Chapter 16
How Useful Is the Term 'Modernism' for Understanding the History of Early Twentieth-Century Mathematics?

Leo Corry

Abstract The present article is intended as a critical assessment of some basic assumptions underlying the analysis of modernism in mathematics in its relationship with the broader aspects of cultural modernism, especially in the period 1890–1930. It discusses the potential historiographical gains of approaching the history of mathematics in the periods under such a perspective and suggests that a fruitful analysis of the phenomenon of modernism in mathematics must focus not on the *common features* of mathematics and other contemporary cultural trends, but rather on the *common historical processes* that led to the dominant approaches in all fields.

16.1 Introduction

It is widely acknowledged that the period roughly delimited by 1890 and 1930 was marked by deep change in mathematics. It was also a time of thoroughgoing transformations that impacted the visual arts, music, architecture, and literature. The latter has often been explained in terms of artistic responses to the sweeping processes of modernization affecting Western societies. The term "modernism" has typically been used to refer to such trends and the ways in which they implied highly innovative—sometimes *avant-garde*—aesthetic conceptions characterized by unprecedented radical breaks with long-standing traditions in each area of cultural expression. A question naturally arising in these circumstances is whether the development of mathematics during said period can be seen as part of the phenomenon of "modernism" considered in its broader context, and whether adopting such a perspective leads to important historical insights.

Herbert Mehrtens' pioneering study, *Moderne-Sprache-Mathematik* (Mehrtens 1990), opened the way to serious discussions on this issue. Following on his footsteps, Jeremy Gray published his well-known book, *Plato's Ghost. The Mod-*

L. Corry (✉)
Cohn Institute for History and Philosophy of Science, Tel Aviv University, Tel Aviv, Israel

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ernist Transformation of Mathematics in 2018.[1] The present article is intended as a critical assessment of some basic assumptions underlying the possible discussions of modernism in mathematics and the potential historiographical gains of pursuing such discussions.

The processes of modernization that affected the content of mathematics during the said period concern the development of new methodologies, the rise of newly investigated mathematical entities and of new sub-disciplines, as well as the reshaping of the internal organization of mathematical knowledge, the transformation relationships between mathematics and its neighboring disciplines, the demise or total abandonment of activity in areas of research that were very important in the previous century, and the adoption of either implicit or formulated new philosophical attitudes. At the institutional level, the meteoric rise of the Göttingen school came to epitomize the substantial changes undergone by the discipline in this period, as other centers, both in the German-speaking world (such as Berlin, Munich, Vienna, Hamburg) and outside (Paris, Cambridge), also underwent a significant transformation. In terms of scientific leadership, the achievements of David Hilbert and his circle embodied, both symbolically and contents-wise, the personal dimension of the spirit of the period, alongside other prominent names, such as Emmy Noether, Giuseppe Peano, and Felix Hausdorff.

The suggestive idea of possible parallel developments and similar sources underlying both the broader cultural manifestations and mathematics arises in a comparable way when examining the dramatic changes that affected the neighboring discipline of physics. The classical theories of mechanics and electromagnetism had reached a climax towards the end of the nineteenth century yet now, its foundational assumptions had been put into question, and thoroughly new directions were leading the physicists' understanding of phenomena at both the microscopic and the astronomic level. In his classical 1971 study on the rise of the new quantum mechanics, Paul Forman postulated an organic association between the contemporary adoption of non-deterministic types of causality in physics and some leading cultural motifs which he associated with the modernist spirit of Weimar Germany such as irrationality, anti-scientism, and acausality (Forman 1971). In the epigraph of the article, Forman cited the German physicist Gustav Mie who, in his 1925 inaugural lecture in Freiburg, very explicitly expressed the kind of attitude that attracted Forman's curiosity, as he indicated that even physics, "a discipline rigorously bound to the results of experiment," evolves in ways that parallel those of the intellectual movements in other areas of modern life.

Forman's article has been widely read and cited, sometimes severely criticized, and also intriguingly reappraised by (Forman 2007; Carson et al. 2011). To the extent that one wants to either accept or reject the thrust of Forman's argumentation, what kind of lesson does it teach us about the issue of "mathematics and modernism", if at all? A similar question can be asked of works dealing with the development of other fields of knowledge and culture at the time, including areas

[1] Some of the main ideas were sketched earlier in (Gray 2004, 2006).

distant from mathematics. The present article suggests ways of addressing those questions and indicates some possible, specific directions in which this analysis might be profitably undertaken. The main pitfall against which I want to call attention is that of shooting the arrow and then tracing a bull’s eye around it. Indeed, one of the main difficulties affecting discussions of “modernism” in general (not just concerning the history of mathematics) is that of finding the proper definition of the concept, to begin with. One might easily start by finding a general definition that can then be made to fit the developments of mathematics in the relevant period just to be able to put together all that we have learnt from historical research and thus affirm that, yes, “modernism” characterizes mathematics as it characterizes other contemporary cultural manifestations. Although this approach has some interest, it does not seem to be very illuminating, and indeed it runs the risk of being misleading since, by its very nature, it may force us to be unnecessarily “flexible” in our approach to making historical facts fit the desired definition.

The article opens with an overview of some prominent ways in which the term “modernism” has been used in the historiography of the arts, and calls attention to certain debates surrounding its relevance in that context. This is followed by a discussion of three concrete examples of works that investigate the relationship between modernism in general and the modern exact sciences: on the one hand, an investigation of the influence of scientific ideas on modern visual arts (in the writings of Linda Henderson), and, on the other hand, two books (by Herbert Mehrtens and Jeremy Gray) that explore the connections of modern mathematics with more general, modernist cultural trends. In the following two sections, I consider two examples of authors who discuss the roots and developments of modernist ideas in specific contexts (modernist painting in the writings of Clement Greenberg and Viennese modernism by Allan Janik and Stephen Toulmin) and examine the possible convenience of using their perspective in discussing modernism and mathematics.

Besides the critical examination of existing debates, on the positive side, a main claim raised discussed in this article is that a fruitful analysis of the phenomenon of modernism in mathematics must focus not on the *common features* of mathematics and other contemporary cultural trends (including other scientific disciplines – mainly physics), but rather on the *common historical processes* that led to the dominant approaches in all fields in the period of time we are investigating. To the extent that the existence of what is described as common, modernist *features* in the sciences and in the arts has been explained in the existing literature, this has been typically done in terms of a mysterious “Zeitgeist” or even “common cultural values” (as suggested, e.g., in (Miller 2000, 480; Yourgrau 2005, 3)). Though useful at first sight, such an approach is, in my view, far from satisfactory because the “Zeitgeist”, if it indeed exists, is what needs to be explained. In contrast, a clearer understanding of the *processes* that led to the rise of modernism in other intellectual fields, may help us look for similar historical processes in mathematics.

