Mathematical Modeling of Thermomechanical Phenomena in Machining of Products Made of Functionally Graded Materials

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Abstract. Mathematical modeling of thermomechanical processes that accompany the mechanical processing of products to control them in technological systems is one reserve for improving the quality of products and their performance in mechanisms. Deterministic modeling of thermomechanical phenomena in the mechanical processing of structurally homogeneous materials using equations based on continuous functions allows us to obtain solutions that are represented as analytical relations in closed form and are convenient for analyzing these processes and based on them to make a rational choice of technological parameters to ensure the required characteristics of the processed surfaces of products. The article presents a numerical and analytical model for determining the thermomechanical state during the mechanical processing of structurally inhomogeneous materials containing inhomogeneities such as interfacial cracks and inclusions. Based on this model, functional dependencies of surface layer quality criteria with technological control parameters are determined to ensure products' processed surfaces' required characteristics.

Keywords: Processing · Layer · Structural · Technological · Defects · Surface · Parameters · Temperature · Stress · Thermal State · Process Innovation · Industrial Growth

1 Introduction

Among the processes characteristic of mechanical processing, the principal place belongs to thermomechanical phenomena occurring in the processed surfaces of products. They significantly affect the formation of the thermomechanical state of the working surfaces of the parts. They are associated with the formation of defects such as cracks, burns, and structural changes in the treated surfaces, which contribute to a decrease in the operational properties of parts. Mechanical processing of modern materials, characterized by heterogeneity, is accompanied by high thermal stress and the formation of a thermoelastic state of the processed surface of the products, which increases the processing error and changes the quality of the surface layer due to the appearance of defects in it. Therefore, mathematical modeling of thermomechanical processes accompanying the mechanical processing of products to control them in technological systems is one of the reserves

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for identifying and forecasting process features that affect the quality of products and their performance in mechanisms. Thus, tooling is the most expensive and vulnerable regarding uptime in producing fiber optic cables operating at elevated temperatures and intense wear. Therefore, its working surfaces are subject to increased requirements for roughness, hardness, dimensional stability, and the absence of defects such as burns and cracks. Modeling the formation of thermomechanical fields during the processing of forming elements of tooling and the establishment of functional relationships between technological parameters, critical temperatures and stresses, and the tool's durability allows you to create optimal conditions for their operation.

Modern materials' mechanical processing, characterized by heterogeneity, is accompanied by high heat stress. The formation of a thermopile state of the processed surface of products, which increases the processing error, changes the surface layer's quality because of the appearance of defects in it. Therefore, mathematical modeling of thermomechanical processes that accompany the mechanical processing of products to control them in technological systems is one reserve for improving the quality of products and their performance mechanisms.

2 Literature Review

Modeling thermomechanical processes in machining products can be divided into two types: modeling processes using continuous functional dependencies and modeling thermomechanical processes with discontinuous functional dependencies. Deterministic modeling of thermomechanical phenomena using equations based on continuous functions [1] allows us to obtain solutions that are represented as analytical relations in closed form and are convenient for analyzing these processes and based on them to make a rational choice of technological parameters to ensure the required characteristics of the processed surfaces of products [2].

Thus, the most expensive and vulnerable uptime in fiber-optic cables is tooling that works in high temperatures and intense wear conditions. Therefore, its working surfaces are subject to increased requirements for roughness, hardness, dimensional stability, and the absence of defects such as cauterization and cracks.

Modeling the formation of thermomechanical fields informing tooling elements and establishing functional relationships between technological parameters, critical values of temperatures and stresses, and tool life allows optimal production conditions.

Mechanical processing of parts made of composite materials characterized by structural heterogeneity associated with functional gradient properties is accompanied by thermomechanical phenomena, which are not always described by continuous dependencies. Thus, the temperature and thermoelastic fields formed in the surface layer of products made of composite materials are subjected to discontinuities in the areas of accumulation of inhomogeneities during operation [3].

The most effective methods for modeling physical processes occurring in environments of inhomogeneous structure and electromagnetic signals in environments with variable characteristics, and the formation of micro-cracks in structural materials that have various types of heterogeneities of ancestral origin, until this time, remain methods using Cauchy integrals [4], boundary value problems of the theory of analytical functions, and the method of singular integral equations [5].

The functionality of products depends on the defects in the structure of their material. In natural materials, there are always numerous different micro-defects, the development of which, under the influence of load [6], leads to the appearance of cracks and their increase and, as a result, local or complete destruction [7]. In this paper, based on the method of singular integral equations, a unified approach to modeling thermomechanical processes in mechanical processing in the surface layer of products [8] whose materials have structural and technological inhomogeneities is presented [9].

