Neutral-Bipolar Fuzzy Sets and Its Applications

G. Aruna and J. Jesintha Rosline

1 Introduction

The notion of ambiguity and haziness in real life can be simply dealt with the flexibility of fuzzy set theory presented by Professor Lotfi. A. Zadeh in the year 1965 [9]. Further, the theory of Fuzzy numbers was established by various authors like Krassimir T. Atanassov [1], Dubois, R. Yager (1993), and Mizumoto, J. Buckley [9]. Wen–Ran Zhang initiated the concept bipolar fuzzy sets in 1998 [8]. The bipolar fuzzy set theory is the blending of both polarization and fuzziness [8]. It delivers a theoretical basis for clustering and conflict determinations. Bui Cong Cuong [2] introduced the concept called Picture fuzzy sets (PFSs). The idea of neutrality invoked the PFS as a dynamic area in fuzzy sets. Muhammad Riaz [6, 7] presented the hybrid idea of bipolar picture fuzzy operators. Fuzzy arithmetic models are useful innovations because of their differences between the ancient numerical models in sciences and symbolic models used in expert systems. Basic description, examples, properties of fuzzy sets defined by authors like Kaufmann, Dubois, Prade, Yager, Garg and Jesintha Rosline et al. [3, 4, 5] are stepping stones for Neutral-Bipolar fuzzy sets (NBFSs). The features of bipolarity and its applications for organic sciences, Psychology, neural networking, decision-making, business, and applied mathematics motivate to frame the conception Neutral-Bipolar fuzzy sets (NBFSs). In human life, the existence of uncertainty involves in decision-making of each possession. Typically, human beings have the nature of neutrality and bipolarity in making decisions. The exploration of neutrality and bipolarity with the impact of fuzzy set paves the way of new notion Neutral-Bipolar fuzzy sets. The Neutral-Bipolar fuzzy set is a fuzzy arithmetic tool, which has a far greater

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expressive power than interval arithmetic by virtue of gradation of membership in a fuzzy set.

The main objective of this chapter is to present the distinctive conception termed as Neutral-Bipolar fuzzy sets (NBFSs). The neutrality degree of membership ranges from $[-1, 1]$ in NBFS. The section two comprises of basic definitions. The Neutral-Bipolar fuzzy set is well defined in the section three. The properties of Neutral-Bipolar fuzzy sets, its operations, and theorems are discussed in section four and five. Finally, the applications of NBFS in MCDM technique of decision-making are illustrated.

2 Preliminaries

Picture Fuzzy Sets [2] A picture fuzzy set (PFS) on the universe U is defined as P = {(x, $\mu(x)$, $\eta(x)$, $\nu(x)$)} where $\mu(x)\epsilon$ $[0, 1]$ is the positive membership degree of x in P, $\eta(x)\epsilon$ $[0, 1]$ is the neutral membership degree and $\nu(x)\epsilon$ $[0, 1]$ is the negative membership degree.

Bipolar Fuzzy Sets [8] Let X be the nonempty fuzzy set. The bipolar fuzzy set $B = \{B^-, B^+\}$ is defined in X. $B^+ \in [0, 1]$ is the satisfaction degree of x in B, $B^- \in [-1, 0]$ is the satisfaction degree of the implicit counter property of x in B.

Operations on Picture Fuzzy Sets [2] Let K_1, $K_2 \in PFS$: (i) $K_1 \subseteq K_2$ if $\mu_{K_1}(x) \leq \mu_{K_2}(x)$, $\eta_{K_1}(x) \leq \eta_{K_2}(x)$ and $\nu_{K_1}(x) \geq \nu_{K_2}(x) \forall x \in X$. (ii) $K_1 = K_2$ if $K_1 \subseteq K_2$ and $K_1 \supseteq K_2$, (iii) $K_1 \cup K_2 = \Big\{ < x, \max\big(\mu_{K_1}(x), \mu_{K_2}(x)\big)$, $\min\big(\eta_{K_1}(x), \eta_{K_2}(x)\big)$, $\min(\nu_{K_1}(x), \nu_{K_2}(x)) > \} \forall x \in X$. (iv)$K_1 \cap K_2 = \Big\{ < x, \min\big(\mu_{K_1}(x), \mu_{K_2}(x)\big), \min\big(\eta_{K_1}(x), \eta_{K_2}(x)\big), \max(\nu_{K_1}(x), \nu_{K_2}(x)) > \} \forall x \in X$. (v) ${K_1}^C = \big\{ < x, \nu_{K_1}(x), \eta_{K_1}(x), \mu_{K_1}(x) > \big\} \forall x \in X$.

