition 6 (Absolute Difference Structure). *Suppose $\mathscr{D}$ is a nonempty set and $\precsim$ a* quaternary relation *on $\mathscr{D}$ (*binary relation *on $\mathscr{D} \times \mathscr{D}$). $(\mathscr{D}, \precsim)$ is an* absolute difference structure *iff, for any $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4, \mathfrak{d}'_1, \mathfrak{d}'_2, \mathfrak{d}'_3 \in \mathscr{D}$, the following* six *conditions are satisfied:*

Condition 1 (Weak Order) *$\precsim$ is a weak order (*Connected *and* Transitive*).*
Condition 2 (Absoluteness) *If $\mathfrak{d}_1 \neq \mathfrak{d}_2$, then $(\mathfrak{d}_1, \mathfrak{d}_1) \sim (\mathfrak{d}_2, \mathfrak{d}_2) \prec (\mathfrak{d}_1, \mathfrak{d}_2) \sim (\mathfrak{d}_2, \mathfrak{d}_1)$, where $(\mathfrak{d}_1, \mathfrak{d}_2) \sim (\mathfrak{d}_3, \mathfrak{d}_4) := (\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}_3, \mathfrak{d}_4)$ and $(\mathfrak{d}_3, \mathfrak{d}_4) \precsim (\mathfrak{d}_1, \mathfrak{d}_2)$, and $(\mathfrak{d}_1, \mathfrak{d}_2) \prec (\mathfrak{d}_3, \mathfrak{d}_4) := (\mathfrak{d}_3, \mathfrak{d}_4) \not\precsim (\mathfrak{d}_1, \mathfrak{d}_2)$.*

Condition 3 (Betweenness)

1. *If* $\mathfrak{d}_2 \neq \mathfrak{d}_3$*,* $(\mathfrak{d}_1, \mathfrak{d}_2), (\mathfrak{d}_2, \mathfrak{d}_3) \precsim (\mathfrak{d}_1, \mathfrak{d}_3)$*, and* $(\mathfrak{d}_2, \mathfrak{d}_3), (\mathfrak{d}_3, \mathfrak{d}_4) \precsim (\mathfrak{d}_2, \mathfrak{d}_4)$*, then* $(\mathfrak{d}_1, \mathfrak{d}_3), (\mathfrak{d}_2, \mathfrak{d}_4) \precsim (\mathfrak{d}_1, \mathfrak{d}_4)$*.*
2. *If* $(\mathfrak{d}_1, \mathfrak{d}_2), (\mathfrak{d}_2, \mathfrak{d}_3) \precsim (\mathfrak{d}_1, \mathfrak{d}_3)$ *and* $(\mathfrak{d}_1, \mathfrak{d}_3), (\mathfrak{d}_3, \mathfrak{d}_4) \precsim (\mathfrak{d}_1, \mathfrak{d}_4)$*, then* $(\mathfrak{d}_1, \mathfrak{d}_3) \precsim (\mathfrak{d}_1, \mathfrak{d}_4)$*.*

Condition 4 (Weak Monotonicity) *Suppose that* $(\mathfrak{d}_1, \mathfrak{d}_2), (\mathfrak{d}_2, \mathfrak{d}_3) \precsim (\mathfrak{d}_1, \mathfrak{d}_3)$*. If* $(\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}'_1, \mathfrak{d}'_2)$ *and* $(\mathfrak{d}_2, \mathfrak{d}_3) \precsim (\mathfrak{d}'_2, \mathfrak{d}'_3)$*, then* $(\mathfrak{d}_1, \mathfrak{d}_3) \precsim (\mathfrak{d}'_1, \mathfrak{d}'_3)$*. Moreover if either* $(\mathfrak{d}_1, \mathfrak{d}_2) \prec (\mathfrak{d}'_1, \mathfrak{d}'_2)$ *or* $(\mathfrak{d}_2, \mathfrak{d}_3) \prec (\mathfrak{d}'_2, \mathfrak{d}'_3)$*, then* $(\mathfrak{d}_1, \mathfrak{d}_3) \prec (\mathfrak{d}'_1, \mathfrak{d}'_3)$*.*

Condition 5 (Solvability) *If* $(\mathfrak{d}_3, \mathfrak{d}_4) \precsim (\mathfrak{d}_1, \mathfrak{d}_2)$*, then there exists* $\mathfrak{d}'_4 \in \mathscr{D}$ *such that* $(\mathfrak{d}'_4, \mathfrak{d}_2) \precsim (\mathfrak{d}_1, \mathfrak{d}_2)$ *and* $(\mathfrak{d}_1, \mathfrak{d}'_4) \sim (\mathfrak{d}_3, \mathfrak{d}_4)$*.*

Condition 6 (Archimedean Property) *If* $\mathfrak{d}_1^{(1)}, \mathfrak{d}_1^{(2)}, \ldots, \mathfrak{d}_1^{(i)}, \ldots$ *is a strictly bounded standard sequence (i.e., there exist* $\mathfrak{d}_2, \mathfrak{d}_3 \in \mathscr{D}$ *such that for any* $i = 1, 2, \ldots$*,* $(\mathfrak{d}_1^{(i)}, \mathfrak{d}_1^{(1)}) \precsim (\mathfrak{d}_1^{(i+1)}, \mathfrak{d}_1^{(1)}) \prec (\mathfrak{d}_2, \mathfrak{d}_3)$ *and* $(\mathfrak{d}_1^{(1)}, \mathfrak{d}_1^{(1)}) \prec (\mathfrak{d}_1^{(2)}, \mathfrak{d}_1^{(1)}) \sim (\mathfrak{d}_1^{(i+1)}, \mathfrak{d}_1^{(i)})$*), then the sequence is finite.*

The following definition [1, p.172] makes Conditions 3-6 easy to understand.

Definition 7 (Betweenness). *Suppose* $(\mathscr{D}, \precsim)$ *satisfies Conditions 1 and 2 of Definition 6. We say that* $\mathfrak{d}_2$ *is between* $\mathfrak{d}_1$ *and* $\mathfrak{d}_3$ *(in symbols,* $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_3$*) iff* $(\mathfrak{d}_1, \mathfrak{d}_2), (\mathfrak{d}_2, \mathfrak{d}_3) \precsim (\mathfrak{d}_1, \mathfrak{d}_3)$*.*

We can replace Conditions 3-6 by the following Conditions 3'-6':

Condition 3' (Betweenness)

1. If $\mathfrak{d}_2 \neq \mathfrak{d}_3$, $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_3$, and $\mathfrak{d}_2|\mathfrak{d}_3|\mathfrak{d}_4$, then both $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_4$ and $\mathfrak{d}_1|\mathfrak{d}_3|\mathfrak{d}_4$.
2. If $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_3$ and $\mathfrak{d}_1|\mathfrak{d}_3|\mathfrak{d}_4$, then $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_4$.

Condition 4' (Weak Monotonicity) If $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_3$, $\mathfrak{d}'_1|\mathfrak{d}'_2|\mathfrak{d}'_3$, and $(\mathfrak{d}_1, \mathfrak{d}_2) \sim (\mathfrak{d}'_1, \mathfrak{d}'_2)$, then $(\mathfrak{d}_2, \mathfrak{d}_3) \precsim (\mathfrak{d}'_2, \mathfrak{d}'_3)$ iff $(\mathfrak{d}_1, \mathfrak{d}_3) \precsim (\mathfrak{d}'_1, \mathfrak{d}'_3)$.