It is pertinent to mention that Mehrtens pointed to this direction, as he stressed the difference between “*Moderne*”, referring to the intellectual trend itself, and “*Modernisierung*”, which refers to the historical *processes* leading to changes in the discipline of mathematics, within its broader social and cultural context. However, there seems to be much room for further exploration in this direction, which could lead to additional insights thus far overlooked by historians. If properly pursued, this might amount, I suggest, to a significant contribution to the historiography of the discipline. Likewise, and no less interestingly, a clearer understanding of the historical processes that led to putative modernist mathematics could shed new light on the essence and origins of modernism in general.

16.2 Modernism: A Useful Historiographical Category?

Despite its ubiquity, the fruitfulness of the concept of "modernism" as an analytic category in the context of general cultural history is far from being self-evident or settled. Indeed, its very meaning and the time span that it covers remains the subject of debate. Ranging from the seminal anthology edited by Malcolm Bradbury and James McFarlane (1976), and the more recent, two-volume collection edited by Astradur Eysteinsson and Vivian Liska (2007), to the volumes of the journal *Modernism/modernity*, by the Modernist Studies Association, the body of research literature is enormous. From this abundance of sources, I want to focus here on Ulrich Weisstein's article, "How Useful is the Term 'Modernism' for the Interdisciplinary Study of Twentieth-Century Art?" (1995), (whose title I have obviously appropriated). Based on the assumption that the idea of "Modernism" has indeed been used in fruitful ways in his own field of research, comparative literature, Weisstein wondered about its possible usefulness in researching other domains, including the visual arts and music. In doing so, he characterized the various kinds of modernist aesthetics in terms of their emphasis on the formal, as opposed to concrete subject matters and intentions, together with a consistent inclination to undertake radical breaks with the accepted norms of the field, by way of rites of passage and inspirational manifestos meant to embody avant-garde attitudes.

To be sure, Weisstein's characterization has both merits and drawbacks as an adequate prism from which to approach modernism in general. Yet the same can be said of many such checklists proposed by other authors pursuing the same task, most partially overlapping with and differing from Weisstein's, as well as from each other.[2] Thus, assessing the extent to which modern mathematics is properly defined as a modernist phenomenon by reference to any specific proposal of this kind—by checking whether or not, and to what extent, the suggested features are manifested—may end up being an unilluminating historiographical exercise. It runs the risk of providing a Procrustean bed into which the historical facts are forced, while shedding little new light on our understanding of the historical processes. "Modernism" may only become a truly useful historiographical category for our topic if it helps interpreting the known historical evidence in innovative ways, or if it

[2] See, e.g., (Calinescu 1987; Childs 2000; Eysteinsson 1990, 2021; Gay 2007).

would lead us to consider new kinds of materials thus far ignored, or underestimated, as part of historical research on the development of mathematics.

The question whether "modernism" can be used as a useful category to study the history of mathematics, moreover, is best understood when seen as part of a broader trend noticeable over the last 30 years, that involved attempts to take advantage and inspiration of historiographic conceptions, originating in neighboring fields, mainly in the historiography of other scientific disciplines (Barany 2020; Remmert et al. 2016). The trend arose, in the first place, in relation with the Kuhnian concepts of "revolution" and "paradigm" (Gillies 1992), Lakatos' "scientific research programs" (Hallett 1979a, b), and with ideas taken from the sociology of knowledge (MacKenzie 1993), which in an extreme version led to David Bloor's "strong program" (Bloor 1991). More recently, it has comprised reliance on ideas such as "research schools" (Parshall 2004), "traditions" (Rowe 2004), "images of science" (Corry 1989; Bottazzini and Dahan-Dalmédico 2001), "epistemic configurations" (Epple 2004), "material culture of science" (Galison 2003),[3] quantitative analyses (Goldstein 1999; Wagner-Döbler and Berg 1993), and various others.

When discussing mathematics in association with literature, art, or music, on the other hand, it is important to stress the obvious, namely, that in fields like art, literature and music, considerations of objectivity, universality, testability, and the like, if appearing at all as part of the aims of the artists or of the audiences, emerge in ways that differ sensibly from those of science (see, e.g., Corry 2007a, b, c; Engelhardt and Tubbs 2021). No less important is to keep in mind the different relationship each of these domains entertain with its own past and history. Many definitions of modernism put at their focus the idea of a "radical break with the past", and such definitions will necessarily apply in sensibly different ways to either the arts or to mathematics. Being guided, above all, by the need to solve problems and to develop mathematical theories, the kinds, and breadth of choices available to a mathematician (and, in particular, choices that may lead to "breaks with the past") are much more reduced and clearly constrained than those available to the artist. In shaping their artistic self-identity and in defining their creative agenda, modernist artists can choose to ignore, and even oppose any aspect of traditional aesthetics and craftsmanship. This implies taking professional risks, of course, especially when it comes to artists at the beginning of their careers, but it can certainly be done and, in fact, has been successfully achieved by prominent modernists.

The choices open before aspiring mathematicians intent on making "a radical break with the past" while remaining part of the mathematical community are much more reduced. Artists might decide to develop their work and career by innovating within the field to the extent that cuts all connection with the contemporary mainstream in the relevant community. The aspiring mathematicians, in contrast, must fully assume the central values of the professional ethos to become part of the guild. They will abide by the rules of classical logic and gain complete control

[3] Although more naturally seen as dealing with the history of physics, Galison's book devotes considerable attention to Poincaré's mathematics as well.

of the accepted "mathematical craftsmanship" in their field of choice. They must publish in the mainstream mathematical journals and will typically strive to do so in those broadly considered to be the leading ones. Moreover, in very few cases will an already established mathematician come up with radical proposals for changes in the standards of the field.[4]

The kind of radical changes that have affected mathematics, especially modern mathematics, touch upon the images of knowledge, and particular to innovative ways of organizing knowledge into sub-disciplines (as in the case of "Modern Algebra" (Corry 2007b)) or developing new methodologies over older ones (as in the case of computer-assisted number theory (Corry 2007c)).

When it comes to the relationship between mathematics and other scientific disciplines, particularly physics, however, there are important points to stress. Thus, given their radical new approach to the basic concepts of physics—time, space, matter, causality—it seems natural that historians undertaking the question of modernism in science and the arts, turned to the theory of relativity and quantum mechanics as a fundamental bridge across domains during the period in question. The aforementioned work of Forman is a foremost example of this trend. Indeed, Forman stressed, while focusing his account specifically on the impact of Oswald Spengler's ideas, that attempts at drawing such bridges were at the very heart of Weimar culture. Spengler's account of Western culture draws fundamental parallels between art, mathematics, science, culture, and society, and the main contribution of Forman's analysis is found in the detailed description of the strong impression this perspective on history caused both on scientists and mathematicians.

