Chapter 20
Diffraction of Light

Abstract Problems on diffraction of light are solved in this last chapter. Diffraction is bending or spreading of light at aperture or obstacle. Problems on diffraction by a single slit and diffraction by a grating and its resolving power are discussed. Solutions obtained by analysis and computer calculation of wxMaxima are presented.

20.1 Basic Concepts and Formulae

(1) When light waves encounter an aperture or an obstacle, the waves spread out as they travel and undergo interference. This is called diffraction. Diffraction of light is due to interference of continuous distribution of coherence sources of light.

(2) The Fraunhofer diffraction pattern of light by a single slit of width a on a screen consists of a bright central region and an alternating dark and bright regions is shown in Fig. 20.1.

The angle, θ, of the dark fringe is given by,

$$\sin\theta = m\frac{\lambda}{a}, \quad m = \pm 1,\ \pm 2,\ \ldots \tag{20.1}$$

where λ is the wavelength of light, a is width of the slit, and m is order number.

(3) The intensity of light, I, on the screen, varies with angle, θ, according to,

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2, \quad \text{where } \beta = \frac{2\pi a \sin\theta}{\lambda}, \tag{20.2}$$

and I_0 is the intensity at $\theta = 0$, as shown in Figure 20.2.

(4) Rayleigh criterion states that two images formed by an aperture are just resolved if the central maximum diffraction pattern of one image falls on the first minimum of the other.

W. M. S. Wan Hassan et al., *Physics—Problems, Solutions, and Computer Calculations*,
https://doi.org/10.1007/978-3-031-43165-4_20

Figure 20.3 shows intensity patterns of two slits that are just resolved according to the Rayleigh criterion. The intensity patterns are drawn separately in (a), while the intensity pattern of both is shown in (b).

The limiting resolving angle for a diffraction by a slit of width, a, is,

$$\theta_{min} = \frac{\lambda}{a}. \tag{20.3}$$

The limiting resolving angle for a circular aperture of diameter, D, is,

$$\theta_{min} = 1.22\frac{\lambda}{D}. \tag{20.4}$$

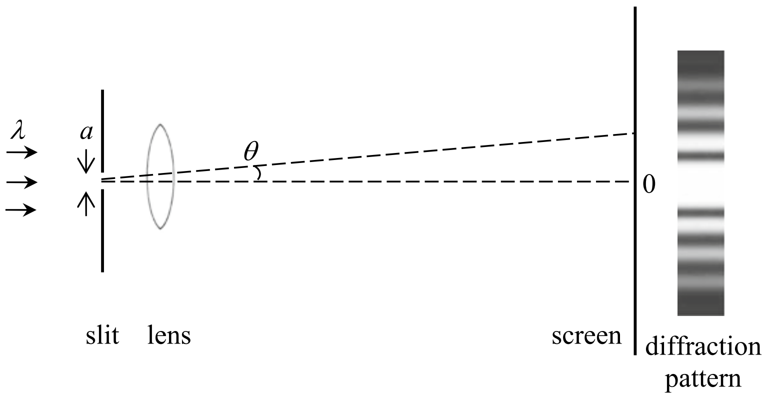
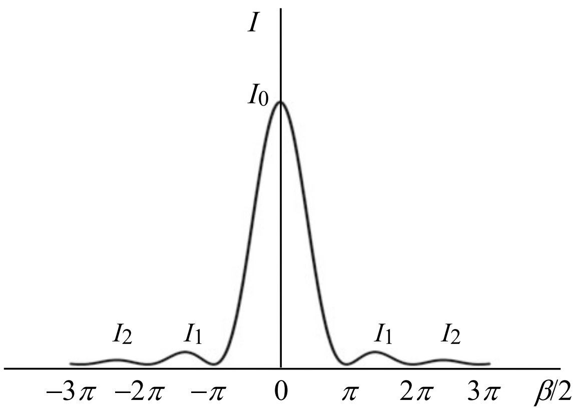


Fig. 20.1 Single slit diffraction. Light of wavelength λ is incident on a narrow slit of width a. Diffraction pattern is observed on a screen. The angle of the dark fringe is θ

Fig. 20.2 Intensity of light I against $\beta/2$ of a single slit diffraction



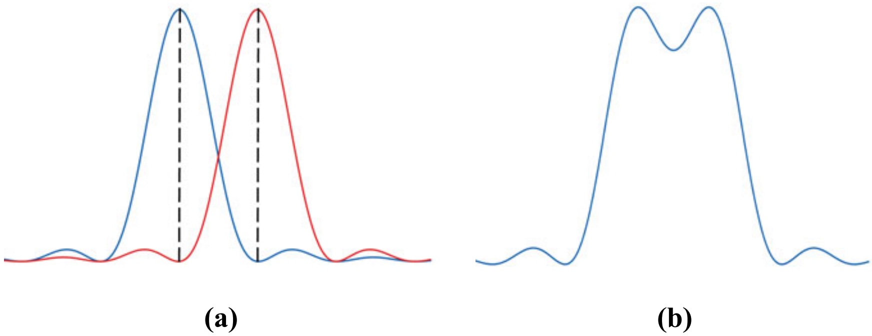


Fig. 20.3 Intensity patterns of two slits that are just resolved according to the Rayleigh criterion. The intensity patterns are shown separately in (**a**), while the intensity pattern as observed on the screen is shown in (**b**)

(5) A diffraction grating consists of packed identical slits. Condition for maximum intensity (bright fringe) is,

$$d \sin\theta = m\lambda, \quad m = 0, 1, 2, \ldots \tag{20.5}$$

where d is the distance between slits, θ is diffraction angle, λ is wavelength of light, and m is order number of the diffraction pattern. Zeroth-order maximum is at angle, $\theta = 0$; first-order maximum corresponding to $m = 1$, is at angle, θ, satisfying $\sin\theta = \lambda/d$; second-order maximum corresponding to $m = 2$, is at angle, θ, satisfying $\sin\theta = 2\lambda/d$; and so on.