Condition 5' (Solvability) If $(\mathfrak{d}_3, \mathfrak{d}_4) \precsim (\mathfrak{d}_1, \mathfrak{d}_2)$ then there exists $\mathfrak{d}'_4 \in \mathscr{D}$ with $\mathfrak{d}_1|\mathfrak{d}'_4|\mathfrak{d}_2$ and $(\mathfrak{d}_1, \mathfrak{d}'_4) \sim (\mathfrak{d}_3, \mathfrak{d}_4)$.

Condition 6' (Archimedean Property) If $\mathfrak{d}_1^{(i+1)}|\mathfrak{d}_1^{(i)}|\mathfrak{d}_1^{(1)}$ for any $i = 1, 2, \ldots$, successive intervals are equal and nonnull, and $(\mathfrak{d}_1^{(i)}, \mathfrak{d}_1^{(1)})$ is strictly bounded, then the sequence is finite.

Krantz et al. [1, pp.173-177] prove the following theorems:

Fact 1 (Representation). *If* $(\mathscr{D}, \precsim)$ *is an* absolute difference structure*, then there exists a real-valued function* f *on* $\mathscr{D}$ *such that, for any* $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4 \in \mathscr{D}$*,* $(\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}_3, \mathfrak{d}_4)$ *iff* $|f(\mathfrak{d}_1) - f(\mathfrak{d}_2)| \leq |f(\mathfrak{d}_3) - f(\mathfrak{d}_4)|$*.*

Fact 2 (Uniqueness). *The above function* f *is a* quasi-interval scale*.*

2.2.2.2 ***Second Step***

The second step is a step to prepare a basic *multidimensional* structure for $\leqslant_{F_1 \times \cdots \times F_n}$. Suppes et al. [7, pp. 160-161] define a basic multidimensional comparison structure, called a *factorial proximity structure*:

Definition 8 (Factorial Proximity Structure).

– $(\mathscr{D}, \precsim)$ *is a* proximity structure *iff the following conditions are satisfied for any* $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$*:*
  - $\precsim$ *is a* weak order.
  - $(\mathfrak{d}_1, \mathfrak{d}_1) \prec (\mathfrak{d}_1, \mathfrak{d}_2)$ *whenever* $\mathfrak{d}_1 \neq \mathfrak{d}_2$.
  - $(\mathfrak{d}_1, \mathfrak{d}_1) \sim (\mathfrak{d}_2, \mathfrak{d}_2)$ *(*Minimality*).*
  - $(\mathfrak{d}_1, \mathfrak{d}_2) \sim (\mathfrak{d}_2, \mathfrak{d}_1)$ *(*Symmetricity*).*

– *The structure is called* n-factorial *iff* $\mathscr{D} := \prod_{i=1}^{n} \mathscr{D}_i$.

– *We use the expression "*$\mathfrak{d}_1 \cdots \mathfrak{d}_n (\in \mathscr{D})$*" for the* n*-tuple of* $\mathfrak{d}_i \in \mathscr{D}_i$ *(*$1 \leq i \leq n$*).*

Remark 2 (Motivation to Introduce Mereology into Model of RMRL). *The motivation to introduce mereology into the model* $\mathfrak{M}$ *of* RMRL *is that the ontological status of this* n*-tuple* $\mathfrak{d}_1 \cdots \mathfrak{d}_n$ *is not clear.*

2.2.2.3 ***Third Step***

In order to make each dimensional factor the *absolute* value of a scale *difference*, we first establish decomposability of a factorial proximity structure $(\mathscr{D}, \precsim)$ into each factor $(\mathscr{D}_i, \precsim_i)$ where $\precsim_i$ is an induced weak order of Definition 10 below. To achieve it, $(\mathscr{D}, \precsim)$ must satisfy *Betweenness*, *Restricted Solvability*, and the *Archimedean Property*. In order to define Betweenness, we need *One-Factor Independence*. Suppes et al. [7, pp.178-181] define these concepts as follows:

Definition 9 (One-Factor Independence). *A factorial proximity structure* $(\mathscr{D}, \precsim)$ *satisfies* One-Factor Independence *iff the following holds for any* $\mathfrak{d}_1, \mathfrak{d}'_1, \mathfrak{d}_2, \mathfrak{d}'_2, \mathfrak{d}_3, \mathfrak{d}'_3, \mathfrak{d}_4, \mathfrak{d}'_4 \in \mathscr{D}$*: If the two elements in each of the pairs* $(\mathfrak{d}_1, \mathfrak{d}'_1), (\mathfrak{d}_2, \mathfrak{d}'_2), (\mathfrak{d}_3, \mathfrak{d}'_3), (\mathfrak{d}_4, \mathfrak{d}'_4)$ *have identical components on all but one factor, and two elements in each of the pairs* $(\mathfrak{d}_1, \mathfrak{d}_3), (\mathfrak{d}'_1, \mathfrak{d}'_3), (\mathfrak{d}_2, \mathfrak{d}_4), (\mathfrak{d}'_2, \mathfrak{d}'_4)$ *have identical components on the remaining factor, then*

$$(\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}'_1, \mathfrak{d}'_2) \quad \textit{iff} \quad (\mathfrak{d}_3, \mathfrak{d}_4) \precsim (\mathfrak{d}'_3, \mathfrak{d}'_4).$$

If we consider all pairs whose elements differ with respect to the i th factor only, then one-factor independence asserts that for any $i (1 \leq i \leq n)$ the induced weak order $\precsim_i$ on $\mathscr{D}_i \times \mathscr{D}_i$ of Definition 10 below is independent of the fixed components of the remaining $\mathscr{D}_i \times \mathscr{D}_i$ for $j \neq i$.

Definition 10 (Betweenness).

– *Let* $(\mathscr{D} := \prod_{i=1}^{n} \mathscr{D}_i, \precsim)$ *be a factorial proximity structure that satisfies One-Factor Independence.*

– *Let* $\precsim_i$ *denote an induced weak order on* $\mathscr{D}_i \times \mathscr{D}_i$.