3 Neutral-Bipolar Fuzzy Sets

The spatial relation is expressed by human mind in the alternate concept of "left" and "right" as opposite direction is as equivalent polarization. But this does not mean that one of them is the negation of the other. The symmetric of 'opposite' captures a notion of semantics rather than a complexity. We are familiar with positive and negative membership degree. In Neutral-Bipolar fuzzy set, the neutral membership refers to the rate of chances between positive and negative side, which exists in the desirable and counter desirable rate. A chance is a reaction, which exists between the stimulation and the responses. Here, the stimulation may come from both positive and negative sides. Hence, the level of indefiniteness in desirable and counter desirable intensifies the neutral level. A human brain not only thinks about the positive and negative poles but also about the neutrality level due to circumstance. The level of neutral need not be zero at all stages. Positive stimulators intensify the neutrality over positive side. Negative stimulators intensify the neutrality over negative side. The intensification range of positive stimulators may result in the diminished rate of negative stimulators and the intensification range of negative stimulators may result in the diminished rate of positive stimulators. The circumstance plays a major role in NBFS. Circumstance sometimes refers to self-motivation, workouts, maintaining good health, people around us, biological diseases, etc. The circumstance constitutes the region with neutrality in positive range and negative range. Hence, we consider the range of neutrality lies in the interval $[-1, 1]$.

Neutral-Bipolar fuzzy set Let X be a nonempty set. A Neutral-Bipolar Fuzzy set (NBFS) is defined as

$$NB = \{< x, \ \mu(x), \ \eta(x), \ \gamma(x), \lambda\ (x) \mid x \epsilon X\},$$

where positive degree of membership $\mu(x) \in [0, 1]$ refers to the desirable rate of x in NB, negative degree of membership $\gamma(x) \in [-1, 0]$ is the counter desirable rate of x in NB, the neutral degree $\eta(x) \in [-1, 1]$ is the rate of satisfaction of the property and counter property to some extent of x in NB, and $\lambda\ (x) \in [0, 1]$ is the undesirable value of x in NB with the conditions $0 \leq \mu(x) + \lambda\ (x) \leq 1$ and $0 \leq |\mu(x)| + |\gamma(x)| + |\eta(x)| \leq 2\ (\forall x \epsilon X)$. Now $2 - (|\mu(x)| + |\gamma(x)| + |\eta(x)|)$ is the refusal degree of x in NB. Let NBFS(x) denote the set of all Neutral-Bipolar fuzzy sets on the universe U.

Neutral-Bipolar fuzzy set can be applied in the circumstances where the neutrality exists. Suppose a person is undergoing a treatment for a particular disease, then

- The rate of curable is referred to as desirable rate.
- The rate of deterioration is referred to as counter desirable rate.
- The rate of chances being positive and negative due to health issues like high immune power system, blood pressure, and diabetes is referred to as neutrality rate.
- If the person is not taking any treatment, it is referred to as undesirable rate.

Degree of NBFS Let $NB_1 \in NB(x)$, then

1. The degree of neutral membership is defined as

$$d\ (\eta(x)) = 2 - [|\mu(x)| + |\gamma(x)|]\ \ (\forall x \epsilon X)$$

2. The degree of positive desirable membership is defined as

$$d_f\ (\mu(x)) = \mu(x) + \mu(x).\mid \eta(x) \mid (\forall x \epsilon X)$$

3. The degree of negative desirable membership is defined as

$$d_f\,(\gamma(x)) = |\,\gamma(x)\,| + |\,\gamma(x)\,|\,.\,|\,\eta(x)\,|\ (\forall x \in X)$$

For example, Let A be the Neutral-Bipolar fuzzy set with $\mu(x) = 0.6$, $|\gamma(x)| = 0.5$ then $d(\eta(x)) = 0.9$, $d_f(\mu(x)) = 1.14$ and $d_f(\gamma(x)) = 0.95$. Suppose if we take the positive response for the product is 0.6, the negative response for the product is 0.5, the neutral response is 0.9, then by using the degree of NBFS we can find the favorable positive response is 1.14 and the favorable negative response is 0.95.

Remark

1. If $\mu(x) \neq 0$, $\gamma(x) = \eta(x) = \lambda(x) = 0$, then NBFS reduces to FS.
2. If $\mu(x) \neq 0$, $\lambda(x) \neq 0$, $\gamma(x) = \eta(x) = 0$,then NBFS reduces to IFS.