The choice of a method for modeling thermomechanical phenomena that accompany the mechanical processing of structural elements depends on the object's size under study. For example, micro-studies [10] are associated with inhomogeneities formed in the surface layer at the workpiece's preparation stage during the manufacturing of structural elements. However, considering defects allows us to adequately consider the loss of functional properties of working surfaces of objects on their performance indicators.

The quality of the surface layer of structural elements during their manufacture is formed under the influence of thermomechanical phenomena accompanying mechanical processing [11]. The presence of stress concentrators in various types of heterogeneities of ancestral origin, introduced in obtaining the workpiece and subsequent types of mechanical processing, are the leading indicators of working surfaces' bearing capacity. The formation of technological origin defects occurs because of heat stress during structural elements' mechanical processing [11, 12]. Based on models of temperature fields, stress fields, and fracture mechanics, the regularities of the formation of defects such as structural changes, micro-cracks, and technological possibilities of their elimination are studied depending on the thermal properties of the processed materials, processing modes, design, and characteristics of the tools used [7, 8]. The problem of stress concentration in defects is solved using material mechanics, which consider the micro-uniformity and defect of their materials when calculating structural elements' load-bearing capacity. Finally, considering defects allows for a more adequately representing mechanism of loss of products' functional properties during machining.

Available models of thermomechanical processing processes are obtained under the assumption of uniformity of materials of structural elements. They do not consider defects in the technological inheritance of products [11]. There are studies of the influence of structural transformations in steels during their mechanical processing on the formation of cracks, according to which the presence of a large amount of austenite in the subsurface layer of parts leads to the formation of tensile stresses, which are realized in the form of brittle cracks [6, 7]. In some cases, structural elements' physical and technical processing is characterized by short duration and high heating and cooling rates. Structural changes are insignificant, and thermomechanical stresses reach limit values [8].

Models of the stress-strain state of parts with coatings have been developed that consider the piecewise heterogeneity of the product coating matrix [8, 9]. However, the lack of studies on the influence of inhomogeneities formed in the surface layer of products during mechanical processing on their functional properties and, in particular, on the bearing capacity or wear resistance determines the relevance of constructing a mathematical model of defect formation in the physical and technical processing of structural elements using the criteria of fracture mechanics. The simulation results allow us to assess the impact of structural inhomogeneities formed during machining in products' working surfaces on the loss of the required properties.

The research aims to develop a numerical-analytical model for determining the thermomechanical state of structurally inhomogeneous materials containing inhomogeneities such as interfacial cracks and inclusions during machining. This model determines the functional relationship of the surface layer quality criteria with the technological control parameters to ensure the required characteristics of the products' processed surfaces.

3 Research Methodology

When choosing and justifying the mathematical model, it was considered that thermal and mechanical phenomena accompany manufacturing parts. However, the combined effect on the stress-strain state of the surface layer is caused by temperature fields. Since the bulk of the surface layer of metal during physical and technical processing is elastic, it can use a thermoelastic body model that reflects the relationship of mechanical and thermal phenomena at finite values of heat flows. For studies of the thermomechanical state of the working surfaces of structural elements, information about the propagation of temperatures and stresses and the depth of the material, taking into account its inhomogeneities, is essential.

For further studies of the kinetics of the formation of thermomechanical processes in the processed material, we will use the following system of differential equations [11, 13] as the central theoretical premise, describing the interaction of the deformation field and the temperature field:

$$G\Delta\overrightarrow{U_j} + (\lambda_t + G)\text{grad div}\overrightarrow{U_j} - \rho\frac{\partial^2\overrightarrow{U_j}}{\partial\tau^2} + P_j = \alpha_t\beta_t\text{grad }T, \tag{1}$$

$$\Delta T - \frac{1}{a}\frac{\partial T}{\partial\tau} - \eta l\frac{\partial}{\partial\tau}\text{div}\overrightarrow{U_j} = -\frac{W}{\lambda} + C_q^{-2}\frac{\partial^2 T}{\partial t^2}, \tag{2}$$

$$\beta_t = 3\lambda_t + 2G;\ l = \frac{1+\tau_r\delta}{\delta};\ \eta = \frac{\alpha_t\beta_t T(\Phi,\tau)}{\lambda}$$