– *We say that* $\mathfrak{d}_2$ *is between* $\mathfrak{d}_1$ *and* $\mathfrak{d}_3$*, denoted by* $\mathfrak{d}_1 | \mathfrak{d}_2 | \mathfrak{d}_3$*, iff*

$$(\mathfrak{d}_1^{(i)}, \mathfrak{d}_2^{(i)}), (\mathfrak{d}_2^{(i)}, \mathfrak{d}_3^{(i)}) \precsim_i (\mathfrak{d}_1^{(i)}, \mathfrak{d}_3^{(i)}) \quad \textit{for any } i.$$

– *A factorial proximity structure* $(\mathscr{D}, \precsim)$ *satisfies* Betweenness *iff the following hold for any* $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4, \mathfrak{d}'_1, \mathfrak{d}'_2, \mathfrak{d}'_3 \in \mathscr{D}$*:*

1. *Suppose that* $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4$ *differ on at most one factor, and* $\mathfrak{d}_2 \neq \mathfrak{d}_3$*, then*
   (a) if $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_3$ *and* $\mathfrak{d}_2|\mathfrak{d}_3|\mathfrak{d}_4$*, then* $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_4$ *and* $\mathfrak{d}_1|\mathfrak{d}_3|\mathfrak{d}_4$*, and*
   (b) if $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_3$ *and* $\mathfrak{d}_1|\mathfrak{d}_3|\mathfrak{d}_4$*, then* $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_4$ *and* $\mathfrak{d}_2|\mathfrak{d}_3|\mathfrak{d}_4$*.*
2. *Suppose that* $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}'_1, \mathfrak{d}'_2, \mathfrak{d}'_3$ *differ on at most one factor,* $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_3$*,* $\mathfrak{d}'_1|\mathfrak{d}'_2|\mathfrak{d}'_3$*, and* $(\mathfrak{d}_2, \mathfrak{d}_3) \sim (\mathfrak{d}'_2, \mathfrak{d}'_3)$*, then*

$$(\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}'_1, \mathfrak{d}'_2) \quad \textit{iff} \quad (\mathfrak{d}_1, \mathfrak{d}_3) \precsim (\mathfrak{d}'_1, \mathfrak{d}'_3).$$

Betweenness (Definition 10) is an extension of the one-dimensional concept of Betweenness (Condition 3) of Definition 6 above. Betweenness (Definition 10) is a one-dimensional property that each induced weak order $\precsim_i$ must satisfy.

Definition 11 (Restricted Solvability). *A factorial proximity structure* $(\mathscr{D}, \precsim)$ *satisfies* Restricted Solvability *iff, for any* $\mathfrak{d}_1, \mathfrak{d}_3, \mathfrak{d}_4, \mathfrak{d}_5, \mathfrak{d}_6 \in \mathscr{D}$*, if* $(\mathfrak{d}_4, \mathfrak{d}_3) \precsim (\mathfrak{d}_5, \mathfrak{d}_6) \precsim (\mathfrak{d}_4, \mathfrak{d}_1)$*, then there exists* $\mathfrak{d}_2 \in \mathscr{D}$ *such that* $\mathfrak{d}_1|\mathfrak{d}_2|\mathfrak{d}_3$ *and* $(\mathfrak{d}_4, \mathfrak{d}_2) \sim (\mathfrak{d}_5, \mathfrak{d}_6)$*.*

Just as the role of Solvability (Condition 5) of Definition 6 above is to determine a class of absolute difference structures of Definition 6 on the basis of which Fact 1 (Representation) above can be proved, so the role of Restricted Solvability is to determine a class of additive difference factorial proximity structures of Definition 15 below on the basis of which Theorem 1 (Representation) below can be proved.

Definition 12 (Archimedean Property). *A factorial proximity structure* $(\mathscr{D}, \precsim)$ *satisfies the* Archimedean Property *iff, for any* $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4 \in \mathscr{D}$ *with* $\mathfrak{d}_1 \neq \mathfrak{d}_2$*, any sequence* $\{\mathfrak{d}_5^{(i)} : \mathfrak{d}_5^{(i)} \in \mathscr{D}, i = 0, 1, \ldots\}$ *that varies on at most one factor, such that*

$$\mathfrak{d}_5^{(0)} = \mathfrak{d}_3,$$

$$(\mathfrak{d}_1, \mathfrak{d}_2) \prec (\mathfrak{d}_5^{(i)}, \mathfrak{d}_5^{(i+1)}) \textit{ and } (\mathfrak{d}_3, \mathfrak{d}_5^{(i)}) \prec (\mathfrak{d}_3, \mathfrak{d}_5^{(i+1)}) \prec (\mathfrak{d}_3, \mathfrak{d}_4) \textit{ for any } i,$$

is finite.

Just as the Archimedean Property (Condition 6) of Definition 6 above is a technically necessary condition to prove Fact 1 (Representation) above and Fact 5 (Representation) below, so the Archimedean Property (Definition 12) is a technically necessary condition to prove Fact 3 (Representation) below. Suppes et al. [7, p. 181] prove the following theorems:

Fact 3 (Representation). *Suppose* $(\mathscr{D}, \precsim)$ *is a factorial proximity structure that satisfies One-Factor Independence (Definition 9), Betweenness (Definition 10), Restrict Solvability (Definition 11), and the Archimedean Property (Definition 12). Then there exist real-valued functions* f_i *defined on* $\mathscr{D}_i$ $(1 \leq i \leq n)$ *and real-valued function* g *that increases in each of* n *real arguments such that*

$$\delta(\mathfrak{d}_1, \mathfrak{d}_2) := g(|f_1(\mathfrak{d}_1^{(1)}) - f_1(\mathfrak{d}_2^{(1)})|, \ldots, |f_n(\mathfrak{d}_1^{(n)})) - f_n(\mathfrak{d}_2^{(1)})|)$$

and

$$(\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}_3, \mathfrak{d}_4) \quad \textit{iff} \quad \delta(\mathfrak{d}_1, \mathfrak{d}_2) \leq \delta(\mathfrak{d}_3, \mathfrak{d}_4).$$

Fact 4 (Uniqueness). *The above functions f_i are interval scales, and the above function g is an ordinal scale.*

2.2.2.4 ***Fourth Step***

In order to represent the *sum* of dimensional factors, a factorial proximity structure $(\mathscr{D}, \precsim)$ should satisfy *Independence* and the *Thomsen Condition* only for the dimensionality $n = 2$. Suppes et al. [7, p. 182] define these concepts as follows:

Definition 13 (Independence). *A factorial proximity structure $(\mathscr{D}, \precsim)$ satisfies* Independence *iff the following holds for any $\mathfrak{d}_1, \mathfrak{d}'_1, \mathfrak{d}_2, \mathfrak{d}'_2, \mathfrak{d}_3, \mathfrak{d}'_3, \mathfrak{d}_4, \mathfrak{d}'_4 \in \mathscr{D}$: If the two elements in each of $(\mathfrak{d}_1, \mathfrak{d}'_1), (\mathfrak{d}_2, \mathfrak{d}'_2), (\mathfrak{d}_3, \mathfrak{d}'_3), (\mathfrak{d}_4, \mathfrak{d}'_4)$ have identical components on one factor, and the two elements in each of $(\mathfrak{d}_1, \mathfrak{d}_3), (\mathfrak{d}'_1, \mathfrak{d}'_3), (\mathfrak{d}_2, \mathfrak{d}_4), (\mathfrak{d}'_2, \mathfrak{d}'_4)$ have identical components on all the remaining factors, then*

$$(\mathfrak{d}_1, \mathfrak{d}_2) \precsim (\mathfrak{d}'_1, \mathfrak{d}'_2) \quad \textit{iff} \quad (\mathfrak{d}_3, \mathfrak{d}_4) \precsim (\mathfrak{d}'_3, \mathfrak{d}'_4).$$

Remark 3 (One-Factor Independence and Independence) Independence *(Definition 13) implies* One-Factor Independence *(Definition 9).*

Just as One-Factor Independence (Definition 9) above is a necessary condition to prove Fact 3 (Representation) above, so Independence (Definition 13) is a necessary condition to prove Fact 5 (Representation) below.