Additionally, there are more or less successful attempts at understanding these bridges that could be mentioned here (Miller 2000; Vargish and Mook 1999).[5] And yet, in spite of the disciplinary closeness between physics and mathematics, there are some important differences that affect our discussion here, particularly concerning

[4] The most prominent example that would come to mind is that of Luitzen J.E. Brouwer, whose doctoral advisor urged him to delete the more philosophical and controversial parts of the dissertation and to focus on the more mainstream aspects of mathematics that it contained. It was only somewhat later, as he became a respected practitioner of a mainstream mathematical domain, that he started publishing and promoting his philosophical ideas, and to devote his time and energies to developing new kinds of radical, intuitionistic mathematics. Brouwer promoted a kind of logic, later called "intuitionistic logic", deviating from the mainstream but not implying a call to abandon classical logic, but rather to revert logic to a previous stage in its evolution, where no considerations of the actual infinite had (wrongfully and dangerously, from his perspective) made deep headway into mainstream mathematics. See (van Dalen 1999, 89–99). Another interesting case is that of Doron Zeilberger's call, after a distinguished carrer in classical disciplines, for an abandonment of "Human-Supremacist", "human-generated, and human-centrist 'conceptual' pure math mathematics" in favor of computer-generated, "experimental mathematics".

[5] In an illuminating article about the use of the terms "classical" and "modern" by physicists in the early twentieth century, Staley (2005) addresses this difference from an interesting perspective. In his opinion, whereas in physics discussions about "classical" theories and their status were more significant for the consolidation and propagation of new theories and approaches than any invocation of "modernity", in mathematics, different views about "modernity" were central to many debates within the mathematical community.

what I have elsewhere called the "reflexive character of mathematical knowledge," on which I want to comment briefly as part of this preliminary discussion.

Mathematics is in a unique position among the sciences to allow an investigation of aspects of the discipline with tools offered by mathematics itself (Corry 1989). Entire mathematical disciplines that arose in the early twentieth century are devoted to this quest: proof theory, complexity theory, category theory, etc. These analyze specific aspects of mathematical practice and mathematical theories, and do so with the help of tools provided by the discipline and with the same degree of precision and clarity that is typical, and indeed unique, to mathematics. Gödel's theorems, for instance, involve results about the limitations of deduction mathematical theories defined by systems of axioms. The way that new methods were explicitly introduced to prove them does not differ from the way this is done in other mathematical situations. Biology, for example, cannot self-analyze the discipline with tools taken from the discipline itself, as biological theories are not biological entities.

On the other hand, literature can become the subject matter of a literary piece; painting can become the subject matter of painting, and so on, for other artistic endeavors. But whatever these domains can express about themselves, they will do so differently from what mathematics can say about itself. This unique feature of mathematics is not remarkable in itself and is also specifically relevant to the discussion of modernism, given the dual fact that (1) the reflexive study of the language and methods of specific cultural fields has very often been taken to be a hallmark of modernism in the arts, as I will stress below, and that (2) this reflexive character of mathematics became so prominently developed in the period that interests us here.

The differences and tensions arising in this complex, triangular relationship between mathematics, the natural sciences and the arts must be considered and stressed explicitly in any serious analysis of mathematics and modernism. This relationship, moreover, is subject to ongoing changes and conditioned by historical circumstances. Hence, a proper examination of the historical processes under which the three realms evolved in the period that interests us here, and their possible interactions, is necessary for such an analysis and to shed new light on the history of modern mathematics. Whatever one may want to say about modernism in mathematics and its relationship with modernism in other fields, one must remember that the changing relationship among the fields must be taken to be part of this historical phenomenon.[6]

[6] An even broader and more comprehensive such analysis should also pay attention to philosophy and the social sciences with their own specificities, but for reasons of space I will leave them outside the scope of the present discussion. See, e.g., (Ross 1994; Vrahimis 2012).

16.3 Modern Mathematics and Modernist Art

I move now on to examine some existing works explicitly addressing the connections between mathematics and the arts in the period between1890–1930 and to comment on them against the background of the ideas discussed in the previous section. First, I focus on an analysis of the possible influences of mathematics and the sciences on the arts. Then, I move to consider the opposite direction, which includes the important contribution of Jeremy Gray.

An outstanding example of analysis of the influence of physics and mathematics on modern art in the early twentieth century appears in the work of Linda Henderson (Henderson 1983, 2004, 2005, 2007). Henderson explored the ways in which certain scientific ideas dominating the public imagination at the turn of the century, provided "the armature of the cultural matrix that stimulated the imaginations of modern artists and writers" (Henderson 2004, 458). Artists who felt the inadequacy of current artistic language to express the complexity of new realities newly uncovered by science (or increasingly perceived by public imagination) were pushed into pursuing new directions of expression, and hence contributing to the creation of a new artistic language; the modernist language of art. But in showing this, Henderson also studiously undermined the all-too-easy, and often repeated image of a putative convergence of modern art and modern science at the turn of twentieth century in the emblematic personae and personalities of Picasso and Einstein.[7] Contrary to a conception first broadly and famously promoted in Sigfried Giedion's *Space, Time and Architecture* (Giedion 1941), Einstein's early ideas on relativity were not at all known to Picasso at the time of consolidating his new cubist conceptions. More generally, it was not before 1919, when in the wake of the famous Eddington eclipse expedition, Einstein was catapulted to world fame, as the popularizations of relativity theory captured public and artistic imagination (Levenson 2003, 218–37). It was only then that ideas of space and time related to relativity did offer new metaphors and opened new avenues of expression that some prominent artists undertook to follow. As Henderson's work illustrates, it was not relativity but central ideas stemming from classical physics in the late nineteenth century that underlie the ways in which science contributed to creating new artistic directions in the early modernist period. These ideas were related above all with the *ether*, but also with other concepts and theories that stressed the existence of supra-sensible, invisible physical phenomena. These "invisible phenomena" comprised the discovery of X-rays, radioactivity, the discourse around the fourth dimension (especially as popularized through the works of the British hyperspace philosopher Howard Hinton (1853–1907)), and the idea of the cosmic consciousness introduced by the Russian esoteric philosopher Pyotr Demianovich Ouspenskii (1878–1947).