where λ_t, G are Lamé constants; ρ is the density of the processed material; α_t is the temperature coefficient of linear expansion of metal; $a = \frac{\lambda}{C_v}$ is the thermal diffusivity; λ is the thermal conductivity; C_v is the volume heat capacity; $\vec{U}(\Phi,\tau)$ is the total vector of movement of the inner point $\Phi(x, y, z)$ of the surface layer under the influence of thermomechanical forces accompanying the processing process; τ_r is the relaxation time; W is the power of the heat source; C_q is the speed of heat propagation in the processed material; τ is the time; P_j is the cutting force; $\text{grad }T(x, y, z) = \frac{\partial T}{\partial x}\vec{i} + \frac{\partial T}{\partial y}\vec{j} + \frac{\partial T}{\partial z}\vec{k}$, $\text{div}\overrightarrow{U_j} = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z}$, $(j \rightleftarrows x, y, z)$.

Since the thermal phenomena prevail over the power ones in the final processing methods, we can ignore the term that considers mechanical energy conversion into thermal energy in the heat equation. Therefore, we will come to a parabolic heat equation.

We will neglect the influence of inertial terms and heat propagation speed limitation for the explicit solvability of the specified system (1)–(2). Moreover, to overcome the analytical difficulties associated with solving spatial problems of thermoelasticity, we will consider the planar problem. This transition is justified because, for studying the thermomechanical state of the treated surfaces, information about the propagation of temperatures and deformations and the depth and direction of the source's movement is essential.

When drawing up the design scheme, we assume the processed product is modeled as a piecewise homogeneous half-plane. It allows us to study thermomechanical processes with several types of coatings with a thickness applied to the main matrix. This scheme determines the thermal and deformation conditions for the interface of layers along their section boundaries a_k.

The influence of structural inhomogeneities in the material during smelting and the technological process will be considered in the model by inclusions and defects such as microcracks in the surface layer.

The system of equations that determine the thermal and stress-strain state when processing the surface of parts with coatings, the upper layer of which has inhomogeneities such as inclusions and cracks, contains:

Equation of non-stationary thermal conductivity:

$$\frac{\partial T}{\partial \tau} = a^2\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \begin{array}{c} 0 \le x < \infty; \\ -\infty < y < \infty. \end{array} \tag{3}$$

The equation of the Lamé elasticity in displacements:

$$\frac{\partial \Theta}{\partial x}\frac{1}{1-2\mu} + \Delta\overline{u} = B^T\frac{\partial T}{\partial x}; \overline{u}(x, y) = \frac{u}{2G}; \overline{v}(x, y) = \frac{v}{2G}; \tag{4}$$

$$\frac{\partial \Theta}{\partial y}\frac{1}{1-2\mu} + \Delta\overline{v} = B^T\frac{\partial T}{\partial y}; B^T = \frac{4G(1+\mu)}{1-2\mu}a_k; \Delta = \frac{\partial^2}{dx^2} + \frac{\partial^2}{dy^2}, \tag{5}$$

where $T(x, y, \tau)$ is the temperature at a point with coordinates (x, y) and at any time τ; a – thermal conductivity of the material; α is the temperature coefficient of linear expansion; μ, G are Lamé constants; u, v are components of the point displacement vector (x, y); $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the Laplace operator.

The initial conditions for this task can be taken as follows:

$$T(x, y, 0) = 0. \tag{6}$$

Boundary conditions for temperature and deformation fields that take into account heat transfer from the surface outside the contact zone of the tool with the part and intense heat generation in the processing zone have the form:

$$\frac{\partial T}{\partial x} = -\frac{q(y, \tau)}{\lambda}, |y| < a^*, -\lambda\frac{\partial T}{\partial x} + \gamma T = 0, |y| > a^*, \tag{7}$$

$$\sigma_x(x, y, \tau)|_{x=0} = \tau_{\tau y}(x, y, \tau)|_{x=0} = 0, \tag{8}$$

where $q(y, \tau)$ is the intensity of the heat flow generated by the interaction of the tool with the part; λ is the coefficient of thermal conductivity of the processed material; $2a^*$ is the length of the tool contact zone with the work surface; γ is the coefficient of heat exchange with the environment; σ_x, τ_{xy} are normal and tangent stresses.