Definition 14 (Thomsen Condition). *A factorial proximity structure $(\mathscr{D}, \precsim)$ with $\mathscr{D} := \mathscr{D}_1 \times \mathscr{D}_2$ satisfies the* Thomsen Condition *iff, for any $\mathfrak{d}_1^{(i)}, \mathfrak{d}_2^{(i)}, \mathfrak{d}_3^{(i)}, \mathfrak{d}_4^{(i)}, \mathfrak{d}_5^{(i)}, \mathfrak{d}_6^{(i)} \in \mathscr{D}_i$ $(i = 1, 2)$,*

$$(\mathfrak{d}_1^{(1)}\mathfrak{d}_5^{(2)}, \mathfrak{d}_2^{(1)}\mathfrak{d}_6^{(2)}) \sim (\mathfrak{d}_5^{(1)}\mathfrak{d}_3^{(2)}, \mathfrak{d}_6^{(1)}\mathfrak{d}_4^{(2)})$$

and

$$(\mathfrak{d}_5^{(1)}\mathfrak{d}_1^{(2)}, \mathfrak{d}_6^{(1)}\mathfrak{d}_2^{(2)}) \sim (\mathfrak{d}_3^{(1)}\mathfrak{d}_5^{(2)}, \mathfrak{d}_4^{(1)}\mathfrak{d}_6^{(2)})$$

imply

$$(\mathfrak{d}_1^{(1)}\mathfrak{d}_1^{(2)}, \mathfrak{d}_2^{(1)}\mathfrak{d}_2^{(2)}) \sim (\mathfrak{d}_3^{(1)}\mathfrak{d}_3^{(2)}, \mathfrak{d}_4^{(1)}\mathfrak{d}_4^{(2)}).$$

Remark 4 (Thomsen Condition Only for Two Dimensionality). *The Thomsen Condition must be assumed only when the dimensionality $n = 2$.*

Suppes et al. [7, p. 183] prove the following theorems:

Fact 5 (Representation). *Suppose that $(\mathscr{D}, \precsim)$ is a factorial proximity structure that satisfies Restrict Solvability (Definition 11) and Independence (Definition 13), and that each structure $(\mathscr{D}_i, \precsim_i)$, where $\precsim_i$ is an induced weak order on $\mathscr{D}_i \times \mathscr{D}_i$, satisfies the Archimedean Property (Condition 6 of Definition 6). If $n \geq 3$, then there exist real-valued functions f_i defined on $\mathscr{D}_i \times \mathscr{D}_i$ $(1 \leq i \leq n)$ such that for any $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4 \in \mathscr{D}$,*

$$(\mathfrak{d}_1^{(1)} \cdots \mathfrak{d}_1^{(n)}, \mathfrak{d}_2^{(1)} \cdots \mathfrak{d}_2^{(n)}) \precsim (\mathfrak{d}_3^{(1)} \cdots \mathfrak{d}_3^{(n)}, \mathfrak{d}_4^{(1)} \cdots \mathfrak{d}_4^{(n)})$$

iff

$$\sum_{i=1}^{n} f_i(\mathfrak{d}_1^{(i)}, \mathfrak{d}_2^{(i)}) \geq \sum_{i=1}^{n} f_i(\mathfrak{d}_3^{(i)}, \mathfrak{d}_4^{(i)}).$$

If $n = 2$, then the above assertions hold provided the Thomsen Condition (Definition 14) is also satisfied.

Fact 6 (Uniqueness). *The above functions f_i are interval scales.*

2.2.2.5 ***Fifth Step***

The fifth step is a step to combine the third and fourth steps. Suppes et al. [7, p. 184] define an *additive difference factorial proximity structure* as follows:

Definition 15 (Additive Difference Factorial Proximity Structure). *When $n \geq 2$ and the* factorial proximity structure $(\mathscr{D}(:= \prod_{i=1}^{n} \mathscr{D}_i), \precsim)$ *satisfies* Betweenness, Restricted Solvability, *the* Archimedean Property, Independence, *and the* Thomsen Condition, *we call it an* additive difference factorial proximity structure.

By combining Facts 3–6, Suppes et al. [7, p. 185] prove the following theorems:

Fact 7 (Representation). *If $(\mathscr{D}, \precsim)$ is an additive difference factorial proximity structure (Definition 15), there exist real-valued functions f_i defined on $\mathscr{D}_i$ $(1 \leq i \leq n)$ such that for any $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4 \in \mathscr{D}$,*

$$(\mathfrak{d}_1^{(1)} \cdots \mathfrak{d}_1^{(n)}, \mathfrak{d}_2^{(1)} \cdots \mathfrak{d}_2^{(n)}) \precsim (\mathfrak{d}_3^{(1)} \cdots \mathfrak{d}_3^{(n)}, \mathfrak{d}_4^{(1)} \cdots \mathfrak{d}_4^{(n)})$$

iff

$$\sum_{i=1}^{n} g_i(|f_i(\mathfrak{d}_1^{(i)}) - f_i(\mathfrak{d}_2^{(i)})|) \leq \sum_{i=1}^{n} g_i(|f_i(\mathfrak{d}_3^{(i)}) - f_i(\mathfrak{d}_4^{(i)})|)$$

Fact 8 (Uniqueness). *The above functions f_i are interval scales and the above functions g_i are interval scales with a common unit.*

2.2.2.6 ***Sixth Step***

The *ontological status* of an n-tuple $\mathfrak{d}_1 \cdots \mathfrak{d}_n$ in Definition 8 is not clear. So in order to describe the *parthood between a particular and its parts* referred to for determining the raking on a resemblance relation, we would like to introduce *mereology*:

Definition 16 (Mereology).

- *A* mereological parthood relation P *(Varzi [9, p.14]) is a binary relation on $\mathscr{D}$ satisfying the following properties:*
  - *For any $\mathfrak{d} \in \mathscr{D}$, $P(\mathfrak{d}, \mathfrak{d})$ (*Reflexivity*).*
  - *For any $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3 \in \mathscr{D}$, if $P(\mathfrak{d}_1, \mathfrak{d}_2)$ and $P(\mathfrak{d}_2, \mathfrak{d}_3)$, then $P(\mathfrak{d}_1, \mathfrak{d}_3)$ (*Transitivity*).*