4 Properties of Neutral-Bipolar Fuzzy Sets (NBFSs)

Subset Let X be a nonempty set and NB_1, $NB_2 \in NBFS(x)$. Then NB_1 is a subset of NB_2 denoted by$NB_1 \subseteq NB_2$ if for each $x \in X$; $\mu_{NB_1}(x) \leq \mu_{NB_2}(x)$, $|\,\eta_{NB_1}(x)\,| \leq |\,\eta_{NB_2}(x)\,|$ and $\gamma_{N_{B1}}(x) \geq \gamma_{NB_2}(x)$.

Complement of NBFS The complement of the set $NB_1 \in NB(x)$ is symbolized as $NB_1{}^C$ and it is indicated by $NB_1{}^C = \left\{1 - \mu_{NB_1}(x), 2 - |\eta_{N_{B1}}(x)|\ , -1 - \gamma_{NB_1}(x)\right\}$ $(\forall x \in X)$.

Equal Set The two nonempty set NB_1, $NB_2 \in NB(x)$ are said to be equal set if for each $x \in X$; $\mu_{NB_1}(x) = \mu_{NB_2}(x)$, $|\,\eta_{NB_1}(x)\,| = |\,\eta_{NB_2}(x)\,|$ and $\gamma_{NB_1}(x) = \gamma_{NB_2}(x)$.

Equivalent The two NBFS set NB_1, $NB_2 \in NB(x)$ are alleged to be equivalent to each other. $NB_1 \sim NB_2$. If there exists function$f : \mu_{NB_1}(x) \rightarrow \mu_{NB_2}(x)$, $f : |\,\eta_{NB_1}(x)\,| \rightarrow |\,\eta_{NB_2}(x)\,|$ and $f : \gamma_{NB_1}(x) \rightarrow \gamma_{NB_2}(x)$, which are bijective. Then the function has a 1–1 correspondence between domain and codomain.

Proper Subset If $\mu_{N_{B1}}(x) \leq \mu_{NB_2}(x)$, $|\,\eta_{NB_1}(x)\,| \leq |\,\eta_{NB_2}(x)\,|$, $\gamma_{NB_1}(x) \geq \gamma_{NB_2}(x) \forall x \in X$ and $NB_1 \subseteq NB_2$, $NB_1 \neq NB_2$. Then $NB_1$is the proper subset of NB_2.

Definition 1 If $NB_1, NB_2, NB_3 \in NB(x)$. Then we have the following:

1. If $NB_1 \precsim NB_1$, then NB_1 is reflexive.
2. If $NB_1 \precsim NB_2$, $NB_2 \precsim NB_1$,then it is symmetric.
3. If $NB_1 \precsim NB_2$, $NB_2 \precsim NB_3$ implies $NB_1 \precsim NB_3$, then it is transitive relation.

5 Operations on Neutral-Bipolar Fuzzy Sets

5.1 Operations and Theorems on NBFS

Operations on NBFS Let $NB_1 = (\mu_1(x),\ \eta_1(x), \gamma_1(x))$ and $NB_2 = (\mu_2(x),\ \eta_2(x), \gamma_2(x))$ be two NBFS. Then

(i) Union: $NB_1 \cup NB_2 = \{ < x, \max\left(\mu_{NB_1}(x), \mu_{NB_2}(x)\right), \max\left(|\eta_{NB_1}(x)|, |\ \eta_{NB_2}(x)|\right), \min(\gamma_{NB_1}(x), \gamma_{NB_2}(x)) >; \forall x \in X \}$

(ii) Intersection: $NB_1 \cap NB_2 = \{ < x, \min\left(\mu_{NB_1}(x), \mu_{NB_2}(x)\right), \min\left(|\eta_{NB_1}(x)|, |\ \eta_{NB_2}(x)|\right), \max(\gamma_{NB_1}(x), \gamma_{NB_2}(x)) >; \forall x \in X \}$

Proposition 1 Bipolar fuzzy set is the special instance of Neutral-Bipolar fuzzy set (NBFS).

Proof For any $x \in X$, consider an NBFS set $NB_1 = (\mu_1(x),\ \eta_1(x), \gamma_1(x))$. Suppose the value of $\eta_1(x)$ is zero, then NBFS diminishes to Bipolar fuzzy set.

Proposition 2 The NBFS is not the generalization of Picture Fuzzy set.

Proof For any $x \in X$, consider an NBFS set $NB_1 = (\mu_1(x),\ \eta_1(x), \gamma_1(x))$.