Conditions for interfacing layers (coatings) [13]:

$$\begin{array}{ll} \text{for temperature fields} & \text{for deformation fields} \\ \overset{k-1}{T}(a_k - 0, y, \tau) = \overset{k}{T}(a_k + 0, y, \tau) & \overset{k-1}{\upsilon}_j(a_k - 0, y) = \overset{k}{\upsilon}_j(a_k + 0, y) \\ \lambda_{k-q} \frac{\partial \overset{k-1}{T}}{\partial x}(a_k - 0, y, \tau) = \lambda_k \frac{\partial \overset{\sigma}{T}}{\partial x}(a_k + 0, y, \tau) & \overset{k-1}{\sigma}_x(a_k - 0, y) = \overset{k}{\sigma}_x(a_k + 0, y) \\ & \overset{k-1}{\tau}_{xy}(a_k - 0, y) = \overset{k}{\tau}_{xy}(a_k + 0, y) \end{array}; \tag{9}$$

where λ_k is the thermal conductivity of the k-th layer; α_k is the thickness of the k-th layer; $\overset{k}{\upsilon}_j$ is the displacement components in the k-th layer.

For surface layers with structural and technological inhomogeneities, the discontinuity conditions of the solution, depending on the defect, will be [13]:

$$\begin{array}{ll} \text{for inclusion} & \text{on crack} - \text{like defects} \\ \langle \overline{\upsilon} \rangle = 0, \langle \sigma_x \rangle \neq 0 & \langle \sigma_x \rangle = 0, \langle \overline{\upsilon} \rangle \neq 0 \\ \langle \overline{v} \rangle = 0, \langle \tau_{xy} \rangle \neq 0 & \langle \tau_{xy} \rangle = 0, \langle \overline{v} \rangle \neq 0 \end{array} \tag{10}$$

where $\langle \overline{\upsilon} \rangle$, $\langle \overline{v} \rangle$, $\langle \sigma_x \rangle$, $\langle \tau_{xy} \rangle$ are jumps in the displacement and stress components.

The deformable surface layer's maximum equilibrium state was tested by classical strength criteria [14, 15]. Of the fracture criteria that consider local physical and mechanical properties of inhomogeneous materials, the most acceptable for this case are the criteria of the force approach [4, 5], associated with the use of the stress intensity factor (SIF). When loading causes the stress intensity to equal the limit value of K_{1c}, the crack-like defect turns into the main crack.

Modeling the effect of the initial piecewise homogeneity of the processed materials (parts with coatings) on thermomechanical processes is carried out using discontinuous solutions [13]. They are solutions that satisfy the Fourier thermal conductivity and Lamé elasticity equations everywhere except for the defect boundaries. When crossing the boundary, the fields of displacement and stress suffer discontinuities of the first kind, i.e., their jumps $\langle \overline{\upsilon} \rangle$, $\langle \overline{v} \rangle$, $\langle \sigma_x \rangle$, $\langle \tau_{xy} \rangle$ appear.

The solution of the thermal problem (3)–(10) is carried out using integral Fourier transforms for the variable y and Laplace transformations for τ to the function $T(x, y, \tau)$ in the first ($k = 0$) layer, which is described in integral form as [13]:

$$T_0(x, y, \tau) = \int_{-a}^{a} d\tau \int_0^{\tau} \chi(t - \tau, x, y - \eta) dt, \tag{11}$$

where $q(t, x, y - \eta) = \frac{1}{2\pi i} \int_r K_p^m(y - \eta, x) e^{pt} dp$, $\chi(y, \tau) = \sum_{m=0}^{\infty} \chi_m(y) 2e^{-\tau} L_m(2\tau)$ are polynomials of the Laguerre; $K_p^m(y - \eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-t\beta(y-\eta)}}{l_{m_1\beta p}} d\beta$; $l_{m_1\beta p}$ is the expression that considers the thermal properties of layers $k = 0 - m$, their thickness, and boundary conditions.

The stress-strain state of the layered half-plane is also estimated by the method of discontinuous solutions. The boundaries of the section $x = a_k (k = 0)$ are defects when passing through which the displacement and stress fields suffer discontinuities.

The construction of discontinuous solutions of the Lamé equations with prescribed jumps is done using the functions of Trefftz [16]:

$$\begin{gathered} \overline{\upsilon} = \psi_1 + (x - a)\psi_0', \\ \overline{v} = \psi_2 + (x - a)\psi_0', \\ \psi_0' = \frac{\psi_1' + \psi_2'}{3 - 4\eta}, \\ e = \psi_1' + \psi_2'' + \psi_0', \end{gathered} \tag{12}$$

where $\Delta\psi_0(x, y) = 0$, $\Delta\psi_j(x, y) = b_k^t T^{(j)}$, $(j = 1, 2)$.