Figure 20.4 shows the diffraction of a monochrome light by a diffraction grating.

From Eq. (20.5) and Fig. 20.4, one writes,

$$\theta_m = \sin^{-1}\left(\frac{m\lambda}{d}\right), \tag{20.6}$$

$$y_m \approx \frac{m\lambda D}{d}. \tag{20.7}$$

Resolving power, R, of a diffraction grating at m-th order diffraction is,

$$R = Nm = \frac{\lambda}{\Delta\lambda}, \tag{20.8}$$

where N is the number of lines of the diffraction grating, $\Delta\lambda$ is wavelength separation of two monochromatic light waves that are barely distinguishable and λ is their mean wavelength.

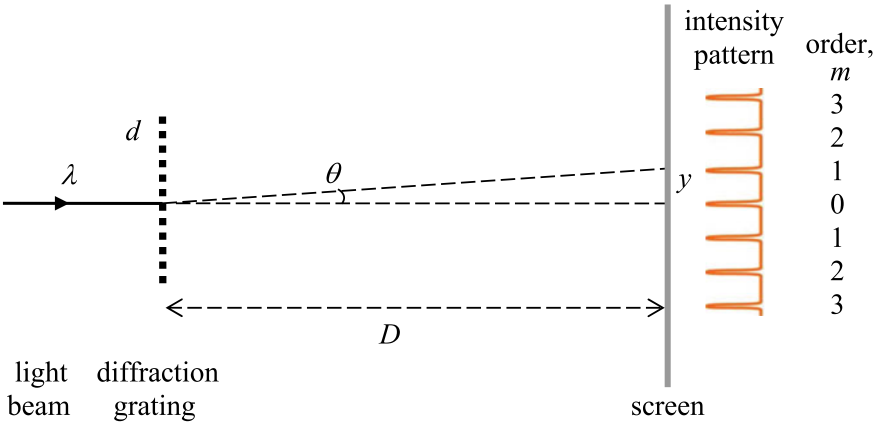


Fig. 20.4 Diffraction of a monochrome light by a diffraction grating. Light of wavelength λ is incident on a diffraction grating with slit separation d. Bright fringe is observed on a screen a distance D away, at angle θ or a distance y from the central maximum

20.2 Problems and Solutions

Problem 20.1 A plane wave of monochromatic light ($\lambda = 5900$ Å) is incident on a slit of width, $a = 0.04$ mm. A converging lens ($f = +70$ cm) is placed behind the slit to focus the light on a screen. What is the separation between the first and the second minima?

Solution

Figure 20.5 shows the slit, lens, screen, and geometry of the single slit diffraction. Also shown on the far right is the diffraction pattern. Here, a is the width of the slit and θ is the diffraction angle.

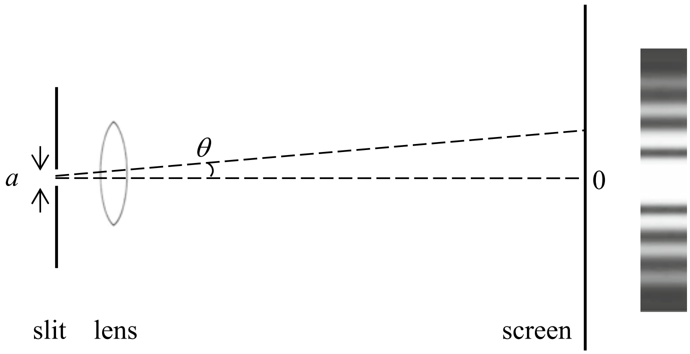


Fig. 20.5 Single slit diffraction experiment, a is slit width and θ is angle of dark fringe, Problem 20.1

For this diffraction, the minimum (dark) is obtained if (Eq. 20.1),

$$a \ \sin\theta = m\lambda, \qquad m = \pm 1, \ \pm 2, \ \ldots$$

As θ is a small angle, $\sin\theta \approx \theta$, one writes,

$$\begin{aligned} a\theta_m &= m\lambda, \\ \theta_m &= m\frac{\lambda}{a}, \quad m = \pm 1, \ \pm 2, \ \ldots \end{aligned}$$

For the first minimum (first dark fringe), the diffraction angle is,

$$\theta_1 = \frac{\lambda}{a} = \frac{5900 \times 10^{-10} \text{ m}}{0.04 \times 10^{-3} \text{ m}} = 1.5 \times 10^{-2} \text{ rad.}$$

For the second minimum, the diffraction angle is,

$$\theta_2 = 2 \times \frac{\lambda}{a} = 2 \times \frac{5900 \times 10^{-10} \text{ m}}{0.04 \times 10^{-3} \text{ m}} = 2.9 \times 10^{-2} \text{ rad.}$$

The angular difference of the two minima is,

$$\Delta\theta = \theta_2 - \theta_1 = 1.5 \times 10^{-2} \text{ rad.}$$

The separation of the two minima is

$$\Delta y = f \cdot \Delta\theta = 0.70 \text{ m} \times 1.5 \times 10^{-2} = 0.01 \text{ m.}$$