Henderson offers a superb example of how, by looking into the development of science, we can gain new insights into the issue of modernism in art. The main

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[7] A typical version of which appears in (Miller 2002).

thread of her account emerges from within the internal development in the arts and focuses on some crucial historical crossroads where substantial questions about the most fundamental assumptions of art and of its language arose at the turn of the twentieth century. Faced with these pressing questions, certain artists sought to come to terms with these by looking for new ideas and directions of thought. Henderson then separately focuses on contemporary developments in science, developments that, in themselves, had nothing to do with modernism or with modernist *Zeitgeist*, and shows how these developments afforded new concepts, a new imagery, and new perspectives that the artists could take as starting point for the new ways they were attempting to develop in their own artistic quest. Thus, in Henderson’s narrative there is no assumption of common ideas or common trends simultaneously arising in both realms for some unknown reason. In fact, whether the main scientific ideas were properly understood by the artists in question is not a truly relevant point in her account. She shows, in this way, how public perception of scientific ideas—not necessarily the truly important or more revolutionary ones at the time—played a central role in the consolidation of major trends and personal styles in modernist art (Cubism, Futurism, Duchamp, Boccioni, Kupka, etc.). Science appears here as offering a broadened world of ideas, metaphors, and images from which the artists could pick according to their needs, tastes, and inclinations.[8]

Henderson’s work thus offers a remarkable example of an approach that, the direction of the impact were to be turned around, has the potential to lead to a truly illuminating attempt at making sense of modern mathematics as part of the broader cultural phenomena of modernism. Such an approach would ideally involve two steps:

1. The historian should first take a fresh look at the overall developments in the discipline of mathematics—its results, its language, its foundational conceptions, its relationships with neighboring disciplines, its institutions and values—to trace those places where the discipline and its practitioners face in this period of time *an inadequacy to address, in terms of the existing disciplinary tools, the complexity of a new reality*. This inadequacy may well manifest itself in terms of a deep crisis or anxiety systematically surfacing in the disciplinary discourse, that historians should identify and articulate.
2. In a second crucial step, the historian should provide an account of the ways in which this inadequacy was addressed by mathematicians following new paths. In this account, external inputs from the arts, music, architecture or philosophy would become instrumental in helping shape the course of events that transformed the discipline at the turn of the twentieth century.

Whether or not such an approach may successfully be applied to understanding in new ways specific situations in the development of modern mathematics is yet to be seen. At this point, I would like to take a brief look at two seminal books that

[8] Similar in this respect, with an emphasis on mathematics, are the account presented in (Gamwell 2015).

undertook the most thoroughgoing analysis to date of modern mathematics as part of the more general cultural phenomenon of modernism and to analyze, relying on the scheme suggested above, the scope and impact of their undertaking. One of them is, as already stated, Jeremy Gray's *Plato's Ghost,* but I start with Herbert Mehrtens' *Moderne-Sprache-Mathematik* (Mehrtens 1990), which pioneered the trend.

Mehrtens explicitly connected some of the basic features commonly associated with modern mathematics to modernization processes and their manifestations in various fields of culture and society. He examined the impact of the rise of new types of industries and professions (e.g., in the insurance area), and of trends in higher education. In his analysis, he incorporated—among other things—semiotic concepts and philosophical insights drawn from authors like Foucault or Lacan. He accorded prime importance to an examination of mathematical language while stressing a three-fold distinction between different aspects of the latter: (1) mathematics *as a* language (*Sprache Mathematik*), (2) the language used in mathematical texts (comprising systems of terms and symbols that are combined according to formal rules stipulated in advance), and (3) the language of mathematicians (*Sprechen der Mathematiker*), which comprises a combination of language used in fully formalized mathematical texts and texts written in natural language.

In these terms, Mehrtens discussed modernism in mathematics by referring to the main kinds of reactions elicited by the development of mathematics by the end of the nineteenth century, notably as they manifested themselves through debates about the source of its meaning in mathematics and about the autonomy of the discipline and its discourse. These reactions comprised a break with more traditional disciplines and a search for disciplinary autonomy of a kind and degree theretofore unknown in the field. In these terms, he identified two groups of mathematicians espousing diverging views. On the one hand, there was a "modern" camp represented by the likes of Georg Cantor (1845–1914), David Hilbert (1862–1943), Felix Hausdorff (1868–1942) and Ernst Zermelo (1871–1953). An increasing estrangement from the classical conception of mathematics was characteristic of their attitude as an attempt to explore some naturally or transcendentally given mathematical entities (such as numbers, geometrical spaces, or functions). They conceived the essence of mathematics to be the analysis of a man-made symbolic language, and the exploration of the logical possibilities spanned by the application of the rules that control this language. Mathematics, in this view, was a free, creative enterprise constrained only by fruitfulness and internal coherence. Hilbert was, in Mehrtens account, leading figure of this camp.

Concurrently, a second camp developed, denominated by Mehrtens as "counter-modern", led by mathematicians such as Felix Klein (1849–1925), Henri Poincaré (1854–1912), and Luitzen J. E. Brouwer (1881–1966). For them, the investigation of spatial and arithmetic intuition (in the classical sense of *Anschauung*) continued to be the primary thrust of mathematics. He also included mathematicians in this camp who lay their stress on real-world applications in physics, technology, economics, etc. The rhetoric of "freedom" of ideas as the basis of mathematics, initiated by Richard Dedekind (1831–1916) and enthusiastically followed by the modernist mathematicians (Corry 2017), was rejected, in Mehrtens' account, by the mathematicians of the counter-modernist camp, who priced above all finiteness, *Anschauung* and "construction." Brouwer appears here as the arch-counter-modernist. His idiosyncratic positions in both mathematical and political matters (as well as the affinities between Brouwer and the national-socialist Berlin mathematician (1886–1982)) allowed Mehrtens to identify what he saw as the common, counter-modernist traits underlying both levels (Mehrtens 1996).

An important and original point underlying Mehrtens' analysis is the emphasis on the simultaneous existence of these two camps and the focus on the ongoing critical dialogue between them, as the main feature of the history of early twentieth-century mathematics. This critical dialogue was, inter alia, at the root of a crisis of meaning that affected the discipline in the 1920s (the so-called "foundational crisis" (pp. 289–330)) and led to a redefinition of its self-identity. Moreover, by contrasting the attitudes of the two camps, Mehrtens implicitly presented the modernist attitude in mathematics *as a matter of choice* rather than one of necessity.

Mehrtens' book has been consistently praised for its pioneering status in the debate on modernism in mathematics and for the original approach, it has put forward. However, its limitations have also been consistently pointed out. Mehrtens' analysis focuses mainly on the programmatic declarations of those mathematicians he discusses and on their institutional activities. These are matters of real interest as sources of historical analysis, and it is worth stressing that the contents of mathematics are influenced by ideological considerations and institutional constraints. But as Moritz Epple remarks, in the final account, "Mehrtens does not attempt to analyze some of the more advanced productions of modernist or counter-modernist mathematicians, and makes, in fact, no claims about the internal construction of modern mathematics" (Epple 1997, 191).[9]

Thus, Mehrtens left many fundamental questions unanswered, and his argumentation was somewhat misleading. For one thing, the critical debate among "moderns" and "countermoderns" would appear to be, in Mehrtens' account, one that referred only to the external or meta-mathematical aspects, while being alien to questions of actual research programs, newly emerging mathematical results, techniques, or disciplines. In addition, the classification of mathematicians into these two camps, and the criteria of belonging to either of them, seems too coarse to stand the test of close historical scrutiny, and in the final account was too strongly circumscribed to the Göttingen mathematical culture. In this sense, Mehrtens' book, for all its virtues, falls short of giving a satisfactory account of "modern mathematics" as a "modernist" undertaking.