  - *For any* $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$, *if* $P(\mathfrak{d}_1, \mathfrak{d}_2)$ *and* $P(\mathfrak{d}_2, \mathfrak{d}_1)$, *then* $\mathfrak{d}_1$ *equals* $\mathfrak{d}_2$ *(*Antisymmetry*).*
- *For any* $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$, *a* mereological proper parthood relation $PP(\mathfrak{d}_1, \mathfrak{d}_2)$ *is such a binary relation on* $\mathscr{D}$ *that* $P(\mathfrak{d}_1, \mathfrak{d}_2)$ *and* $\mathfrak{d}_1$ *does not equal* $\mathfrak{d}_2$.
- *For any* $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$, *a* mereological overlap relation $O(\mathfrak{d}_1, \mathfrak{d}_2)$ *is such a binary relation on* $\mathscr{D}$ *that there exists* $\mathfrak{d}_3 \in \mathscr{D}$ *such that* $P(\mathfrak{d}_3, \mathfrak{d}_1)$ *and* $P(\mathfrak{d}_3, \mathfrak{d}_2)$.
- *For any* $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$, *if* $PP(\mathfrak{d}_1, \mathfrak{d}_2)$, *then there exists* $\mathfrak{d}_3 \in \mathscr{D}$ *such that* $P(\mathfrak{d}_3, \mathfrak{d}_2)$ *and not* $O(\mathfrak{d}_3, \mathfrak{d}_1)$ *(*Supplementation*) (Varzi [9, pp.51-52]).*
- *For any* $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3 \in \mathscr{D}$, *a* mereological product relation $PR(\mathfrak{d}_3, \mathfrak{d}_1, \mathfrak{d}_2)$ *is such a ternary relation on* $\mathscr{D}$ *that* $P(\mathfrak{d}_4, \mathfrak{d}_3)$ *iff* $P(\mathfrak{d}_4, \mathfrak{d}_1)$ *and* $P(\mathfrak{d}_4, \mathfrak{d}_2)$, *for any* $\mathfrak{d}_4 \in \mathscr{D}$.
- *For any* $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$, *then there exists* $\mathfrak{d}_3 \in \mathscr{D}$ *such that* $PR(\mathfrak{d}_3, \mathfrak{d}_1, \mathfrak{d}_2)$ *(*Product*).*
- *For any* $\mathfrak{d}_1, \mathfrak{d}_2 \in \mathscr{D}$, *we define* $\mathfrak{d}_1 \bigotimes \mathfrak{d}_2$ *as the uniquely existential object bearing the relation* PR *with* $\mathfrak{d}_1$ *and* $\mathfrak{d}_2$, *in symbols,* $\imath\mathfrak{d}_3 PR(\mathfrak{d}_3, \mathfrak{d}_1, \mathfrak{d}_2)$ *(Varzi [9, pp.51-52]).*

Example 2 (Rodriguez-Pereyra-Yi Problem and Mereology). *In Rodriguez-Pereyra-Yi Problem, by means of a mereological parthood function* P, *we would like to describe the parthood between a particular and its parts referred to for determining the raking on resemblance relations with respect to carminity and vermillionity and triangularity. In this case, neither carminity, vermillionity nor triangularity themselves is reified.*

2.2.2.7 ***Final Step***

By connecting *measurement-theoretic* concepts with *mereological* concepts, we define a *mereorogical additive difference factorial proximity structured model* $\mathfrak{M}$ of RMRL:

Definition 17 (Mereorogical Additive Difference Factorial Proximity Structured Model).

- *The* mereological additive difference factorial proximity structured frame *of* RMRL *is a structure*

$$\mathfrak{F} := (\mathscr{W}, R, \mathscr{D}, \{\precsim_F\}_{F \in \mathscr{P}}, \{\mathscr{D}_{\precsim_F}\}_{F \in \mathscr{P}}, P, \{\precsim_{F_1 \times \cdots \times F_n}\}_{F_1 \ldots, F_n \in \mathscr{P}}),$$

  where
  - $\mathscr{W}$ *is a non-empty class of worlds,*
  - R *a binary accessibility relation on* $\mathscr{W}$,
  - $\mathscr{D}$ *a non-empty class of particulars,*
  - $\{\precsim_F\}_{F \in \mathscr{P}}$ *a family of such quaternary relations* $\precsim_F$ *on* $\mathscr{D}$ *that* $(\mathscr{D}, \precsim_F)$ *is an absolute difference structure and* $\precsim_F$ *satisfies* Maximality *of Definition 8,*
  - $\{\mathscr{D}_{\precsim_F}\}_{F \in \mathscr{P}}$ *a non-empty class of* $\mathscr{D}_{\precsim_F}$ *which is a non-empty class of* the parts of particulars referred to for determining the ranking on $\precsim_F$ *and which postulates that there exists a unique* F*-part of a particular belonging to* $\mathscr{D}$,

- P *a mereological parthood relation on* $\mathscr{D} \cup \bigcup_{F \in \mathscr{P}} \mathscr{D}_{\precsim_F}$ *of Definition 16*
- $\{\precsim_{F_1 \times \cdots \times F_n}\}_{F_1 \ldots, F_n \in \mathscr{P}}$ *a family of such quaternary relations* $\precsim_{F_1 \times \cdots \times F_n}$ *on* $\mathscr{D}_{F_1} \times \cdots \times \mathscr{D}_{F_n}$ *that* $(\mathscr{D}_{F_1} \times \cdots \times \mathscr{D}_{F_n}, \precsim_{F_1 \times \cdots \times F_n})$ *is an additive difference factorial proximity structure.*

– *A function* I *is an* interpretation *of* $\mathfrak{F}$ *if* I
- *assigns to each* $a \in \mathscr{C}$ *and each* $w \in \mathscr{W}$ *some object that is a member of* $\mathscr{D}$ *that satisfies* Transworld Identity*: for any* w, w',

$$I(a, w) = I(a, w'),$$

and
- *assigns to each four-place resemblance predicate symbol* $\leqslant_F$ *and each* $w \in \mathscr{W}$ *such a quaternary relation* $\precsim_F$, *and*
- *assigns to each four-place resemblance predicate symbol* $\leqslant_{F_1 \times \cdots \times F_n}$ *and each* $w \in \mathscr{W}$ *such a quaternary relation* $\precsim^*_{F_1 \times \cdots \times F_n}$ *that it is defined as follows:*

if, for any particular $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4 \in \mathscr{D}$, $\imath \mathfrak{d}_{i+4} P(\mathfrak{d}_{i+4}, \mathfrak{d}_1)$, $\imath \mathfrak{d}_{i+5} P(\mathfrak{d}_{i+5}, \mathfrak{d}_2)$, $\imath \mathfrak{d}_{i+6} P(\mathfrak{d}_{i+6}, \mathfrak{d}_3)$, $\imath \mathfrak{d}_{i+7} P(\mathfrak{d}_{i+7}, \mathfrak{d}_4) \in \mathscr{D}_{F_i}$ *are such uniquely existential parts of* $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3, \mathfrak{d}_4$ *respectively, then*

$$(\mathfrak{d}_1, \mathfrak{d}_2) \precsim^*_{F_1 \times \cdots \times F_n} (\mathfrak{d}_3, \mathfrak{d}_4)$$

iff

$$\begin{aligned}&(\bigotimes_{i=1}^{n} \imath \mathfrak{d}_{i+4} P(\mathfrak{d}_{i+4}, \mathfrak{d}_1), \bigotimes_{i=1}^{n} \imath \mathfrak{d}_{i+5} P(\mathfrak{d}_{i+5}, \mathfrak{d}_2))\\ &\precsim_{F_1 \times \cdots \times F_n}\\ &(\bigotimes_{i=1}^{n} \imath \mathfrak{d}_{i+6} P(\mathfrak{d}_{i+6}, \mathfrak{d}_3), \bigotimes_{i=1}^{n} \imath \mathfrak{d}_{i+7} P(\mathfrak{d}_{i+7}, \mathfrak{d}_4)).\end{aligned}$$