- wxMaxima codes:

```
(%i4) fpprintprec:5; lambda:5900e-10; a:0.04e-3; f:70e-2;
(fpprintprec)     5
(lambda)    5.9*10^-7
(a)   4.0*10^-5
(f)   0.7
(%i5) theta1: lambda/a;
(theta1) 0.01475
(%i6) theta2: 2*lambda/a;
(theta2) 0.0295
(%i7) delta_theta: theta2-theta1;
(delta_theta) 0.01475
(%i8) delta_y: f*delta_theta;
(delta_y) 0.010325
```

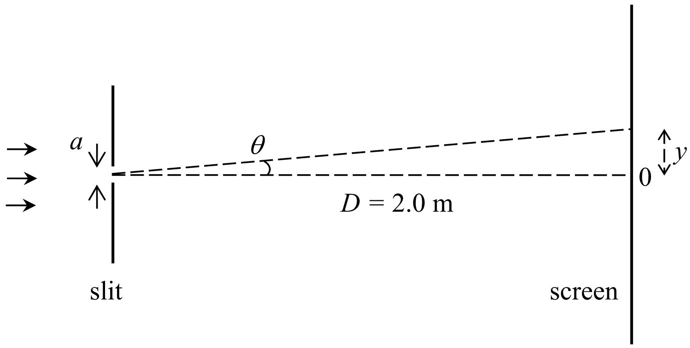


Fig. 20.6 Single slit diffraction experiment, a is slit width, θ is angle of dark fringe, y is on-screen distance of dark fringe, and D is slit-screen distance, Problem 20.2

Comments on the codes:

(%i4)	Set floating point print precision to 5, assign values of λ, a, and f.
(%i5), (%i6)	Calculate θ_1 and θ_2.
(%i7), (%i8)	Calculate $\Delta\theta$ and Δy.

Problem 20.2 A light of wavelength 580 nm is shined on a slit of width 0.30 mm. A screen is positioned 2.0 m away from the slit. Determine,

(a) the location of the first dark fringe,
(b) the width of the central bright fringe,
(c) the width of the first bright fringe.

Solution

(a) Figure 20.6 shows the slit, lens, and geometry of the problem. Here, a is the width of the slit, θ is angle of diffraction, and D is distance between the slit and the screen.

The first dark fringe satisfies $a \sin\theta_1 = \lambda$ (Eq. 20.1). This means that,

$$\sin\theta_1 = \pm\frac{\lambda}{a} = \pm\frac{y_1}{D},$$
$$y_1 = \pm\frac{D\lambda}{a} = \pm\frac{(2.0\text{ m})(580\times10^{-9}\text{ m})}{0.30\times10^{-3}\text{ m}} = \pm 3.9\times10^{-3}\text{ m}.$$

(b) The width of the central bright fringe is two times y_1,

$$2y_1 = 7.7\times10^{-3}\text{ m}.$$

(c) The first-order bright fringe is located between the first and second dark fringes, that is, between y_1 and y_2. Calculate y_2,

$$y_2 = \frac{2D\lambda}{a} = \frac{2(2.0\ \text{m})(580 \times 10^{-9}\ \text{m})}{0.30 \times 10^{-3}\ \text{m}} = 7.7 \times 10^{-3}\ \text{m}.$$

The width of the first-order bright fringe is,

$$y_2 - y_1 = 7.7 \times 10^{-3}\ \text{m} - 3.9 \times 10^{-3}\ \text{m} = 3.9 \times 10^{-3}\ \text{m}.$$

- wxMaxima codes:

```
(%i4) fpprintprec:5; lambda:580e-9; a:0.3e-3; D:2;
(fpprintprec)     5
(lambda)    5.8*10^-7
(a)   3.0*10^-4
(D)   2
(%i5) y1: D*lambda/a;
(y1)  0.0038667
(%i6) width_of_central_bright_fringe: 2*y1;
(width_of_central_bright_fringe) 0.0077333
(%i7) y2: 2*D*lambda/a;
(y2)  0.0077333
(%i8) y2-y1;
(%o8) 0.0038667
```

Comments on the codes:

(%i4) Set floating point print precision to 5, assign values of λ, a, and D.
(%i5) Calculate y_1.
(%i6) Calculate the width of the central bright fringe.
(%i7) Calculate y_2.
(%i8) Calculate the width of first-order bright fringe.

Figure 20.7 shows the intensity of the diffraction pattern. Diffraction angle of the first dark fringe is θ_1 and the location of the fringe is y_1. Diffraction angle of the second dark fringe is θ_2 and the location of the second dark fringe is y_2. The width of central bright fringe is $2y_1$ and the width of the first bright fringe is $y_2 - y_1$.

Problem 20.3 Figure 20.8 shows a curve of intensity, I, against $\beta/2$ of a single slit diffraction. The intensity is given by,

$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$, where $\beta = \frac{2\pi a \sin\theta}{\lambda}$,

and I_0 is the maximum intensity of the central bright fringe. Calculate the intensity ratio of first- and second-order maxima (I_1 and I_2) to that of central maximum, I_0, that is, calculate I_1/I_0 and I_2/I_0.