Having said that, I think that two fundamental elements of Mehrtens' analysis are highly relevant to any prospective, insightful analysis of modernism in mathematics. First is the possible, simultaneous existence of alternative approaches to mathematics that are open to choose, according to considerations that do not strictly derive from the body of mathematics itself. Some of the elements that Mehrtens identifies in distinguishing between moderns and counter-moderns seem to me

[9] See also (Epple 1996).

highly relevant, but I think they could be more fruitfully used by historians if approached in a less schematic way, namely, by realizing that in the work of one of the same mathematician (or, alternatively, in the works of several mathematicians associated with one and the same school or tradition) we can find elements of both the modern and the counter-modern trend. These various elements may interact and continuously change their relative weight along the historical process.

The second point refers to the historical processes that Mehrtens indicated as leading to the rise of modernist approaches in mathematics, namely the rapid growth of the discipline (together with other branches of sciences) by the late nineteenth century, and the enormous diversity and heterogeneity that suddenly appeared at various levels of mathematical activity (technical, language-related, philosophical, institutional). In this sense, Mehrtens follows the lead of those accounts of the rise of modernism in the arts that have presented it as a reaction to certain sociological and historical processes (such as urbanization, industrialization, or mechanization), and that in my view, if identified within the history of mathematics, may lead to new insights about the development of the discipline.

The second book to be mentioned here is Jeremy Gray's more recent *Plato's Ghost. The Modernist Transformation of Mathematics.* Gray's book provides a thoroughgoing account of the main transformations mathematics underwent in the period of our discussion, while comparing the main traits of these developments with the conceptions that previously dominated the discipline and that he schematically summarizes as "the consensus in 1880". Gray claims that the developments so described are best understood as a "modernist transformation". This concerns not just the changes that affected the contents of the leading mathematical branches but also additional aspects related to the discipline, such as its foundational conceptions, its language, its disciplinary relationship with physics, or even the ways in which the history of mathematics was now written or in which mathematics was popularized. Thus, for instance, Gray provides an illuminating survey of works written at the turn of the century, many of them by leading mathematicians, aimed at educated audiences of teachers, philosophers, psychologists, lawyers, and members of the Church. Such audiences, Gray suggests, reflected a new kind of growing interest of audiences that were ambiguous about science but wanted to hear about current developments (Gray 2008, 346–65). On the other hand, Gray remains less sure about the connection between modernist trends and the renewed interest in historical writing about mathematics (Gray 2008, 365–372).

Naturally, Gray is well-aware that "if the idea of mathematical modernism is to be worth entertaining, it must be clear, it must be useful, and it must merit the analogy it implies with contemporary cultural modernism." In addition, "there should be mathematical developments that do not fit at the very least those from earlier periods, and one might presume some contemporary ones as well." Accordingly, Gray's book opens with a characterization of modernism meant to provide the underlying thread of his analysis. In his own words:

> Here modernism is defined as an autonomous body of ideas, having little or no outward reference, placing considerable emphasis on formal aspects of the work and maintaining a complicated – indeed anxious – rather than a naïve relationship with the day-to-day world,

> which is the de facto view of a coherent group of people, such as a professional or discipline-based group that has a high sense of the seriousness and value of what it is trying to achieve. (Gray 2008, 1)

Gray intends this definition, not as a straitjacket determined by a strict party line but rather as an idea of a broad cultural field providing a perspective that may help historians integrate issues traditionally treated separately (including both technical aspects of certain sub-disciplines and prevailing philosophical conceptions about mathematics), or stressing new historical insights on previously unnoticed developments. Thus, for instance, the interactions with ideas of artificial languages, the importance of certain philosophers hitherto marginalized in the history of mathematics, the role of popularization, or the interest in the history of mathematics which had a resurgence in the said period.

One issue of particular interest raised by Gray in this context is that of "anxiety" (pp. 266–277). The development of mathematics in the nineteenth century is usually presented as a great success story, which certainly it is, and Gray does not dispute it. But at the same time, a growing sense of anxiety of a new kind, about the reliability of mathematics, the nature of proof, or the pervasiveness of error, was a recurrent theme in many discussions about mathematics, and this is an aspect that has received much less attention. Gray raises the point in direct connection with the anxiety that is often associated with modernism as a general cultural trait of the turn of the century. As an example of this anxiety, he calls attention to certain texts with such a manifest concern that historians previously overlooked or just regarded as isolated texts. Gray makes a clear and explicit connection between these texts with one another and with the broader topics of modernism.

Gray's book complements Mehrtens' in presenting a much broader and nuanced characterization of the discipline of mathematics in the period 1890–1930. On the other hand, in comparison with Mehrtens, Gray devotes much more attention to describing these characteristic features than to explaining the motivations and causes of the processes that ultimately led mathematics to become the kind of discipline that he aptly describes.[10]

Mehrtens' and Gray's books are, then, two significant attempts to approach the history of modern mathematics while relying on the idea of modernism as a historiographical category with significant explanatory added value. Against the background of my brief account and the many additional reviews of the books cited above, I return to my claim that for such attempts to be successful, it is necessary to focus more compellingly on showing (if possible) that the external processes that led to modernism in general and modern mathematics are similar and have common cultural roots.[11] One should not rule out the possibility that such kinds of external processes indeed took place and were meaningful in shaping the history

[10] For additional discussions on Gray's book, see (Feferman 2009; Rowe 2013; Schappacher 2012; Scholz 2010).

[11] An alternative, but not very convincing, way to connect mathematics with the general phenomenon of modernism appears in (Everdell 1997), where Cantor and Dedekind are presented

of mathematics. But in terms of existing research, little evidence of anything of the sort has been put forward by historians in rigorous detail thus far. The question, therefore, arises whether it is possible and illuminating to do so.

16.4 Greenberg's Modernist Painting and Modernist Mathematics

I proceed to discuss in this and in the next section two specific kinds of analysis of modernism that, while being completely unrelated to mathematics, do suggest directions that might be followed to turn the type of general directives delineated in the previous sections into concrete historical research. First, I discuss some ideas found in the writings of the celebrated and highly controversial art critic Clement Greenberg (1909–1994). For some historians of art, I should stress at the outset, Greenberg is total anathema and the foremost example of how the history of modern art should *not* be written and understood. Art historian Caroline A. Jones, for instance, described his views on modernism as "extraordinarily narrow" and as not proving "capacious enough for much painting of the modern period (even much "great painting", *pace* Greenberg)" (Jones 2000, 494). Jones published the most comprehensive account to date of Greenberg's writings and influence (Jones 2005). The reader willing to take the challenge of her ambitious book will get the direct taste of the kind of passionate opposition (and attraction) that the "Greenberg effect" has aroused among its critics.