Stresses are found by the equations:

$$\begin{gathered} \sigma_x = (1 - \mu)\psi_0' + \psi_1' + (x - a_k)\psi_0''; \\ \sigma_y = -\mu\psi_0' + \psi_2' + (x - a_k)\psi_0''; \\ \tau_{xy} = \psi_1^2 + 2(x - a_k)\psi_0' + \psi_2' + \psi_0'. \end{gathered} \tag{13}$$

where $\Delta\psi_0(x, y) = 0$, $\Delta\psi_j(x, y) = b_k^t T^{(j)}$, $(j = 1, 2)$.

Application of generalized Fourier transforms for variables x, y to Eqs. (2), (3), (6), (7), taking into account (10), allows us to obtain recurrent relations that link the displacements and stresses in the k-th layer with the stresses and displacements formed in the first layer under the action of non-stationary temperature fields [17].

Under conditions of uneven heating, thermal deformations occur in the surface layer, which causes temperature stresses. Crack formation occurs under the influence of these stresses, concentrated in defects [18].

The most interesting is the behavior of stresses near the vertices of defects such as cracks, sharp inclusions, and structural imperfections, i.e., stress features at $y \to \pm l_k$. The stress field's nature near the end of the defect obtained in the classical's framework elasticity theory is determined by the stress intensity coefficients $K_I + iK_{II}$.

The influence of the tool design parameters on the thermomechanical state of the surface layer was determined using the model problem (1), (4) and boundary conditions in the form of:

$$q(y, \tau) = \frac{c\sqrt{\tau}}{\lambda}\left[H(y) - H\left(y - 2a^*\right)\right]\sum_{k=0}^{n} \sigma\left(y + kl - v_{kp}\tau\right) \tag{14}$$

where $H(y)$ is the function of Heaviside; $\sigma(y)$ is the Dirac Delta function; n is the number of cutting edges of the tool that pass through the contact zone during $\tau = \frac{\sqrt{\pi t_{gr}}}{\upsilon_{kp}}$; λ is the thermal conductivity of the product material; $c\sqrt{\tau}$ is the heat flow from a single edge; v_g, v_{kp}, t_{gr} are processing modes; $2a^*$ is the arc length of the tool contact with the part; l^* is the distance between the cutting edges.

The maximum values of the instantaneous temperature T_M, from the individual grains to the constant component $-T_K$, were obtained theoretically and confirmed experimentally. These values were later used as criteria for predicting the conditions for forming cauterization-type defects and their depth.

The study of the role of heterogeneity of the coating structure in the mechanism of reducing crack resistance was carried out using the local fracture criterion established by the model in the form of the following inequality:

$$l_0 < \frac{Da\lambda^2 \upsilon_q^u K_c^2}{\pi^2\left[c\upsilon_{kp} G(1+\nu)a_t\left(1 - exp\left(\frac{\upsilon_q\sqrt{Dt}}{a\tau}\right)\right)\right]}, \tag{15}$$

where υ_{kp}, υ_q, t are processing modes; D, C are tool parameters; λ, a are thermophysical characteristics of the treated coating; K_C is the crack resistance of this coating; G is the elastic modulus; ν is the Poison's ratio; a_t is the temperature coefficient of linear expansion; l_0 is the characteristic linear size of the structural parameter (structure defect).

The technique of a choice of tools and modes of processing wear-resistant coatings provides no cracks or chips on the treated surfaces based on the conditions (14), inequalities (15), and constraints on stresses of the detachment of the coating from the matrix:

$$\sigma_{max}\left(M, a_t, E, \nu, a, \lambda, \upsilon_{kp}, \upsilon_q, t_{gr}\right) \le [\sigma]_{SI} \tag{16}$$

The model allows optimal performance technological parameters for defect-free parts processing with coatings based on the coating's initial thermal and mechanical properties—detail system.

4 Results and Discussion

The quality of the processed surfaces will be ensured using the technological control parameters to choose such processing modes. The tool's characteristics are that the current values of the processing temperature and heat flow $q(y, \tau)$, stress $\sigma(M)$, and cutting forces P_y, P_z will not exceed their limit values.