Solution

The first-order intensity maximum, I_1, is located approximately in the middle of $\beta/2 = \pi$ and $\beta/2 = 2\pi$, that is, at $\beta/2 = 3\pi/2$. The intensity ratio of the first maximum

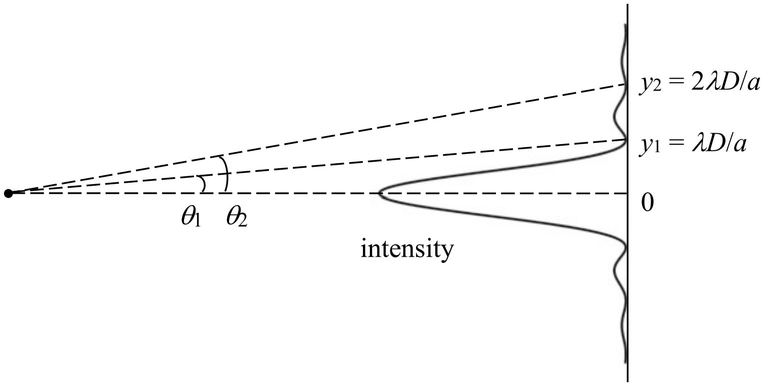
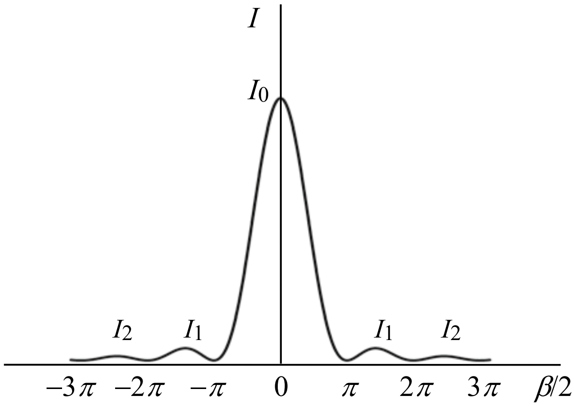


Fig. 20.7 Intensity of the diffraction pattern

Fig. 20.8 Intensity I against $\beta/2$ of a single slit diffraction



to that of the central maximum is,

$$\frac{I_1}{I_0} = \left[\frac{\sin(3\pi/2)}{3\pi/2}\right]^2 = 0.045.$$

The second-order intensity maximum, I_2, is located approximately in the middle of $\beta/2 = 2\pi$ and $\beta/2 = 3\pi$, that is, at $\beta/2 = 5\pi/2$. The intensity ratio of the second maximum to that of the central maximum is,

$$\frac{I_2}{I_0} = \left[\frac{\sin(5\pi/2)}{5\pi/2}\right]^2 = 0.016.$$

This means that, I_1 and I_2 are approximately 4.5% and 1.6% of I_0, respectively.

- wxMaxima codes:

```
(%i1) fpprintprec: 5;
(fpprintprec)     5
(%i3) I1_over_I0: (sin(3*%pi/2)/(3*%pi/2))^2; float(%);
(I1_over_I0) 4/(9*%pi^2)
(%o3) 0.045032
(%i5) I2_over_I0: (sin(5*%pi/2)/(5*%pi/2))^2; float(%);
(I2_over_I0) 4/(25*%pi^2)
(%o5) 0.016211
```

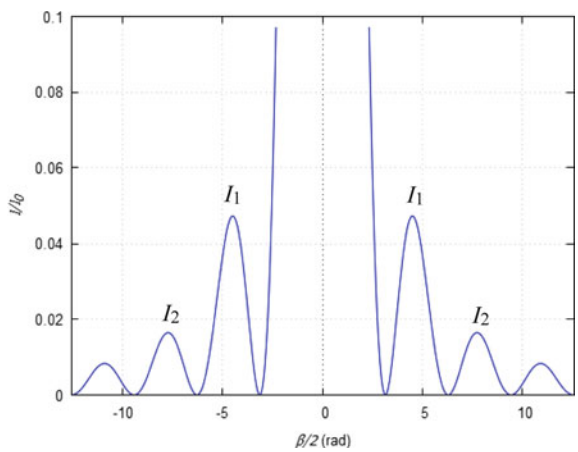
Comments on the codes:

(%i1) Set floating point print precision to 5.
(%i3), (%i5) Calculate I_1/I_0 and I_2/I_0.

Further question: Plot the intensity, I, against $\beta/2$ of a single slit diffraction to check the results.

- Plot of I against $\beta/2$ for $-4\pi \le \beta/2 \le 4\pi$ rad by wxMaxima:

```
(%i2) I0:1; I:I0*(sin(betaovertwo)/betaovertwo)^2;
(I0)   1
(I)    sin(betaovertwo)^2/betaovertwo^2
(%i3) wxplot2d(I, [betaovertwo, -4*%pi, 4*%pi], [y,0, 0.1], grid2d,
[xlabel,"{/Symbol-Italic b/2} (rad)"], [ylabel,"{/Helvetica-Italic
I/I_0}"]);
```



Comments on the codes:

(%i2) Assign $I_0 = 1$ and define $I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$.
(%i3) Plot I against $\beta/2$ for -4π rad $\leq \beta/2 \leq 4\pi$ rad.

Problem 20.4 In a single slit diffraction experiment, a light of wavelength 580 nm is incident on a slit of width 0.30 mm. A screen is located 2.0 m away from the slit. By setting the intensity of central maximum as $I_0 = 1.00$, plot the curve of

(a) intensity, I, versus angle of diffraction, θ, in radian,
(b) intensity, I, versus angle of diffraction, θ, in degree,
(c) intensity, I, versus distance on the screen, y.