Although NBFS encompasses the neutrality property, it contrasts in ranges from PFS. Moreover, the length of neutrality is functional in NBFS. So, NBFS is not the generalization of PFS.

Theorem 1 Let $NB_1, NB_2, NB_3 \in NB(x)$ in the universe U. Then, we have various operational rules:

(i) Idempotent Rule: $NB_1 \cup NB_1 = NB_1$ and $NB_1 \cap NB_1 = NB_1$
(ii) Commutative Rule: $NB_1 \cup NB_2 = NB_2 \cup NB_1$ and $NB_1 \cap NB_2 = NB_2 \cap NB_1$.
(iii) Complementary Rule: $({NB_1}^C)^C = NB_1$
(iv) Associative Rule: $(NB_1 \cup NB_2) \cup NB_3 = NB_1 \cup (NB_2 \cup NB_3)$ and

$$(NB_1 \cap NB_2) \cap NB_3 = NB_1 \cap (NB_2 \cap NB_3)$$

(v) Distributive Rule: $NB_1 \cup (NB_2 \cap NB_3) = (NB_1 \cup NB_2) \cap (NB_1 \cup NB_3)$ and $NB_1 \cap (NB_2 \cup NB_3) = (NB_1 \cap NB_2) \cup (NB_1 \cap NB_3)$
(vi) De Morgan's Rule: $(NB_1 \cup NB_2)^C = {NB_1}^C \cap {NB_2}^C$

$$(NB_1 \cap NB_2)^C = {NB_1}^C \cup {NB_2}^C$$

Proof

(i)

$$NB_1 \cup NB_1 = \left\{ <x, \max\left(\mu_{NB_1}(x), \mu_{NB_1}(x)\right), \max\ \left(|\eta_{NB_1}(x)|, |\ \eta_{NB_1}(x)|\right), \min\left(\gamma_{NB_1}(x), \gamma_{NB_1}(x)\right) >; \forall x \in X \right\}$$

$$= NB_1 .i.e, NB_1 \cup NB_1 = NB_1.$$

Similarly, $NB_1 \cap NB_1 = NB_1$.

(ii)

$$NB_1 \cup NB_2 = \left\{ < x, \max\left(\mu_{NB_1}(x), \mu_{NB_2}(x)\right), \max\ \left(|\eta_{N_{B1}}(x)|, |\ \eta_{NB_2}(x)|\right), \min\left(\gamma_{NB_1}(x), \gamma_{NB_2}(x)\right) >; \forall x \in X \ \right\}$$

$$=\{<x, \max\left(\mu_{NB_2}(x), \mu_{NB_1}(x)\right), \max\left(|\eta_{NB_2}(x)|, |\eta_{NB_1}(x)|\right), \min(\gamma_{NB_2}(x), \gamma_{NB_1}(x))>; \forall x \in X \ \} = NB_2 \cup NB_1.$$

Similarly, $NB_1 \cap NB_2 = NB_2 \cap NB_1$.

(iii)

$$NB_1{}^C = \left\{ < x, 1 - \mu_{NB_1}(x), 2 - |\eta_{NB_1}(x)|\ , -1 - \gamma_{N_{B1}}(x) >; \forall x \in X \right\}.$$

$$\left(NB_1{}^C\right)^C = \left\{ < x, 1 - \mu_{NB_1{}^C}(x), 2 - |\eta_{NB_1{}^C}(x)|\ , -1 - \gamma_{N_{B1}{}^C}(x) >; \forall x \in X \right\}.$$

Hence, $\left(NB_1{}^C\right)^C = NB_1$.

Proof is obvious for (iv) and (v).

Example

Let NB_1, $NB_2 \in NB(x)$ be two NBFS sets in the universe U. If $NB_1 = (0.8, 0.5, -0.7)$ and $NB_2 = (0.7,\ 0.3, -0.6)$. Then the arithmetic results follow:

$$NB_1 \cup NB_2 = (0.8,\ 0.5, -0.7)\ , NB_1 \cap NB_2 = (0.7,\ 0.3, -0.6)\ , \left(NB_1{}^C\right) = (0.2, 1.5, -0.3)\ ,\ \left(NB_1{}^C\right)^C = (0.8, 0.5, -0.7)\ \text{Hence, } \left(NB_1{}^C\right)^C = NB_1.$$

Definition 2 Let $NB_1 = (\mu_1(x),\ \eta_1(x), \gamma_1(x))$ and $NB_2 = (\mu_2(x),\ \eta_2(x), \gamma_2(x))$ be two NBFS. Then