Still, I find it pertinent to call attention to some of Greenberg's texts for their high suggestivity for the main aim of this article. Being an outsider to the world of art criticism, I can bypass the question of whether Greenberg's characterization of modernism in art is comprehensive enough. Likewise, I can certainly ignore the ways in which he allegedly turned his view from *descriptive* to *normative*, i.e., that he did not limit himself to providing a historical explanation of the process that led to the creation and predominance of certain styles in twentieth century art, but he also wanted to determine, along the same train of ideas, what good art is and should be.[12] Greenberg was certainly not just a detached commentator but a main figure, strongly involved in the art scene in New York who had the power and the tools to build and destroy at will the careers of many an artist. His support of Jackson Pollock is a well-known chapter of his achievements in this regard, and so is his very negative attitude towards Marcel Duchamp and Ad Reinhardt.

as the true (unaware) initiators of modernism because the way in which they treated the continuum in their mathematical work. See also (Pollack-Milgate 2021).

[12] One can find in Greenberg's own texts support for such a view, but in other places he emphatically denied that his analysis was ever intended as anything beyond pure description. See, e.g., (Greenberg 1983): "I wrote a piece called 'Modernist Painting' that got taken as a program when it was only a description."

Good examples of Greenberg’s insights that I deem valuable for the present discussion are found in a famous article of 1960, “Modernist Painting”, where he characterized the essence of modernism in terms that, if unaware of the context, one could easily take to be a description of modern mathematics. He thus wrote:

> The essence of Modernism lies, as I see it, in the use of characteristic methods of a discipline to criticize the discipline itself, not in order to subvert it but in order to entrench it more firmly in its area of competence. ... The self-criticism of Modernism grows out of, but is not the same thing as, the criticism of the Enlightenment. The Enlightenment criticized from the outside, the way criticism in its accepted sense does; Modernism criticizes from the inside, through the procedures themselves of that which is being criticized. (Greenberg 1995, 85)

Indeed, the reflexive character of mathematics (discussed above) reached a distinctive peak at the turn of the twentieth century and became the main tool for discussing and indeed criticizing the discipline. Think of the foundational works of Frege, Russell, Hilbert, Brouwer, Weyl or Gödel. As in Greenberg’s description, this “criticism” worked from within, using the tools of the discipline, meant not to subvert it, but rather to entrench its status.[13]

For Greenberg, the source of this new kind of criticism coming from within could be traced back to Kant. It would seem natural that, given the essentially critical nature of the discipline, philosophy would engage in this kind to self-criticism, and Kant took it to new heights in his critical philosophy exploring the conditions of production of philosophy itself. However, Greenberg raised an interesting historical point here, relevant to our account. As the eighteenth century wore on, more rational justifications started to emerge in other disciplines as well, eventually reaching the arts. The latter, according to Greenberg, had been denied by the Enlightenment a serious task and the arts were thus gradually reduced to “pure and simple” entertainment. A type of Kantian self-criticism that would explore the conditions of production of art from within art itself (and here he meant mainly the visual arts) appeared as a possible way to redefine the kind of experience that would stress what is valuable in art in its own right and, particularly, what could not be obtained from any other kind of activity. Herein lies Greenberg’s explanation of the origin, the essence, and indeed the justification of modernist art:

> Each art had to determine, through its own operations and works, the effects exclusive to itself. ... It quickly emerged that the unique and proper area of competence of each art coincided with all that was unique in the nature of its medium. (Greenberg 1995, 86)

And in the case of painting this led Greenberg to characterize modernism in terms of a preoccupation with two main dimensions of this artistic activity, namely, (1)

[13] It is worth stressing, however, that the issue of self-criticism and the ability of an individual (or a collective for that matter) to effectively distance himself from the normative framework in which he functions in order to be self-critical and innovative is a truly complex one, when considered from a broader philosophical point of view. For a through discussion that examines the views of philosophers like Brandom, Friedman, Davidson, Habermas, Rorty, and others, see (Fisch and Benbaji 2011).

the intrinsic fact of painting's *flatness* and the inherent physical delimitation of this flatness and (2) the gradual tendency of painting (recognizable since the last third of the nineteenth century) to estrange itself from the classical task of representation while occupying itself increasingly with questions of its nature. Thus, these two main characteristic features—painting's preoccupation with the question of flatness and its limitations—appear here as a direct consequence of the self-critical processes that Greenberg described above:

> It was the stressing of the ineluctable flatness of the surface that remained, however, more fundamental than anything else to the processes by which pictorial art criticized and defined itself under Modernism. For flatness alone was unique and exclusive to pictorial art. The enclosing shape of the picture was a limiting condition, or norm, that was shared with the art of the theater; color was a norm and a means shared not only with the theater, but also with sculpture. Because flatness was the only condition painting shared with no other art, Modernist painting oriented itself to flatness as it did to nothing else. (Greenberg 1995, 86)

Greenberg's focus exclusively on the question of flatness as *the* defining feature of modernist art has been one of the main points of criticism directed against him. We need not enter a debate about that here. What I do learn from Greenberg's analysis, however, is a possible underlying explanation of the historical conditions for the rise to pre-eminence of what Greenberg sees as Kantian-like self-criticism (art analyzing art with the tools of art alone) and which appears as a primary characteristic trait of modernist art. Since as already indicated, this kind of critical approach is also strongly distinctive of modern mathematics (and especially of the foundational quests typical of the turn of the twentieth century: mathematics analyzing the foundations and the limitations of mathematics with the tools of mathematics alone, and without the help of external, philosophical and metaphysical arguments) we are led to wonder about a possible new focal point of analysis airising from Greenberg's approach to the question: was the rise of a new kind of modern mathematics related to a search for what was unique and exclusive to mathematics and the peculiar nature of its medium? And if so: why did mathematicians engage in this search? What happened in, say, the last part of the nineteenth century, and not before that, that prompted at that time this kind of search, and what were the consequences of it?

We may then ask these questions for mathematics in general and not just for those places that are typically associated with modernist trends, namely the new kind of foundational research that appeared in the works of Frege, Russell, Hilbert, and others at the turn of the twentieth century. I will return briefly to these questions in the concluding section. At this point, I want to stress that an analogy with Greenberg's analysis might, in principle, help us understand the origins and causes of the processes (social, institutional, disciplinary, philosophical, internal, etc.) behind the rise of modern mathematics and not just to check against a checklist of features characteristic of modernism in art.