Solution

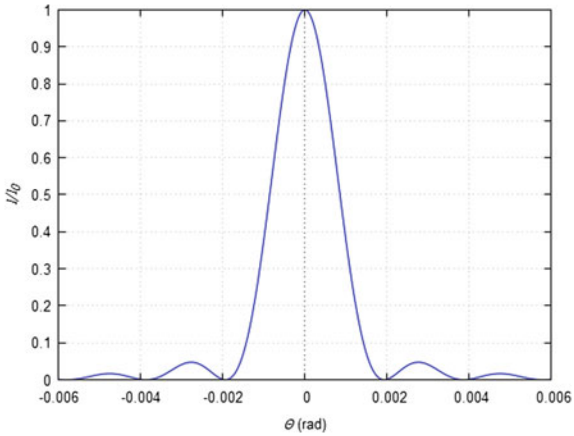
(a) Intensity, I, at angle of diffraction, θ, is given by,

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2, \text{ where } \beta = \frac{2\pi a \sin\theta}{\lambda}.$$

To plot the curve by wxMaxima, first, assign the values of wavelength, λ, slit width, a, slit-screen distance, D, and intensity of the central maximum, I_0. Next, define β in terms of θ (radian) and I in terms of β. Lastly, plot I against θ (radian) using the wxplot2d function.

- Plot by wxMaxima:

```
(%i5) fpprintprec:5; lambda:580e-9; a:0.3e-3; D:2; I0:1;
(fpprintprec)     5
(lambda)    5.8*10^-7
(a)   3.0*10^-4
(D)   2
(I0)  1
(%i6) beta: 2*%pi*a*sin(theta)/lambda;
(beta) 1034.5*%pi*sin(theta)
(%i7) I: I0*sin(beta/2)^2/(beta/2)^2;
(I)  (3.7378*10^-6*sin(517.24*%pi*sin(theta))^2)/(%pi^2*sin(theta)^2)
(%i8) wxplot2d(I, [theta,-0.006,0.006], grid2d, [xlabel,"{/Symbol-Italic Q}
(rad)"], [ylabel,"{/Helvetica-Italic I/I_0}"]);
```



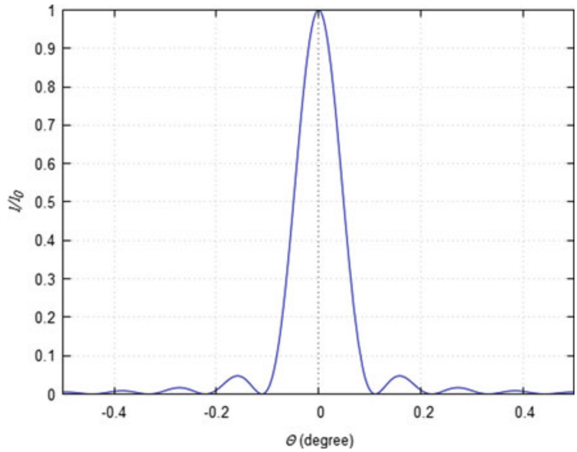
Comments on the codes:

(%i5)	Set floating point print precision to 5, assign values of λ, a, D, and I_0.
(%i6), (%i7)	Define β and I.
(%i8)	Plot I against θ for $-0.006 \leq \theta \leq 0.006$ rad.

(b) Assign the values of wavelength, λ, slit width, a, slit-screen distance, D, and intensity of the central maximum, I_0. Define β in terms of θ (degree) and I in terms of β. Plot I against θ (degree) using the wxplot2d function.

- Plot by wxMaxima:

```
(%i5) fpprintprec:5; lambda:580e-9; a:0.3e-3; D:2; I0:1;
(fpprintprec)     5
(lambda)    5.8*10^-7
(a)   3.0*10^-4
(D)   2
(I0)  1
(%i6) beta: 2*%pi*a*sin(degree*%pi/180)/lambda;
(beta) 1034.5*%pi*sin((%pi*degree)/180)
(%i7) I: I0*sin(beta/2)^2/(beta/2)^2;
(I) (3.7378*10^-6*sin(517.24*%pi*sin((%pi*degree)/180))^2)
/(%pi^2*sin((%pi*degree)/180)^2)
(%i8) wxplot2d(I, [degree,-0.5,0.5], grid2d, [xlabel,"{/Symbol-Italic Q}
(degree)"], [ylabel,"{/Helvetica-Italic I/I_0}"]);
```



Comments on the codes:

(%i5)	Set floating point print precision to 5, assign values of λ, a, D, and I_0.
(%i6), (%i7)	Define β and I.
(%i8)	Plot I against θ for $-0.5° \leq \theta \leq 0.5°$.

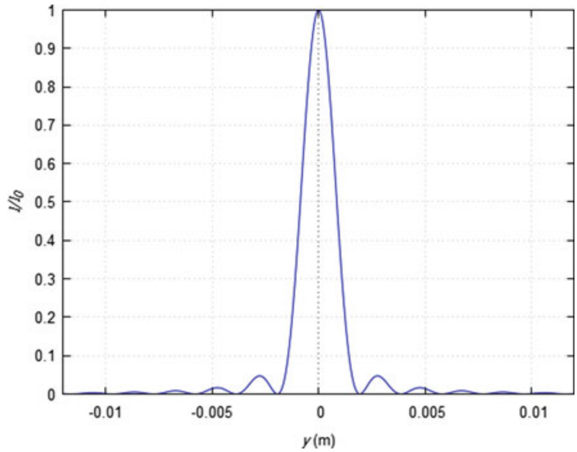
(c) The intensity, I, at position, y, is

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2, \text{ where } \beta = \frac{2\pi a \sin\theta}{\lambda} = \frac{2\pi a y}{\lambda D},$$

because $\sin\theta = y/D$. To plot I against y, assign the values of wavelength, λ, slit width, a, slit-screen distance, D, and intensity of the central maximum, I_0. Next, define β in terms of y and I in terms of β. Lastly, plot I against y using the wxplot2d function.