(i) $NB_1 \bigvee NB_2 = \Big\{ \max\left(\mu_{NB_1}(x), \mu_{NB_2}(x)\right), \max\left(|\eta_{NB_1}(x)|, |\eta_{NB_2}(x)|\right), \min(\gamma_{NB_1}(x), \gamma_{NB_2}(x)\Big\}$

(ii) $NB_1 \wedge NB_2 = \Big\{ \min\left(\mu_{NB_1}(x), \mu_{NB_2}(x)\right), \min\left(|\eta_{NB_1}(x)|, |\eta_{NB_2}(x)|\right), \max(\gamma_{NB_1}(x), \gamma_{NB_2}(x)\Big\}$

(iii) For $\lambda > 0$, $NB_1{}^{\lambda} = \left(\left(\mu_{NB_1}(x)\right)\right)^{\lambda}, \left(|\eta_{NB_1}(x)|\right)^{\lambda}, \left(1 - \left(1 - \left(-\gamma_{NB_1}(x)\right)^{\lambda}\right)\right)$

(iv) For $\lambda > 0$, $NB_1 \bigotimes NB_2 = \{\mu_{NB_1}(x).\mu_{NB_2}(x)\ ,\ |\eta_{NB_1}(x)|.|\eta_{NB_2}(x)|,$

$$\{ 1 - \left(1 - \left(-\gamma_{NB_1}(x)\right)\right\} + \{ 1 - (1 - (- \gamma_{NB_2}(x))) \\ - \{1 - \left(1 - \gamma_{NB_1}(x)\right)\} \{1 - (1 - \gamma_{NB_2}(x))\} \}$$

5.2 Certain Results on Neutral-Bipolar Fuzzy Operations

Definition 3 Let $NB_1 = (\mu_1(x),\ \eta_1(x), \gamma_1(x))$ be the NBFS. Then score valued function $\delta(NB_1)$ and the accuracy valued function $\alpha(NB_1)$ of NBFS is defined as

(i) $\delta\,(NB_1) = \frac{1}{2}\,(\mu_1 + |\eta_1| + \gamma_1)$where $\delta(NB_1) \in [-1, 1]$
(ii) $\alpha\,(NB_1) = \frac{1}{2}\,(\mu_1 + |\eta_1| - \gamma_1)$where $\delta(NB_1) \in [0, 1]$.

Definition 4 Let $NB_1, NB_2 \in NBFS$, the results as follows:

- If $\delta(NB_1) < \delta(NB_2)$ then $NB_1 < NB_2$.
- If $\delta(NB_1) > \delta(NB_2)$ then $NB_1 > NB_2$.
- If $\delta(NB_1) = \delta(NB_2)$,then we can find the grades using accuracy function. Case 1: If $\alpha(NB_1) < \alpha(NB_2)$ then $NB_1 < NB_2$. Case 2: If $\alpha(NB_1) > \alpha(NB_2)$ then $NB_1 > NB_2$. Case 3: If $\alpha(NB_1) = \alpha(NB_2)$ then $NB_1 = NB_2$.

Neutral-Bipolar Weighted Geometric Mean (NBWGM) The Neutral-Bipolar Weighted Geometric Mean (NBWGM) is defined as NBWGM $(NB_1, NB_2, \ldots.. NB_n) = NB_1{}^{q_1} \bigotimes NB_2{}^{q_2} \bigotimes \cdots \bigotimes NB_n{}^{q_n}$, where q $= (q_1, q_2, \ldots., q_n)$, $q_k \in [0, 1]$ is the weight vector of NB_k and $\sum_{k=1}^{n} q_k = 1$.

Theorem 3 The Neutral-Bipolar Weighted Geometric Mean of NBFS $NB_i (i = 1, 2, \ldots, n)$ of NBWGM operator is

$$\text{NBWGM}\,(NB_1, NB_2, \ldots..NB_n) = \Big\{\prod_{k=1}^{n} \left(\mu_{NB_k}\right)^{q_k}, \prod_{k=1}^{n} |\left(\eta_{NB_k}\right)^{q_k}|\ , \left(- \prod_{k=1}^{n} (- \ \left(-\gamma_{NB_k}\right)^{q_k}\right)\Big\}$$