It is enlightening to consider some additional points raised by Greenberg, which are relevant to our discussion. Thus, for instance, strongly connected with the previous issue, Greenberg stressed the centrality of the quest for the autonomy of art. The impact of the process of self-criticism was translated, in Greenberg's analysis, to a focused search for "purity" in art as the guarantee for the preservation of the necessary standards,[14] and consequently, the status of the medium of art was transformed. In Greenberg’s words:

> Realistic, naturalistic art had dissembled the medium, using art to conceal art; Modernism used art to call attention to art. The limitations that constitute the medium of painting – the flat surface, the shape of the support, the properties of the pigment – were treated by the Old Masters as negative factors that could be acknowledged only implicitly or indirectly. Under Modernism these same limitations came to be regarded as positive factors and were acknowledged openly. Manet’s became the first Modernist pictures by virtue of the frankness with which they declared the flat surfaces on which they were painted. The Impressionists, in Manet’s wake, abjured underpainting and glazes, to leave the eye under no doubt as to the fact that the colors they used were made of paint that came from tubes or pots. Cézanne sacrificed verisimilitude, or correctness, in order to fit his drawing and design more explicitly to the rectangular shape of the canvas. (Greenberg 1995, 86)

Again, the analogy with mathematics seems to me highly suggestive, but we need to analyze its validity very carefully. The search for autonomy, and eventually even segregation, is an acknowledged characteristic of at least certain essential parts of modern mathematics. In this sense, the analogy with modern art is evident and has often been mentioned. But what were the reasons for this? We are well aware of important internal, purely mathematical dynamics of ideas leading to the rise of a new kind of approach and practice that stressed the need for the autonomy of mathematical discourse and mathematical methods. Here perhaps with the help of a perspective similar to that suggested by Greenberg for art, we may look for some other, more external kinds of causes in the case of mathematics. The increased search for purity in mathematics can be related to a specific attempt to “guarantee its standards of quality”. But what about “limitations that constitute the medium” of mathematics, that were treated by the Old Masters as negative factors that could be acknowledged only implicitly or indirectly”, and that in modern mathematics could come “to be regarded as positive factors” and to be “acknowledged openly”? This appears as a remarkable, and far from self-evident characterization of modern art that Greenberg’s analysis brings to the fore. But given the already mentioned, essentially inevitable, need to rely on historical continuity in mathematics (as opposed to the arts), a transposition of this kind of argumentation to mathematics is far from straightforward and requires additional care.

In further exploring this point, however, one might try to bring to bear ideas from sociologists of science such as Rudolf Stichweh, who has highlighted the systemic, interrelated character of discipline formation by the end of nineteenth century. Stichweh’s analysis meant to show how the emergence and consolidation of an autonomous self-understanding of the various academic disciplines depended always on similar processes taking place in the neighboring disciplines at the same time (Stichweh 1984). Stichweh’s perspective might open interesting avenues of research also for our topic, but at this point, I leave this as an open thread calling for further thought concerning the question of modernism in mathematics.

[14] A discussion of “purity” and its centrality in modernism, from a different perspective appears in (Cheetham 1991).

In referring to the "necessity of formalism" as the "essential, defining side" of modernism (at least in the case of painting and sculpture), Greenberg added another interesting explanation that seems very suggestive for mathematics as well:

> Modernism defines itself in the long run not as a "movement", much less a program, but rather as a kind of bias or tropism: towards aesthetic value, aesthetic value as such and as an ultimate. ... This more conscious, this almost exacerbated concern with aesthetic value emerges in the mid-nineteenth century in response to an emergency. The emergency is perceived in a growing relaxation of aesthetic standards at the top of Western society, and in the threat this offers to serious practice of art and literature. (Greenberg 1971, 171)

Keeping in mind that terms such as "formal", "abstract" or "aesthetic" have significantly different meanings and elicit different contexts in mathematics and in the arts, one can still ask whether the idea of associating the entrenchment of formalist approaches as part of the consolidation of modern mathematics with a reaction to an emergency, as described here by Greenberg for the arts, may bring with it new insights. Moreover, we can also ask if the "emergency" in question was not only similar but perhaps even the same one in both cases. I already mentioned the issue of "anxiety" discussed by Gray concerning the development of mathematics at the turn of the nineteenth century, which he related to what some mathematicians conceived as a relaxation of standards. There is no doubt that formalism in mathematics can be associated with a possible reaction to such a relaxation. Thus, formalism may appear here not just as a common trait perceived in mathematics and art but also as motivated by similar concerns in both cases. More on this I will say in the next section.

Finally, I would like to mention yet another suggestion of Greenberg that may be relevant for historians of mathematics in their field, as it touches upon the supposed radical break with the past that appears in so many characterizations of modernism. In an article entitled "Modern and Postmodern," Greenberg wrote:

> Contrary to the common notion, Modernism or the avant-garde didn't make its entrance by breaking with the past. Far from it. Nor did it have such a thing as a program, nor has it really ever had one. It's been in the nature, rather, of an attitude and an orientation: an attitude and orientation to standards and levels: standards and levels of aesthetic quality in the first and also the last place. ...
>
> And where did the Modernists get their standards and levels from? From the past, that is, the best of the past. But not so much from particular models in the past – though from these too – as from a generalized feeling and apprehending, a kind of distilling and extracting of aesthetic quality as shown by the best of the past. (Greenberg 1980)

I find it remarkable that Greenberg would stress this point in opposition to what so many considered an unavoidable trait of modernism. As I said above, truly radical breaks with the past seem rather unlikely in mathematics. As Greenberg stresses here, modernism may arise not from a radical break but from a conscious process of distilling and extracting quality from what proved to be the best practice in the past. I think that in laying the central elements of modern mathematics, some of the most influential mathematicians of the turn of the century acted precisely in this way. This was undoubtedly the case, as I have discussed in detail elsewhere, with

Dedekind’s early introduction of structuralist concerns in the algebra (Corry 2017) and with Hilbert’s introduction of the modern axiomatic approach (Corry 2004).

As already stated, however, there are also good reasons to react to Greenberg’s views with great care. It is not only that they are very much debated among art historians, but also that Greenberg did not write systematic, scholarly texts with all due footnotes and references. Most of his writings appeared as scattered articles, conferences, etc., and they sometimes follow a somewhat associative style. Thus, one must not be surprised to find deep changes and possibly conflicting views in them throughout the years.