- Plot by wxMaxima:

```
(%i5) fpprintprec:5; lambda:580e-9; a:0.3e-3; D:2; I0:1;
(fpprintprec)     5
(lambda)    5.8*10^-7
(a)   3.0*10^-4
(D)   2
(I0)  1
(%i6) beta: 2*%pi*a*y/lambda;
(beta) 1034.5*%pi*y
(%i7) I: I0*sin(beta/2)^2/(beta/2)^2;
(I) (3.7378*10^-6*sin(517.24*%pi*y)^2)/(%pi^2*y^2)
(%i8) wxplot2d(I, [y,-0.012,0.012], grid2d, [xlabel,"{/Helvetica-Italic y}
(m)"], [ylabel,"{/Helvetica-Italic I/I_0}"]);
```



Comments on the codes:

(%i5) Set floating point print precision to 5, assign values of λ, *a*, *D*, and I_0.
(%i6), (%i7) Define β and *I*.
(%i8) Plot *I* against *y* for $-0.012 \le y \le 0.012$ m.

Problem 20.5 A light of wavelength, $\lambda = 580$ nm, is incident to a slit of width, $a_1 = 29\ \mu\text{m} = 50\lambda$. The screen is located a distance, $D = 0.8$ m, away from the slit. A diffraction pattern is observed on the screen. The experiment is repeated using different slits of width, $a_2 = 58\ \mu\text{m} = 100\lambda$, and $a_3 = 87\ \mu\text{m} = 150\lambda$. How do the diffraction patterns change?

Solution

This problem is solved by plotting the intensities of the three diffraction patterns from slits of different widths. We plot these three curves of intensity *I* against diffraction angle θ (degree),

$$I_1 = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2, \text{ where } \beta = \frac{2\pi a_1 \sin\theta}{\lambda},$$

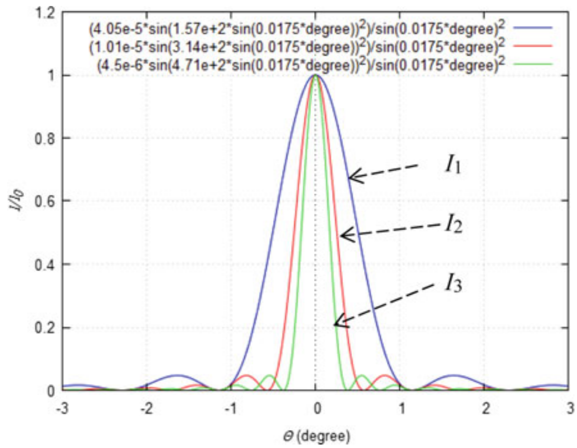
$$I_2 = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2, \text{ where } \beta = \frac{2\pi a_2 \sin\theta}{\lambda},$$

$$I_3 = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2, \text{ where } \beta = \frac{2\pi a_3 \sin\theta}{\lambda}.$$

Assign the values of wavelength, λ, slit widths, a_1, a_2, and a_3, slit-screen distance, *D*, and intensity of the central maximum, I_0. Define β in terms of θ (degree) and *I* in terms of β. Plot *I* against θ (degree) using the wxplot2d function.

- Plot by wxMaxima:

```
(%i7) fpprintprec:3; lambda:580e-9; a1:50*lambda; a2:100*lambda;
a3:150*lambda; D:0.8; I0:1;
(fpprintprec)      3
(lambda)     5.8*10^-7
(a1)   2.9*10^-5
(a2)   5.8*10^-5
(a3)   8.7*10^-5
(D)    0.8
(I0)   1
(%i8) beta: float(2*%pi*a1*sin(degree*%pi/180)/lambda);
(beta) 3.14*10^2*sin(0.0175*degree)
(%i9) I1: I0*sin(beta/2)^2/(beta/2)^2;
(I1)   (4.05*10^-5*sin(1.57*10^2*sin(0.0175*degree))^2)/sin(0.0175*degree)^2
(%i10) beta: float(2*%pi*a2*sin(degree*%pi/180)/lambda);
(beta) 6.28*10^2*sin(0.0175*degree)
(%i11) I2: I0*sin(beta/2)^2/(beta/2)^2;
(I2)   (1.01*10^-5*sin(3.14*10^2*sin(0.0175*degree))^2)/sin(0.0175*degree)^2
(%i12) beta: float(2*%pi*a3*sin(degree*%pi/180)/lambda);
(beta) 9.42*10^2*sin(0.0175*degree)
(%i13) I3: I0*sin(beta/2)^2/(beta/2)^2;
(I3)   (4.5*10^-6*sin(4.71*10^2*sin(0.0175*degree))^2)/sin(0.0175*degree)^2
(%i14) wxplot2d([I1, I2, I3], [degree,-3,3], [y,0,1.2], grid2d,
[xlabel,"{/Symbol-Italic Q} (degree)"], [ylabel,"{/Helvetica-Italic
I/I_0}"]);
```



Comments on the codes:

(%i7)	Set floating point print precision to 5, assign values of λ, a_1, a_2, a_3, D, and I_0.
(%i8), (%i9)	Define β and I_1.
(%i10), (%i11)	Define β and I_2.
(%i12), (%i13)	Define β and I_3.
(%i14)	Plot I_1, I_2, and I_3 against θ for $-3° \leq \theta \leq 3°$.

The diffraction fringe widths decrease as the slit widths increase. This means that narrow slit gives wide diffraction. Table 20.1 gives the angular and linear widths of the central bright fringe (central maxima) of the three experiments. The angular and linear widths of the central maxima are calculated as $2\lambda/a \times 180/\pi$ and $2\lambda D/a$, respectively.

Problem 20.6 Calculate separation distance of two points on the moon that are just resolved by the Palomar Mountain telescope. Diameter of the telescope aperture is 5.0 m, earth-moon distance is 3.86×10^5 km, and $\lambda = 5500$ Å.