Implementing a system of limiting inequalities in the values of the temperature itself and the depth of its propagation in the form of:

$$T(x, y, \tau) = \frac{C}{2\pi\lambda}\sum_{k=0}^{n} H\left(\tau - \frac{kl}{\upsilon_{kp}}\right) \times H\left(\frac{L+kl}{\upsilon_{kp}}\right)\int_{\gamma_1}^{\gamma_2} f\left(x, y, \tau, \tau'\right)d\tau' \le [T]_M \tag{17}$$

$$\begin{gathered} T_k(0, y, \tau) = \frac{C\upsilon_{kp}}{\pi\lambda l\sqrt{\upsilon_g}}\int_a^{\tau}\int_{-e}^{e}\frac{\chi(\eta, t)e^{\frac{(y-\eta)^2}{4(\tau-t)}}}{2\sqrt{\pi(\tau-t)}}\times \\ \times\left\{\frac{1}{\sqrt{\pi(\tau-1)}} + \gamma e^{\gamma^2(\tau-t)}\left[1 + \Phi\left(\gamma\sqrt{\tau-t}\right)\right]\right\}d\eta dt \le [T] \end{gathered} \tag{18}$$

$$T_k^{\max}(L, 0) = \frac{C\upsilon_{kp}\alpha}{\lambda l \upsilon_q^2}\sqrt{\frac{\alpha}{\pi}}\left[1 - exp\left(-\frac{\upsilon_q\sqrt{Dt_{gr}}}{\alpha}\right)\right] \le [T] \tag{19}$$

avoids the formation of structural changes. It can serve as a basis for the design of processing cycles based on the thermal criterion.

Processing of materials and alloys without cracks can be provided if the stresses formed in the intensive cooling zone are limited to the maximum values:

$$\sigma_{max}(x,\tau) = 2G\frac{1+\nu}{1-\nu}\alpha_t T_k erf\left(\frac{x}{2\sqrt{\alpha\tau}}\right) \le [\sigma_i] \tag{20}$$

With a dominant influence of hereditary inhomogeneity on the intensity of crack-like defects, it is necessary to use criteria that include deterministic relationships of technological parameters and properties of the inhomogeneities themselves. As such, it can use the restriction of the stress intensity coefficient:

$$K = \frac{1}{\pi\sqrt{l}}\int_{-e}^{e}\sqrt{\frac{l+t}{l-t}}\sigma_x, \sigma_y dt \le K_{Ic} \tag{21}$$

Alternatively, providing with the help of control technological parameters of the limit value of the heat flow, at which the balance of structural defects is maintained:

$$q^* = \frac{P_z \upsilon_{kp}\alpha_s}{\sqrt{Dt_{gr}}} \le \frac{\sqrt{3}\lambda K_{Ic}}{Hl\sqrt{\pi l \sigma}} \tag{22}$$

Conditions for defect-free processing can be implemented using information about the structure of the processed material. So, with the general nature of structural imperfections of length $2l$, their regular location relative to the contact zone of the tool with the part, it can use the condition of defect equilibrium as a criterion ratio in the form of:

$$l_0 < \frac{K_C^2}{x[GT_k(1+\nu)\alpha_t]^2} \tag{23}$$

In this formula, the process part is contained in the contact's relation temperature value Tк to the processing conditions.

These inequalities link the limiting characteristics of the temperature and force fields with the control and technological parameters. They specify the range of combinations of these parameters that meet the obtained thermomechanical criteria. Finally, it considers the properties of the processed material and guarantees the required quality of products.

Based on the obtained criteria, an algorithm for ensuring the surface layer of parts during machining, considering the maximum performance, is constructed.

It is known that the quality of processing the working surfaces of tooling for producing fiber-optic cables significantly affects their functionality [19]. Reducing the roughness of processing, all other things being equal, increases static strength, especially brittle, and significantly limits endurance. It was found [19] that samples with low surface roughness have a higher impact strength. The influence of the micro geometry of working surfaces on technological stresses explains these facts. It is believed that the irregularities formed during surface treatment are effective concentrators, leading to surface cracks on the form-forming surfaces of tooling. Optical fibers that are used in communication systems are made of quartz glass. This material provides a low loss level in a fairly broad frequency range, allowing large distances between regenerators. An optical fiber is a two-layer dielectric waveguide through which one or more modes (types of electromagnetic waves) propagate.

The main reason leading to the scattering of light energy in optical fibers (OF) is microscopic inhomogeneities in the OF material that occur during their manufacture and the fiber-coating interface's roughness. Therefore, one of the main requirements for the tool in producing fiber-optic cables is the low roughness of its working surfaces, the high geometric accuracy of the calibration part, and compliance with special conditions in OF production. Using the correlation between the roughness of the working area of the die and the defect value $(-\alpha, \alpha)$ in various finishing operations: finish grinding and super finish grinding, polishing, the SIF dependence $K_{III} = f(\alpha)$ is found (Fig. 1).