Proof The proof follows by the method of mathematical induction. For k = 2

$$NB_1{}^{q_1} \bigotimes NB_2{}^{q_2} = \Big(\ (\mu_{NB_1})^{q_1} . \ (\mu_{NB_2})^{q_2}, \ \ |(\eta_{NB_1})^{q_1}|.|(\eta_{NB_2})^{q_2}|,$$
$$\left(1-\left(1-\left(-\gamma_{NB_1}^{q_1}\right)\right)\right)+\left(1-\left(1-\left(-\gamma_{NB_2}^{q_2}\right)\right)\right)$$
$$-\left(\left(1-\left(1-\left(-\gamma_{NB_1}^{q_1}\right)\right).\left(1-\left(1-\left(-\gamma_{NB_2}^{q_2}\right)\right)\right)\right)\right)$$

$$= \prod_{k=1}^{2} \ (\mu_{NB_k})^{q_k}, \prod_{k=1}^{2} \ |\ (\eta_{NB_k})^{q_k}\ |, \left(1-\left(1-(-\gamma)_{NB_1}^{q_1}\right)+1\right.$$
$$-\left(1-(-\gamma)_{NB_2}^{q_2}\right)-1+\left(1-(-\gamma)_{NB_1}^{q_1}\right)+\left(1-(-\gamma)_{NB_2}^{q_2}\right)-\left(1-(-\gamma)_{NB_1}^{q_1}\right)$$
$$\left.\left(1-(-\gamma)_{NB_2}^{q_2}\right)\right)$$

$$= \left\{\prod_{k=1}^{2} \ (\mu_{NB_k})^{q_k}, \prod_{k=1}^{2} \ |(\eta_{NB_k})^{q_k}| \ , \left(1-\prod_{k=1}^{2}\left(1- \ (-\gamma_{NB_k})^{q_k}\right)\right\}\right.$$

Let us assume it holds for k = n.$NB_1{}^{q_1} \otimes NB_2{}^{q_2} \otimes \cdots \otimes NB_n{}^{q_n} = \left\{\prod_{k=1}^{n} \ (\mu_{NB_k})^{q_k}, \prod_{k=1}^{n} \ |(\eta_{NB_k})^{q_k}| \ , \left(-\prod_{k=1}^{n}\left(- \ (-\gamma_{NB_k})^{q_k}\right)\right\}\right.$

We prove for k = n + 1,

$$NB_1{}^{q_1} \bigotimes NB_2{}^{q_2} \bigotimes \cdots \bigotimes NB_n{}^{q_n} \bigotimes NB_{n+1}{}^{q_{n+1}} = \bigotimes_{k=1}^{n} NB_k{}^{q_k} \bigotimes NB_{k+1}{}^{q_{k+1}}$$

Consider, $\bigotimes_{k=1}^{n} NB_k{}^{q_k} \otimes NB_{k+1}{}^{q_{k+1}} = \Big(\ \bigotimes_{k=1}^{n} (\mu_{NB_k})^{q_k} . \ (\mu_{NB_{k+1}})^{q_{k+1}},$
$\bigotimes_{k=1}^{n} \ |\ (\eta_{NB_k})^{q_k}\ | \ . \ |\ (\eta_{NB_{k+1}})^{q_{k+1}}\ |, \bigotimes_{k=1}^{n}\left(1-\left(1-\left(-\gamma_{NB_k}^{q_k}\right)\right)\right)$

$$+\left(1-\left(1-\left(-\gamma_{NB_{k+1}}^{q_{k+1}}\right)\right)\right)-\left(\bigotimes_{k=1}^{n}\left(1-\left(1-\left(-\gamma_{NB_k}^{q_k}\right)\right).\right.\right.$$
$$\left.\left.\left(1-\left(1-\left(-\gamma_{NB_{k+1}}^{q_{k+1}}\right)\right)\right)\right)\right)$$
$$= \left\{\prod_{k=1}^{n+1} \ (\mu_{NB_k})^{q_k}, \prod_{k=1}^{n+1} \ |(\eta_{NB_k})^{q_k}| \ , \left(1-\prod_{k=1}^{n+1}\left(1- \ (-\gamma_{NB_k})^{q_k}\right)\right\}\right.$$

Hence, the theorem is proved.

Theorem on Idem-potency Let $NB_i (i = 1, 2, \ldots, n)$is the set of all NBFS sets. If $NB_k = NB$ for each k, then NBFSWA$(NB_1, NB_2, \ \ldots\ldots NB_n) = NB$.