Solution

For circular aperture of the telescope, the Rayleigh resolving criterion is (Eq. 20.4),

$$\theta_{min} = 1.22\frac{\lambda}{D}.$$

The resolving angle is,

$$\theta_{min} = 1.22 \times \frac{5500 \times 10^{-10}\ \text{m}}{5.0\ \text{m}} = 1.3 \times 10^{-7}\ \text{rad}.$$

The separation distance so that two points on the moon can be resolved is,

$$\begin{aligned}\Delta x &= d \cdot \theta_{min} = 3.86 \times 10^5\ \text{km} \times 1.3 \times 10^{-7}\ \text{rad} = 0.052\ \text{km} \\ &= 52\ \text{m}.\end{aligned}$$

Table 20.1 Angular and linear widths of the central maxima of a single slit diffraction

Slit width a (μm)	Angular width (degree)	Linear width (mm)
29	2.3	32
58	1.1	16
87	0.76	11

- wxMaxima codes:

```
(%i4) fpprintprec:5; lambda:5500e-10; D:5; d:3.86e8;
(fpprintprec)     5
(lambda) 5.5*10^-7
(D)    5
(d)    3.86*10^8
(%i5) theta_min: 1.22*lambda/D;
(theta_min) 1.342*10^-7
(%i6) delta_x: d*theta_min;
(delta_x) 51.801
```

Comments on the codes:

(%i4)	Set floating point print precision to 5, assign values of λ, D, and d.
(%i5), (%i6)	Calculate θ_{min} and Δx.

Problem 20.7

(a) Estimate the limiting resolving angle of human eyes. The diameter of the pupil is 2.0 mm, index of refraction of the eye is 1.33, and the wavelength of light in air is 550 nm.
(b) What is the spatial resolution of the eye at 25 cm away?

Solution

(a) The wavelength of light in human eye is (Eq. 4.3),

$$\lambda = \frac{\lambda_0}{n} = \frac{550\ \text{nm}}{1.33} = 414\ \text{nm}.$$

The limiting resolving angle of the eye is (Eq. 20.4),

$$\theta_{min} = 1.22\frac{\lambda}{D} = 1.22 \times \frac{414 \times 10^{-9}\ \text{m}}{2.0 \times 10^{-3}\ \text{m}} = 2.5 \times 10^{-4}\ \text{rad}.$$

(b) At a distance of 25 cm from the eye, spatial resolution of the eye is,

$$\Delta x = d \cdot \theta_{min} = (25 \times 10^{-2}\ \text{m})(2.5 \times 10^{-4}\ \text{rad}) = 6.3 \times 10^{-5}\ \text{m}.$$

This means that, at 25 cm away, two points that are less than $6.3 \times 10{-}5$ m apart cannot be resolved by the eye.

- wxMaxima codes:

```
(%i5) fpprintprec:5; lambda0:550e-9; n:1.33; D:2e-3; d:25e-2;
(fpprintprec)     5
(lambda0)   5.5*10^-7
(n)   1.33
(D)   0.002
(d)   0.25
(%i6) lambda: lambda0/n;
(lambda) 4.1353*10^-7
(%i7) theta_min: 1.22*lambda/D;
(theta_min) 2.5226*10^-4
(%i8) delta_x: d*theta_min;
(delta_x) 6.3064*10^-5
```

Comments on the codes:

(%i5)	Set floating point print precision to 5, assign values of λ_0, n, D, and d.
(%i6), (%i7)	Calculate λ and θ_{min}, part (a).
(%i8)	Calculate Δx, part (b).

Problem 20.8 A microscope uses light of sodium lamp of wavelength 589 nm to probe subjects. The aperture of the objective is 1.0 cm in diameter. Calculate the limiting resolving angle.

Solution

The limiting resolving angle of the microscope is (Eq. 20.4),

$$\theta_{min} = 1.22\frac{\lambda}{D} = 1.22 \times \frac{589 \times 10^{-9}\ \text{m}}{1.0 \times 10^{-2}\ \text{m}} = 7.2 \times 10^{-5}\ \text{rad}.$$

This means that two points subtending less than 7.2×10^{-5} rad at the objective of the microscope cannot be resolved.

- wxMaxima codes:

```
(%i3) fpprintprec:5; lambda:589e-9; D:1e-2;
(fpprintprec)     5
(lambda) 5.89*10^-7
(D)   0.01
(%i4) theta_min: 1.22*lambda/D;
(theta_min) 7.1858*10^-5
```

Comments on the codes:

(%i3) Set floating point print precision to 5 and assign value of λ.
(%i4) Calculate θ_{min}.

Problem 20.9 A helium neon laser light of wavelength 632.8 nm is incident to a diffraction grating that has 7000 lines per cm. At what angles do maximum intensities be observed?