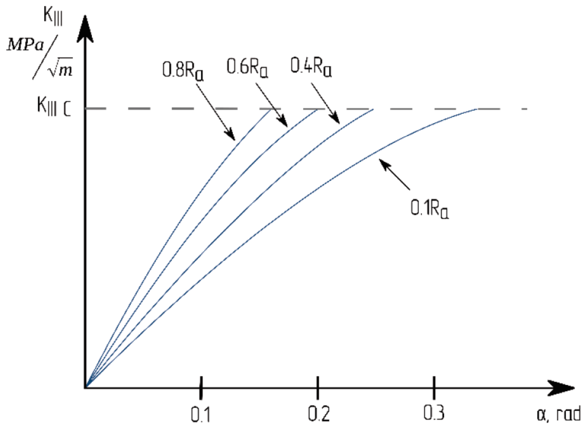

Fig. 1. Dependence of SIF on the size of the defect and roughness of the working area of the tool R_a.

At existing operating stresses on the forming surfaces, tooling is necessary to choose the type and processing modes of the working surfaces, which provide the necessary roughness for preserving the functional properties of fiber-optic products (Fig. 2).

Thus, to ensure the required quality characteristics of the fiber-optic cables, it is necessary to ensure a roughness of $0.8 \leq R_a \leq 1.2$ microns on their working surfaces when manufacturing tools. This roughness can be achieved through grinding operations and subsequent finishing polishing.

Based on the nature of mandrel molds' operation and dies for their manufacture, the most acceptable are hard alloys. Therefore, it is essential that when manufacturing these tools, there are no micro defects on their working surfaces, the roughness corresponds to the required values, and the error of the calibration part is within the tolerance.

The determination of rational technological parameters in the manufacture of mandrel dies should begin from the conditions of their choice in the range that will avoid the development of micro defects in cracks when machining the tool's working surfaces. Existing methods for evaluating the strength of tools obtained by sintering according to the level of the ultimate stress state formed during machining could be more effective. Such calculations ignore the presence of hereditary defects in the processed product, as well as the presence of growing subcritical microcracks, which leads to a significant overestimation of the allowable stresses and, as a result, to an incorrect assessment of

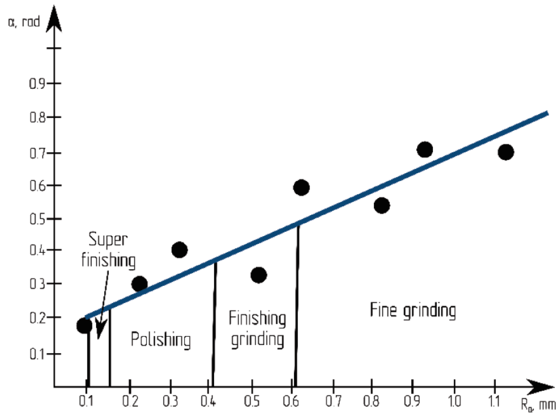

Fig. 2. Influence of working surface treatment modes of tool equipment on its functional properties.

the tool resource, which can be represented as:

$$N_p = N_1 + N_2 \tag{24}$$

where N_1 is the percentage of durability due to the growth of micro defects to a critical size during operation, N_2 is the percentage of tool durability associated with the growth of a macrocrack to the limit state during its manufacture.

Given that tools made of superhard materials are inherently highly defective, the N_2 component can be the central part of their durability, i.e., it ultimately determines the tool's life.

To determine the tool life, the characteristics of the material's resistance to the growth of a fatigue crack formed at the stage of mechanical processing of forming surfaces are used.

To avoid structural transformations in the tool's surface layer, it is necessary to limit the contact temperature, which is formed when processing the working surface of the tool, to the temperature of structural changes (18). Given that the current values of processing temperatures are determined by the processing modes and characteristics of the tool being processed, consistent assignment of modes, and rational selection of characteristics of the processing tool, it is possible to approach the performance of defect-free processing to T_{opt} iteratively – the optimal processing temperature, which should not exceed the temperature of structural transformations in the material of tools for manufacturing fiber-optic cables.

Fractographic analysis of die and mandrel matrix fractures showed that thermal fatigue characterizes their working surfaces' cracks. A significant localization of deformation is observed under the influence of variable stresses formed during the machining of these tools due to uneven heating and cooling. Internal defects formed at the stage of sintering dies and mandrel matrices, being stress concentrators, often serve as a place of origin for cracks. Therefore, when choosing the material for obtaining die blanks, and mandrel matrices, the low coefficient of thermal expansion α_t, high thermal conductivity

λ_m, and low elastic modulus E_m were considered. This approach to choosing the physical and mechanical properties of the material for tool blanks in producing fiber-optic cables also creates a reserve of durability.