Proof We know that NBWGM $(NB_1, NB_2, \ldots\ldots NB_n) = \left\{\prod_{k=1}^{n} \ (\mu_{NB_k})^{q_k},\right.$
$\left.\prod_{k=1}^{n} \ |(\eta_{NB_k})^{q_k}| \ , \left(1-\prod_{k=1}^{n}\left(1- \ (-\gamma_{NB_k})^{q_k}\right)\right\}\right.$

If $NB_k = NB$ (for each k) $= \Big\{ \ (\mu_{NB})^{\sum_{k=1}^{n} q_k}, \ \ |(\eta_{NB})^{\sum_{k=1}^{n} q_k}| \ ,$ $\left(1-\left(1- \ (-\gamma_{NB})^{\sum_{k=1}^{n} q_k}\right)\right\}$. NBFSWA$(NB_1, NB_2, \ldots\ldots NB_n) = \{\mu_{NB}, \ \ |(\eta_{NB})|,$ $1-(1- \ (-\gamma_{NB})) \ \ \} =$ NB.

Theorem on Monotonicity Let NB_k and $NB_{k'}$ be the pair of sets in NBFS. If $NB_k < NB_{k'}$ for each k, then NBFSWA$(NB_1, NB_2, \ldots\ldots NB_n) < (NB_{1'}, NB_{2'}, \ldots\ldots NB_{n'})$.

Proof If $NB_k, NB_{k'} \in$ NBFS and $NB_k < NB_{k'}$ for each value of k, then

$$\Big\{ \ (\mu_{NB})^{q_k}, \ \ |(\eta_{NB})|^{q_k}| \ , \left(1-\left(1- \ (-\gamma_{NB})^{q_k}\right)\right\} < \Big\{ \ (\mu_{NB'})^{q_k}, \ |(\eta_{NB}) \, , |^{q_k}, \left(1-\left(1- \ (-\gamma_{NB'})^{q_k}\right)\right\}$$

Hence, NBFSWA $(NB_1, NB_2, \ldots\ldots NB_n) < (NB_{1'}, NB_{2'}, \ldots\ldots NB_{n'})$.

Theorem on Boundedness Let $NB_i (i = 1, 2, \ldots, n)$is the set of all NBFS sets. Let$(NB)^- =$ min $(NB_1, NB_2, \ldots\ldots NB_n)$ and $(NB)^+ =$ max $(NB_1, NB_2, \ldots\ldots NB_n)$. Then $(NB)^- \leq (NB_1, NB_2, \ldots\ldots NB_n) \leq (NB)^+$.

Proof The proof is obvious.

6 Multicriteria Decision-Making in NBFS

In this section, Multicriteria decision-making method using the geometric operators defined for NBFS is presented. Presume that $R = \{R_1, R_2, R_3, \ldots, R_q\}$is the collection of alternatives and $C = \{C_1, C_2, C_3, \ldots, C_r\}$ is the collection of criteria. $\sum_{k=1}^{n} q_k = 1 \ (k = 1, 2, 3, \ldots n)$ and it represents the weight of C_k. The decision-makers examined the alternatives with the mentioned conditions and presented the details in the form of Neutral-Bipolar fuzzy sets.

We demonstrate the application of NBFS in bipolar disorder. Manic-depressive illness is a psychological health condition that grounds extreme moods swings. It is a lasting condition one can handle our emotional instability and other indicators by a proper health care. Bipolar type 1, bipolar type 2 and cyclothemic are the major types of bipolar disorder. It refers the three different behaviors manic behavior, depression, hypomanic with depression combinations. Manic behavior refers elevated emotions, highly energetic and active. Depression is loss of attention and persistent hopelessness. Cyclothemic is the combination of hypomanic and depression. We can easily compare this with NBFS like manic behavior as positive value of membership, depression as negative value of membership, and the combination level as neutral value. Experts believed that this biological difference arises due to the substance variations in the brain. The main reasons of bipolar disorder are hereditary factor, environmental influence, physical complaint, and medication.

Table 1 The alternatives with age factor

Alternatives	Age factors
R_1	below 25
R_2	around 25–45
R_3	above 45

Table 2 Decision matrix of NBFS

C_1	C_2	C_3	C_4
R_1(0.7, 0.2, −0.8)	(0.8, 0.3, −0.6)	(0.7, 0.3, −0.5)	(0.7, 0.7, −0.2)
R_2(0.8, 0.1, −0.7)	(0.3, 0.7, −0.4)	(0.9, 0.7, −0.1)	(0.5, 1.0, −0.5)
R_3 (0.4, 0.3, −0.6)	(0.8, 0.3, −0.7)	(0.5, 0.4,-0.6)	(0.6, 0.8, −0.6)