Solution

There are 7000 lines or slits in one cm, so the width of a slit is,

$$d = \frac{1.0}{7000}\ \text{cm} = \frac{1.0 \times 10^{-2}}{7000}\ \text{m} = 1.429 \times 10^{-6}\ \text{m}.$$

For a diffraction grating, to get maximum intensities (bright fringes) (Eq. 20.5),

$d \sin\theta = m\lambda, \quad m = 0, 1, 2, \ldots$

The first-order maximum, $m = 1$,

$$\sin\theta_1 = \frac{\lambda}{d} = \frac{632.8 \times 10^{-9}\ \text{m}}{1.429 \times 10^{-6}\ \text{m}} = 0.316,$$
$$\theta_1 = 0.321\ \text{rad} = 18.4°.$$

The second-order maximum, $m = 2$,

$$\sin\theta_2 = \frac{2\lambda}{d} = \frac{2 \times 632.8 \times 10^{-9}\ \text{m}}{1.429 \times 10^{-6}\ \text{m}} = 0.633,$$
$$\theta_2 = 0.685\ \text{rad} = 39.3°.$$

The third-order maximum, $m = 3$,

$$\sin\theta_3 = \frac{3\lambda}{d} = \frac{3 \times 632.8 \times 10^{-9}\ \text{m}}{1.429 \times 10^{-6}\ \text{m}} = 0.949,$$
$$\theta_3 = 1.25\ \text{rad} = 71.7°.$$

For $m = 4$, calculation gives

$$\sin\theta_4 = \frac{4\lambda}{d} = \frac{4 \times 632.8 \times 10^{-9}\ \text{m}}{1.429 \times 10^{-6}\ \text{m}} = 1.27.$$

This cannot be because it is greater than 1. This means that the diffraction patterns that can be observed by this laser light are first- , second- , and third-order maxima.

- wxMaxima codes:

```
(%i3) fpprintprec:5; lambda:632.8e-9; d:1/5000*1e-2;
(fpprintprec)     5
(lambda) 6.328*10^-7
(d)   2.0*10^-6
(%i6) sintheta1:lambda/d; theta1:asin(sintheta1);
theta1_deg:float(theta1*180/%pi);
(sintheta1) 0.3164
(theta1) 0.32193
(theta1_deg) 18.445
(%i9) sintheta2:2*lambda/d; theta2:asin(sintheta2);
theta2_deg:float(theta2*180/%pi);
(sintheta2) 0.6328
(theta2) 0.68516
(theta2_deg) 39.257
(%i12) sintheta3:3*lambda/d; theta3:asin(sintheta3);
theta3_deg:float(theta3*180/%pi);
(sintheta3) 0.9492
(theta3) 1.2507
(theta3_deg) 71.659
(%i13) sintheta4: 4*lambda/d;
(sintheta4) 1.2656
```

Comments on the codes:

(%i3) Set floating point print precision to 5, assign values of λ and d.
(%i6) Calculate θ_1 and convert the angle to degree.
(%i9) Calculate θ_2 and convert the angle to degree.
(%i12) Calculate θ_3 and convert the angle to degree.

Problem 20.10 The first-order spectrum lines are obtained at 30° when a light is incident to a diffraction grating with 6000 lines per cm. What is the wavelength of the light?

Solution

For a diffraction grating, a condition to get maximum intensity (bright bands) is (Eq. 20.5),

$d \sin\theta = m\lambda, \quad m = 0, 1, 2, \ldots$

For this problem,

$$d \sin\theta = m\lambda,$$
$$\left(\frac{1.0 \times 10^{-2}\ \text{m}}{6000}\right) \sin\left(30 \times \frac{\pi}{180}\right) = (1)\lambda,$$
$$\lambda = 8.3 \times 10^{-7}\ \text{m}.$$

The wavelength of the light is 8.3×10^{-7} m.

- wxMaxima codes:

```
(%i4) fpprintprec:5; d:1/6000*1e-2; theta:float(30/180*%pi); m:1;
(fpprintprec)     5
(d)   1.6667*10^-6
(theta) 0.5236
(m)   1
(%i5) lambda: d*sin(theta);
(lambda) 8.3333*10^-7
```

Comments on the codes:

(%i4) Set floating point print precision to 5, assign values of d, θ, and m.
(%i5) Calculate λ.

20.3 Summary

- In a single slit diffraction, the condition for destructive interference is

$$a \sin\theta = m\lambda, \quad m = \pm 1,\ \pm 2,\ \pm 3,\ \ldots$$

where a is the width of the slit and θ is diffraction angle. The intensity at a point on the screen is given by $I_\theta = I_0 \left[\frac{\sin(\beta/2)}{\beta/2}\right]^2$, where $\beta = \frac{2\pi a \sin\theta}{\lambda}$.

- The condition for intensity maxima for a diffraction grating whose slits are separated by a distance d is

$$d \sin\theta = m\lambda, \quad m = 0,\ \pm 1,\ \pm 2,\ \pm 3,\ \ldots$$

where θ is the diffraction angle and m is order number.

20.4 Exercises

Exercise 20.1 In a single slit diffraction experiment, a light of wavelength 600 nm is incident on a slit of width 1.90 μm. What are the diffraction angles of the first and second dark fringes?

(Answer: $\theta_1 = 18.4°$, $\theta_2 = 39.2°$)

Exercise 20.2 In a single slit diffraction experiment, a light of wavelength 610 nm is incident on a slit of width 3.1×10^{-5} m, and diffraction pattern is formed on a screen located 2.5 m away from the slit. Calculate the distance from the central maximum to the first and second minima on the screen.

(Answer: $y_1 = 0.049$ m, $y_2 = 0.098$ m)

Exercise 20.3 An astronomical telescope has a diameter of 5.60 m. Calculate the maximum angle of resolution for this telescope at a wavelength of 600 nm.

(Answer: 1.31×10^{-7} rad)

Exercise 20.4 A beam of light of wavelength 540 nm is incident normally on a diffraction grating with a slit spacing of 1.70×10^{-6} m. What are the angles for the first- and second-order maxima?

(Answer: $\theta_1 = 18.5°$, $\theta_2 = 39.4°$)

Exercise 20.5 A diffraction grating just resolves the wavelengths 610.0 and 610.2 nm in the first order. What is the number of slits in the grating?

(Answer: 3050)