(Refer to Definition 16 for the definition of $\otimes$*.)*

– *A* property class $I(F, w)$ *is defined as a* maximal resemblance class *in terms of a* resemblance relation $\precsim_F$*:* $A \subsetneq \mathscr{D}$ *is a* property class $I(F, w)$ *iff* $(\mathscr{D}, \precsim_F)$ *is an absolute difference structure and for any* $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3 \in A$ *and for any* $\mathfrak{d}_4 \in \overline{A}$,

$$(\mathfrak{d}_1, \mathfrak{d}_2) \prec_F (\mathfrak{d}_3, \mathfrak{d}_4) \quad \text{(Maximality)}.$$

– *The mereological additive difference factorial proximity structured model of* RMRL *is a structure*

$$\mathfrak{M} := (\mathscr{W}, R, \mathscr{D}, \{\precsim_F\}_{F \in \mathscr{P}}, \{\mathscr{D}_{\precsim_F}\}_{F \in \mathscr{P}}, P, \{\precsim_{F_1 \times \cdots \times F_n}\}_{F_1 \ldots, F_n \in \mathscr{P}}, I).$$

Remark 5 ($\precsim^*_{F_1 \times \cdots \times F_n}$ **and** $\precsim_{F_1 \times \cdots \times F_n}$)**.** *In this definition, we consider the comparison* ($\precsim^*_{F_1 \times \cdots \times F_n}$) *of differences of resemblance between particulars with respect to* F_1*-ness and ... and* F_n*-ness to be the comparison* ($\precsim_{F_1 \times \cdots \times F_n}$) *of difference of resemblance between the* n*-tuple products of parts of a particular referred to for determining the raking on resemblance relations with respect to* F_1*-ness and ... and with respect to* F_n*-ness, respectively.*

Remark 6 (Not Absoluteness But Conditionality). *The mereological additive difference factorial proximity structured model of* RMRL *does not require that* $\precsim_F$ *should absolutely satisfy such conditions above as Betweenness and the Archimedean Property and so on, but requires that* if $\precsim_F$ *satisfies them,* then *Theorems 1–4 below can be proven.*

Remark 7 (Nominalism about Universals). $\mathfrak{M}$ *is* nominalistic *both about such universals as properties and about* $\precsim_F$ *and* R neither *of which are* reified*, whereas it is* Platonistic *about such abstract objects as classes, functions, numbers and possible worlds. As Rodriguez-Pereyra [5] observes, Realism/Nominalism about universals is* independent *of Platonism/Nominalism about abstract objects.*

Remark 8 (Non-Circularity of Resemblance Relation). *Since a resemblance relation* $\precsim_F$ *depends* not *on a* property class $I(F, w)$ *defined by* $\precsim_F$ but *on a* predicate symbol F*, where* $I(F, w)$ *is the semantic value of* F*. In this sense,* $\precsim_F$ *is not circular.*

Remark 9 (Reducibility and Resemblance Nominalism). $\mathfrak{M}$ *is resemblance-nominalistic in the sense that* $I(F, w)$ *is* reducible *to* $\precsim_F$*.*

2.2.3 Satisfaction Definition

We define an (extended) assignment as follows:

Definition 18 ((Extended) Assignment).

- *We call* $s : \mathscr{V} \to \mathscr{D}$ *an* assignment*.*
- $\tilde{s} : \mathscr{V} \cup \mathscr{C} \to \mathscr{D}$ *is defined as follows:*
  1. *For each* $x \in \mathscr{V}$*,* $\tilde{s}(x) = s(x)$*,*
  2. *For each* $a \in \mathscr{C}$ *and each* $w \in \mathscr{W}$*,* $\tilde{s}(a) = I(a, w)$*.*

  We call $\tilde{s}$ *an extended assignment.*

We provide MRL with the following satisfaction definition relative to $\mathfrak{M}$, define the truth (at a world) in $\mathfrak{M}$ by means of satisfaction and then define validity as follows:

Definition 19 (Satisfaction).

- *What it means for* $\mathfrak{M}$ *to satisfy* $\varphi \in \Phi_{\mathcal{L}}$ *at* $w \in \mathscr{W}$ *with* s*, in symbols* $(\mathfrak{M}, w) \models \varphi[s]$ *is inductively defined as follows:*
  - $(\mathfrak{M}, w) \models (t_1, t_2) \leqslant_F (t_3, t_4)[s]$ *iff* $(\tilde{s}(t_1), \tilde{s}(t_2)) \precsim_F (\tilde{s}(t_3), \tilde{s}(t_4))$,
  - $(\mathfrak{M}, w) \models (t_1, t_2) \leqslant_{F_1 \times \cdots \times F_n} (t_3, t_4)[s]$ *iff* $(\tilde{s}(t_1), \tilde{s}(t_2)) \precsim^*_{F_1 \times \cdots \times F_n} (\tilde{s}(t_3), \tilde{s}(t_4))$,
  - $(\mathfrak{M}, w) \models F(t)[s]$ *iff* $\tilde{s}(t) \in I(F, w)$*, where I(F,w) is defined by Definition 17,*
  - $(\mathfrak{M}, w) \models t_1 = t_2[s]$ *iff* $\tilde{s}(t_1) = \tilde{s}(t_2)$,
  - $(\mathfrak{M}, w) \models \top[s]$,
  - $(\mathfrak{M}, w) \models \neg\varphi[s]$ *iff* $(\mathfrak{M}, w) \not\models \varphi[s]$,

- $(\mathfrak{M}, w) \models \varphi \wedge \psi[s]$ *iff* $(\mathfrak{M}, w) \models \varphi[s]$ *and* $(\mathfrak{M}, w) \models \psi[s]$,
- $(\mathfrak{M}, w) \models \Box\varphi[s]$ *iff for all* $w \in \mathscr{W}$ *such that* $R(w, w')$, $(\mathfrak{M}, w') \models \varphi[s]$,
- $(\mathfrak{M}, w) \models \forall x\varphi[s]$ *iff for any* $\mathfrak{d} \in \mathscr{D}$, $\mathfrak{M} \models \varphi[s(x|\mathfrak{d})]$, *where* $s(x|\mathfrak{d})$ *is the function that is exactly like* s *except for one thing: for the individual variable* x, *it assigns the object* $\mathfrak{d}$. *This can be expressed as follows:*

$$s(x|d)(y) \;:=\; \begin{cases} s(y) & \text{if} \quad y \neq x \\ \mathfrak{d} & \text{if} \quad y = x. \end{cases}$$

– *If* $(\mathfrak{M}, w) \models \varphi[s]$ *for all* s, *we write* $(\mathfrak{M}, w) \models \varphi$ *and say that* φ *is true at* w *in* $\mathfrak{M}$.
– *If* $(\mathfrak{M}, w) \models \varphi$ *for all* $w \in \mathscr{W}$, *we write* $\mathfrak{M} \models \varphi$ *and say that* φ *is true in* $\mathfrak{M}$.
– *If* φ *is true in any model based on the frame of* MRL, *we write* $\models \varphi$ *and say that* φ *is valid.*

The next corollary follows from Definitions 17 and 19:

Corollary 1 (Property Class and Resemblance Relation).

$$(\mathfrak{M}, w) \models F(t)[s]$$

iff

$$\tilde{s}(t) \in I(F, w)$$

iff for any $\mathfrak{d}_2, \mathfrak{d}_3 \in I(F, w)$ *and for any* $\mathfrak{d}_3 \in \overline{I(F, w)}$,

$$(\tilde{s}(t), \mathfrak{d}_1) \prec_F (\mathfrak{d}_2, \mathfrak{d}_3).$$

Remark 10 (Definability by Resemblance Relation). *The satisfaction clause of* $F(t)$ *can be defined by a resemblance relation* $\prec_F$.