The nature of cracks in the machining of working surfaces of tools for fiber-optic cable production indicates that the grain boundaries of the tool material are more damaged than the grain body. The grid of cracks forming on the tool surface indicates the locality of technological stresses and, most importantly, their relatively rapid relaxation. Theoretical studies show that the most significant temperature stresses are formed when grinding the dies' working surfaces. Moreover, their level increases with increasing heat transfer coefficient. The time of maximum stress formation is equal to the time of contact of the processing tool with the processed surface of dies, mandrel molds, and other equipment for fiber-optic cable production.

When sintering dies blanks, mandrel dies, and other tools for producing signal transmission equipment, it is essential to ensure their material is minimally defective. Otherwise, in the presence of crack-like defects and depending on their density, "hot" prominent cracks appear on their forming surfaces under the influence of temperature stresses.

Based on the modeling of the thermomechanical state of the working surfaces of tooling during its manufacture and operating conditions, it is established that the optimal material for forming and calibration tools is tungsten-cobalt hard alloys. Hard alloys' physical and mechanical characteristics and wear resistance are mainly determined by the ratio of tungsten carbide and cobalt volume fractions, whose properties differ significantly.

In cases where wear-resistant coatings are applied to the tooling's calibration surface, the problem arises of preserving its functionality during operation. Ensuring the fiber-optic cables' quality characteristics significantly depends on the coating condition, which is prone to cracking during finishing operations under the influence of thermomechanical processes due to its partial detachment from the primary material of the calibration tool (Fig. 3).

Therefore, based on the developed model and the obtained criterion ratio (23), it is possible to prevent cracks and chips on the forming surfaces of tooling by the established functional relationships of the crack resistance criterion (21) with the control technological parameters of finishing operations.

The designations used in Fig. 3: 1 is wear-resistant coating; 2 is the matrix of the primary material; q is heat flow entering the coating-matrix system when processing the calibration surface of tooling; $(-l, l)$ is the partial separation of the coating from the matrix; σ^t is the tensile stresses formed in the matrix; σ^c – compressive stresses in the coating; Δ_p is the coating thickness; $q_1 > q_2$.

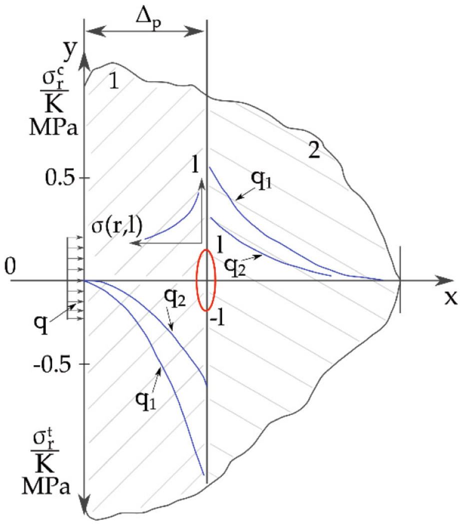

Fig. 3. Calculation scheme for determining the stress-strain state of the coating-matrix system at the boundary, which partially separates the coating from the matrix, size 2l.

5 Conclusions

A mathematical model has been developed that describes thermomechanical processes in the surface layer when processing parts made of materials and alloys, taking into account their inhomogeneities that affect the formation of performance characteristics. The calculated dependencies between the stress intensity factor and the primary control technological parameters are obtained. According to the known characteristics of inherited defects, the limit values of heat flow during machining are determined to ensure the processed surfaces' required quality.

Modeling of thermomechanical processes that form in the surface layer of processed products makes it possible to obtain criteria ratios, the implementation of which, by selecting technological parameters of processing on their basis, makes it possible to provide the required quality indicators for the working surfaces of products, in particular, to avoid defects such as burns and cracks.

The practical use of the obtained results is that there is information about the distribution of hereditary defects in the presence of electron microscopy of the surface layer of products before finishing operations. Choosing among them the defect of the most significant dimensions l_0 – the characteristic linear size of the structural parameter (structure defect) and using the obtained criteria relations, tooling, and processing modes are assigned to link the limiting characteristics of the temperature and force fields with control and technological parameters. They set the range of combinations of these parameters that satisfy the obtained thermomechanical criteria. At the same time, the processed material's properties are considered, and the required quality of products is guaranteed.

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