Table 3 NBFS normalized matrix

C_1	C_2	C_3	C_4
R_1(0.7, 0.2, −0.8)	(0.8, 0.3, −0.6)	(0.7, 0.3, −0.5)	(0.3, 1.3, −0.8)
R_2(0.8, 0.1, −0.7)	(0.3, 0.7, −0.4)	(0.9, 0.7, −0.1)	(0.5, 1.0, −0.5)
R_3 (0.4, 0.3, −0.6)	(0.8, 0.3, −0.7)	(0.5, 0.4,-0.6)	(0.4, 1.2, −0.4)

Assume that the main reasons of bipolar disorder as criteria $C = \{C_1, C_2, C_3, C_4\}$ (C_1= hereditary factor, C_2= environmental influence,C_3= physical complaint, C_4= medication). The age factors as the set of alternatives$R = \{R_1, R_2, R_3\}$ (R_1= below 25, R_2= around 25–45, R_3= above 45) are presented in Table 1.

Reasons for Alternatives

- Below 25: The person below 25 ages mostly belongs to the student's community. They might be taking care by their parents, surrounded by the friends. And those peoples are on the safe side of life.
- Around 25–45: The person around 25–45 ages mostly belong to those who just start their career. They are in the urge to settle down in life. They should take care of their self and their surroundings. They are in the struggling to get a progression and enduring in the career. This stage is the learning process so they are at risk side of life.
- Above 45: The persons around 45 age belongs to those who settled down in a career and those who are in retired life. This is the stage with full of experience, so they can easily manage the complications in life. These persons are on the safe side of life. Now by using the NBFS, we can find the most affected age group people for the mentioned criteria.

Step 1. Construct the NBFS decision matrix $(NB_{ij})_{q \times r}$ for the given condition relates to the alternatives. The NBFS decision matrix is given in Table 2.

Step 2. Normalized$(NB_{ij})_{q \times r}$ matrix is given in Table 3. By taking the complement of cost value and benefit value as presented. Here, C_4= medication is the cost value.

Case 1 The weight vector for this particular case is $q_1 = (0.4, 0.2, 0.2, 0.2)$

Step 3. Compute the value of N_i for the given data using NBWGM for each alternative. NBWGM(N_1) = {0.6068775, 0.342018,-0.6562456}, NBWGM(N_2) = {0.6127774, 0.3451749,- 0.524532}, NBWGM(N_3) = {0.4804497, 0.4192962, −0.590466}.

Step 4. Estimate$\delta(N_i)$,$\delta(N_1) = 0.146325$, $\delta(N_2) = 0.216710$, $\delta(N_3) = 0.154640$.

Step 5. Arrange the N_i according to δ value to choose the appropriate assessment.$\delta(N_2) > \delta(N_3) > \delta(N_1)$.

Result. The persons with age group between 25 and 40 were affected due to high hereditary factor.

Case 2 The weight vector for this particular case is $q_2 = (0.2, 0.4, 0.2, 0.2)$. We can have the same data as given in step 1 and step 2. By arranging $\delta(N_2) > \delta(N_1) > \delta(N_3)$. *Result*. The persons with age group between 25 and 40 were highly influenced by the environment.

Case 3 The weight vector for this particular case is $q_2 = (0.2, 0.2, 0.4, 0.2)$. We can have the same data as given in step 1 and step 2. We get $\delta(N_2) > \delta(N_1) > \delta(N_3)$. The persons with age group between 25 and 40 were highly affected with physical complaints.

Case 4 The weight vector for this particular case is $q_4 = (0.2, 0.2, 0.2, 0.4)$. We can have the same data as given in step 1 and step 2. By arranging the $\delta(N_i)$, we get $\delta(N_2) > \delta(N_3) > \delta(N_1)$. In this case, the persons with age group between 25 and 40 were highly pretentious with medicines.

Conclusion Evidently the persons between 25 and 45 were affected with all the mentioned criteria. Since, they are at the risk side of life. Fuzziness on Neutral-Bipolar fuzzy set (NBFS) supports us to find the people who are to be cared first.

7 Conclusion

The new notion of Neutral-Bipolar fuzzy sets (NBFSs) has been introduced and it is defined with basic properties. Also, we have discussed the NBFS operations, score function value, accuracy value, and aggregated values. In addition, the demonstration of NBFS in bipolar disorder using the operators of NBFS was provided. In MCDM problem, Neutral-Bipolar fuzzy set supports us to find the desirable alternative. NBFS can be effective in the areas such as the neutral thought of human brains in certain situation, neutral results in medicines due to some health factor, neutral financial status in business, and neutral thoughts in decision-making. In forthcoming work, we can elongate the applications in various fields like neural networking and artificial intelligence.

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