2.2.4 Representation and Uniqueness Theorems

Then the next theorems follows from Facts 1 and 2 and Definition 19.

Theorem 1 (Representation).
If $(\mathscr{W}, R, \mathscr{D}, \{\precsim_F\}_{F \in \mathscr{P}}, \{\mathscr{D}_{\precsim_F}\}_{F \in \mathscr{P}}, P, \{\precsim_{F_1 \times \cdots \times F_n}\}_{F_1 \ldots, F_n \in \mathscr{P}}, I)$ *is a mereological additive difference factorial proximity structured model of* RMRL, *then there exists a function* $f : \mathscr{D} \to \mathbb{R}$ *satisfying*

$$(\mathfrak{M}, w) \models (t_1, t_2) \leqslant_F (t_3, t_4)[s]$$

iff

$$(\tilde{s}(t_1), \tilde{s}(t_2)) \precsim_F (\tilde{s}(t_3), \tilde{s}(t_4))$$

iff

$$|f(\tilde{s}(t_1)) - f(\tilde{s}(t_2))| \leq |f(\tilde{s}(t_3)) - f(\tilde{s}(t_4))|.$$

Proof.
Suppose that $(\mathscr{W}, R, \mathscr{D}, \{\precsim_F\}_{F\in\mathscr{P}}, \{\mathscr{D}_{\precsim_F}\}_{F\in\mathscr{P}}, P, \{\precsim_{F_1\times\cdots\times F_n}\}_{F_1\ldots,F_n\in\mathscr{P}}, I)$ is a mereological additive difference factorial proximity structured model of RMRL (Definition 17). Then because, by Definition 17, $(\mathscr{D}, \precsim_F)$ is an absolute difference structure (Definition 6), by Fact 1, there exists a function $f : \mathscr{D} \to \mathbb{R}$ satisfying

$$(\tilde{s}(t_1), \tilde{s}(t_2)) \precsim_F (\tilde{s}(t_3), \tilde{s}(t_4))$$

iff

$$|f(\tilde{s}(t_1)) - f(\tilde{s}(t_2))| \leq |f(\tilde{s}(t_3)) - f(\tilde{s}(t_4))|.$$

On the other hand, by Definition 19, we have

$$(\mathfrak{M}, w) \models (t_1, t_2) \leqslant_F (t_3, t_4)[s]$$

iff

$$(\tilde{s}(t_1), \tilde{s}(t_2)) \precsim_F (\tilde{s}(t_3), \tilde{s}(t_4)).\square$$

Theorem 2 (Uniqueness). *The above function f is a quasi-interval scale.*

By Facts 7 and 8 and Definition 19, we can prove the following *representation* and *uniqueness theorems* for $\precsim_{F_1\times\cdots\times F_n}$:

Theorem 3 (Representation).
If $(\mathscr{W}, R, \mathscr{D}, \{\precsim_F\}_{F\in\mathscr{P}}, \{\mathscr{D}_{\precsim_F}\}_{F\in\mathscr{P}}, P, \{\precsim_{F_1\times\cdots\times F_n}\}_{F_1\ldots,F_n\in\mathscr{P}}, I)$ is a mereological additive difference factorial proximity structured model of RMRL*, then there exist functions $f_{\precsim_{F_i}}(1 \leq i \leq n) : \mathscr{D}_{\precsim_{F_i}} \to \mathbb{R}_{\geq 0}$ and monotonically increasing functions $g_{\precsim_{F_i}}(1 \leq i \leq n) : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ such that*

$$(\mathfrak{M}, w) \models (t_1, t_2) \leqslant_{F_1\times\cdots\times F_n} (t_3, t_4)[s]$$

iff

$$(\tilde{s}(t_1), \tilde{s}(t_2)) \precsim^*_{F_1\times\cdots\times F_n} (\tilde{s}(t_3), \tilde{s}(t_4))$$

iff

$$(4) \quad \sum_{i=1}^{n} g_{\precsim_{F_i}}(|f_{\precsim_{F_i}}(\imath\mathfrak{d}_i P(\mathfrak{d}_i, \tilde{s}(t_1))) - f_{\precsim_{F_i}}(\imath\mathfrak{d}_{i+1} P(\mathfrak{d}_{i+1}, \tilde{s}(t_2)))|)$$
$$\leq$$
$$\sum_{i=1}^{n} g_{\precsim_{F_i}}(|f_{\precsim_{F_i}}(\imath\mathfrak{d}_{i+2} P(\mathfrak{d}_{i+2}, \tilde{s}(t_3))) - f_{\precsim_{F_i}}(\imath\mathfrak{d}_{i+3} P(\mathfrak{d}_{i+3}, \tilde{s}(t_4)))|),$$

where $\imath\mathfrak{d}_i P(\mathfrak{d}_i, \tilde{s}(t_1)), \imath\mathfrak{d}_{i+1} P(\mathfrak{d}_{i+1}, \tilde{s}(t_2)), \imath\mathfrak{d}_{i+2} P(\mathfrak{d}_{i+2}, \tilde{s}(t_3)),$ $\imath\mathfrak{d}_{i+3} P(\mathfrak{d}_{i+3}, \tilde{s}(t_4)) \in \mathscr{D}_{\precsim_{F_i}}$.

Proof.
Suppose that $(\mathscr{W}, R, \mathscr{D}, \{\precsim_F\}_{F\in\mathscr{P}}, \{\mathscr{D}_{\precsim_F}\}_{F\in\mathscr{P}}, P, \{\precsim_{F_1\times\cdots\times F_n}\}_{F_1\ldots,F_n\in\mathscr{P}}, I)$ is a mereological additive difference factorial proximity structured model of RMRL (Definition 17). Then because, by Definition 17, $(\mathscr{D}_{F_1} \times \cdots \times \mathscr{D}_{F_n}, \precsim_{F_1\times\cdots\times F_n})$ is an additive difference factorial proximity structure (Definition 15), by Fact

7, there exist functions $f_{\precsim_{F_i}}(1 \le i \le n) : \mathscr{D}_{\precsim_{F_i}} \to \mathbb{R}_{\ge 0}$ and monotonically increasing functions $g_{\precsim_{F_i}}(1 \le i \le n) : \mathbb{R}_{\ge 0} \to \mathbb{R}_{\ge 0}$ such that

$$(\tilde{s}(t_1), \tilde{s}(t_2)) \precsim^*_{F_1 \times \cdots \times F_n} (\tilde{s}(t_3), \tilde{s}(t_4))$$

iff

$$\sum_{i=1}^{n} g_{\precsim_{F_i}}(|f_{\precsim_{F_i}}(\imath \mathfrak{d}_i P(\mathfrak{d}_i, \tilde{s}(t_1))) - f_{\precsim_{F_i}}(\imath \mathfrak{d}_{i+1} P(\mathfrak{d}_{i+1}, \tilde{s}(t_2)))|)$$
$$\le$$
$$\sum_{i=1}^{n} g_{\precsim_{F_i}}(|f_{\precsim_{F_i}}(\imath \mathfrak{d}_{i+2} P(\mathfrak{d}_{i+2}, \tilde{s}(t_3))) - f_{\precsim_{F_i}}(\imath \mathfrak{d}_{i+3} P(\mathfrak{d}_{i+3}, \tilde{s}(t_4)))|).$$

On the other hand, by Definition 19, we have

$$(\mathfrak{M}, w) \models (t_1, t_2) \leqslant_{F_1 \times \cdots \times F_n} (t_3, t_4)[s]$$

iff

$$(\tilde{s}(t_1), \tilde{s}(t_2)) \precsim^*_{F_1 \times \cdots \times F_n} (\tilde{s}(t_3), \tilde{s}(t_4)). \Box$$

Remark 11 (Mereological Parthood Relation). *One of the points of this theorem is that it is formulated by the help of a mereological parthood relation P.*

Theorem 4 (Uniqueness). *The above functions $f_{\precsim_{F_i}}$ are interval scales and the above functions $g_{\precsim_{F_i}}$ are interval scales with a common unit.*

We define the degree of unresemblance and its weight in terms of Theorems 3 and 4:

Definition 20 (Degree of Unresemblance and Its Weight). *The* degrees of unresemblance *with respect to $\precsim_{F_i}$ are defined by*

$$|f_{\precsim_{F_i}}(\imath \mathfrak{d}_i P(\mathfrak{d}_i, \tilde{s}(t_1))) - f_{\precsim_{F_i}}(\imath \mathfrak{d}_{i+1} P(\mathfrak{d}_{i+1}, \tilde{s}(t_2)))|$$

and

$$|f_{\precsim_{F_i}}(\imath \mathfrak{d}_{i+2} P(\mathfrak{d}_{i+2}, \tilde{s}(t_3))) - f_{\precsim_{F_i}}(\imath \mathfrak{d}_{i+3} P(\mathfrak{d}_{i+3}, \tilde{s}(t_4)))|$$

of (4), and their weights *are defined by*

$$g_{\precsim_{F_i}}$$

of (4), where the existence and uniqueness of $f_{\precsim_{F_i}}$ and $g_{\precsim_{F_i}}$ are guaranteed by Theorems 3 and 4 respectively.

3 Concluding Remarks

Suppose that
$Cx := x$ is carmine,
$Vx := x$ is vermillion,
$Tx := x$ is triangular, and
$(x, y) <_{C\times V\times T} (z, w) := x$ resembles y more than z resembles w with respect to carminity and vermillionity and triangularity. Then the RMRL-logical form of (1) is

$$\forall x\forall y\forall z((Cx \wedge Vy \wedge Tz) \rightarrow (x, y) <_{C\times V\times T} (x, z)).$$

Its semantic value (satisfaction condition) is given by the following corollary that follows from Theorem 3 and Definition 19:

Corollary 2 (Solution to Rodriguez-Pereyra-Yi Problem by RMRL). *If $(\mathscr{W}, R, \mathscr{D}, \{\precsim_C, \precsim_V, \precsim_T\}, \{\mathscr{D}_{\precsim_C}, \mathscr{D}_{\precsim_V}, \mathscr{D}_{\precsim_T}\}, P, \{\precsim_{C\times V\times T}\}, I)$ is a mereological additive difference factorial proximity structured model of* RMRL*, then there exist $f_{\precsim_C} : \mathscr{D}_{\precsim_C} \rightarrow \mathbb{R}_{\geq 0}$ and $f_{\precsim_V} : \mathscr{D}_{\precsim_V} \rightarrow \mathbb{R}_{\geq 0}$ and $f_{\precsim_T} : \mathscr{D}_{\precsim_T} \rightarrow \mathbb{R}_{\geq 0}$ and $g_{\precsim_C}, g_{\precsim_V}, g_{\precsim_T} : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$ such that*

$$(\mathfrak{M}, w) \models \forall x\forall y\forall z((Cx \wedge Vy \wedge Tz) \rightarrow (x, y) <_{C\times V\times T} (x, z))[s]$$

iff there is no $\mathfrak{d}_1, \mathfrak{d}_2, \mathfrak{d}_3 \in \mathscr{D}$ such that $\mathfrak{d}_1 \in I(C, w)$ and $\mathfrak{d}_2 \in I(V, w)$ and $\mathfrak{d}_3 \in I(T, w)$ such that

$$\begin{aligned}
&(g_{\precsim_C}(|f_{\precsim_C}(\imath\mathfrak{d}_4 P(\mathfrak{d}_4, \mathfrak{d}_1)) - f_{\precsim_C}(\imath\mathfrak{d}_5 P(\mathfrak{d}_5, \mathfrak{d}_2))|)\\
&+g_{\precsim_V}(|f_{\precsim_V}(\imath\mathfrak{d}_6 P(\mathfrak{d}_6, \mathfrak{d}_1)) - f_{\precsim_V}(\imath\mathfrak{d}_7 P(\mathfrak{d}_7, \mathfrak{d}_2))|)\\
&+g_{\precsim_T}(|f_{\precsim_T}(\imath\mathfrak{d}_8 P(\mathfrak{d}_8, \mathfrak{d}_1)) - f_{\precsim_T}(\imath\mathfrak{d}_9 P(\mathfrak{d}_9, \mathfrak{d}_2))|))\\
&\geq\\
&(g_{\precsim_C}(|f_{\precsim_C}(\imath\mathfrak{d}_4 P(\mathfrak{d}_4, \mathfrak{d}_1)) - f_{\precsim_C}(\imath\mathfrak{d}_{10} P(\mathfrak{d}_{10}, \mathfrak{d}_3))|)\\
&+g_{\precsim_V}(|f_{\precsim_V}(\imath\mathfrak{d}_6 P(\mathfrak{d}_6, \mathfrak{d}_1)) - f_{\precsim_V}(\imath\mathfrak{d}_{11} P(\mathfrak{d}_{11}, \mathfrak{d}_3))|)\\
&+g_{\precsim_T}(|f_{\precsim_T}(\imath\mathfrak{d}_8 P(\mathfrak{d}_8, \mathfrak{d}_1)) - f_{\precsim_T}(\imath\mathfrak{d}_{12} P(\mathfrak{d}_{12}, \mathfrak{d}_3))|)).
\end{aligned}$$

We have the following conclusion: When we choose as the weight-assignment functions such functions $g_{\precsim_C}, g_{\precsim_V}, g_{\precsim_T}$ that the value of $g_{\precsim_T}$ is much greater than those of $g_{\precsim_C}$ and $g_{\precsim_V}$, Corollary 2 can give a solution to Rodriguez-Pereyra-Yi Problem by Definition 20 in terms of giving the satisfaction condition of (1) in RMRL so that

> the weighted sum of the degrees of unresemblance of carmine particulars to triangular particulars may be greater than that of carmine particulars to vermillion particulars,

instead of using Definition 1 that is the main culprit of this problem. In so doing, RMRL obtains the capacity to deal with multidimensionality in general beyond Rodriguez-Pereyra-Yi Problem.

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