And yet, even if the criticism directed at him is well taken, especially when one tries to apply his view to analyzing in detail the works of specific individual artists, this does not mean that the essential structure of the processes he describes cannot be reconstructed for the purposes I am pursuing here, and then followed in a more scholarly solid fashion. If one were able to develop explanations of these kinds for mathematics, then it may turn out that it is not only justified and valuable to use the term modernism in the context of the history of mathematics but also that it is not just a coincidence that modernism appears in mathematics as well as in the arts nearly contemporarily, and that this coincidence can be made sense of in more or less tangible terms.[15]

16.5 Wittgenstein’s Vienna and Modernist Mathematics

The second source I want to refer to in the search for ideas relevant to a possible fruitful discussion of modernism in mathematics is the book *Wittgenstein’s Vienna* by Allan Janik and Stephen Toulmin. The main topic of this book is an interpretation of the roots and meaning of Wittgenstein’s *Tractatus*. Contrary to accepted views—according to which the fundamental questions underlying the treatise were epistemology, philosophy of science, and logic taken for their own sake—the authors aimed at presenting Wittgenstein as a thinker deeply rooted in the intellectual life of Vienna at the turn of the twentieth century, for whom the question of language and its limitations was mainly an *ethical* concern and not merely a linguistic-analytic one. These ethical concerns, they contended, can

[15] Greenberg, of course, is not the only one to discuss modernism in terms of the processes that led to its rise, rather than by just providing a checklist of characteristic features. Also worthy of mention here is the work of Dan Albright (Albright 1997, 2000), who stresses the crisis of values in art that led to modernism. In his view, if in previous centuries, artists, writers, and musicians could be inherently confident about the validity of the delight and edification they provided to their audiences, during the twentieth century art found itself in a new and odd situation, plagued with insecurity. Faced with the crisis, radical claims about the locus of value in art were advanced in various realms at nearly the same time. The various radical modernist manifestoes thus produced reflect the need of the artist not only to create, as was always the case in the past, but also to promote new standards of value and to provide some new kind of justification to the very existence of art.

only be fully understood against the background of Viennese modernism in its manifold manifestations. Like in the case of Greenberg, I do not intend to come up here with an appraisal of Janik and Toulmin's analysis as a contribution to the Wittgenstein scholarship, but rather to focus on ideas potentially relevant to the topic of mathematics and modernism.

The basic question that Janik and Toulmin pursued thorough the book appears right at the beginning, phrased in the following terms:

> Was it an absolute coincidence that the beginnings of twelve-tone music, 'modern' architecture, legal and logical positivism, nonrepresentational painting and psychoanalysis – not to mention the revival of interest in Schopenhauer and Kierkegaard – were all taking place simultaneously and were largely concentrated in Vienna? (Janik and Toulmin 1973, 18)

The central hypothesis of the book is that "to be a *fin-de-siècle* Viennese artist or intellectual conscious of the social realities of Kakania [a term coined by Robert Musil to describe Austro-Hungarian society disparagingly (L.C.)] one had to face the problem of the nature and limits of language, expression, and communication. (p. 117)" Accordingly, they offered an account of the deep changes that affected art, philosophy, and other aspects of cultural life around 1900 in Vienna, as interrelated attempts to meet the challenges posed by questions of communication (language), authenticity, and symbolic expression. These challenges, in turn, derived from the deep social changes that affected the capital city of the Habsburgs: a medley of interacting tongues, the tension between the central imperial rule and the local and national aspirations, liberalization alongside decentralization of the traditional centers of power, changes in the production processes and social structures. And in this context, the most crucial instance of the philosophical side of this sweeping cultural phenomenon arose in the person whose writings, in their view, embodied the crucial influence on Wittgenstein, Fritz Mauthner (1849–1923), who developed a unique doctrine of "Critique of Language" (*Sprachkritik*) in several interesting books and is one of the few persons mentioned by name in the *Tractatus*.

In the received interpretation of Wittgenstein, the importance of this reference to Mauthner is often downplayed, but Janik and Toulmin made it the centerpiece of their analysis. Their alternative (and, as I see it, enlightening) approach to Wittgenstein affords a useful perspective for our discussion since the authors did not limit themselves to indicating general analogies between various fields of activity or a common, putative underlying ethereal *Zeitgeist* but instead emphasized concrete historical processes that were motivated by similar concerns stemming from the specific historical circumstances of turn-of-the-century Vienna.

Incidentally, an important focus of attention for Janik and Toulmin is found in contemporary science, and in the works of Ernst Mach (1838–1916), Heinrich Hertz (1857–1894), and Ludwig Boltzmann (1844–1906). For these three scientists, as it is well known, metaphysics had no place in science, and they devoted conscious and systematic efforts at finding those places where metaphysics had subtly but mistakenly been incorporated. This task, however, was not pursued in the same way by the three of them. Janik and Toulmin describe them as representing significantly different stages in a continuous process. Mach represents a first stage where the limits of physics were set "externally", as it were, employing a more philosophical analysis. Hertz and Boltzmann, on the contrary, by following an approach that can retrospectively be described as "axiomatic", pursued the same task "from within." Hertz and Boltzmann sought to set the correct limits of physical science through an introspective analysis using the tools of science (and here, of course, we find a remarkable similarity with Greenberg's stress on "criticism from within," as explained above).

The interesting point in their analysis, however, is that they embedded this two-stage process in the more general, broad historical processes that underlie all other manifestations of Viennese modernism. First and foremost, among these manifestations were, for them, the processes leading from Mauthner to Wittgenstein. The philosophical critique of language undertaken by Mauthner as a response to the need mentioned above to establish the "limits of language, expression, and communication" starts from a point that is similar to that of Mach's attempt in physics. And very much like Hertz and Boltzmann had further pursued Mach's quest, but by way of an alternative, more internally focused path, so did Wittgenstein in relation to Mauthner. Hertz and Boltzmann, according to Janik and Toulmin, "had shown how the logical articulation and empirical application of systematic theories in physical science give one a direct *bildliche Darstellung* of the world, namely, a mathematical model which, when suitably applied, can yield true and certain knowledge of the world. And they had done so, furthermore, in a way that satisfied Kant's fundamentally antimetaphysical demands – namely, by mapping the limits of the language of physical theory entirely "from within" (p.166). In similar terms, Janik and Toulmin presented the philosophical work of Wittgenstein as a continuation of Mauthner's, in which the limits of language, in general, were mapped from within. They also examined and laid all the necessary stress on the ethical outlook, which in their interpretation, was so central to Wittgenstein's undertaking and arose from the writings of Kierkegaard and Tolstoy. (Of course, this main element played no role in the story about Mach, Hertz and Boltzmann.)

The sociocultural elements underlying both aspects of the story, as described above, are expanded subtly to cover other fields of activity along the same lines: music, architecture, journalism, law, painting, and literature. And in all these fields Janik and Toulmin also added a third stage that was produced along the lines of commonly characterized historical processes. Thus, for instance, the three stages in music are represented by Gustav Mahler, then Arnold Schönberg, then Paul Hindemith. In the case of architecture, it is Otto Wagner, then Adolf Loos, and then Bauhaus.[16] And in the case of philosophy, the stage after Wittgenstein (who came

[16] (Galison 1990) presents an analysis that complements this view and locates the Bauhaus movement in relation with logical positivism, as part of Viennese modernism.

after Mauthner) is that of logical positivism.[17] The process which is common to all these threads can be briefly described as follows:

> In architecture as in music, then, the technical innovations worked out before 1914 by the 'critical' generation of Schönberg and Loos were formalized in the 1920s and 1930s, so becoming the basis for a compulsory antidecorative style which eventually became as conventional as the overdecorative style which it displaced. And we might pursue these parallels still further if we pleased – into poetry and literature, painting and sculpture, and even into physics and pure mathematics. In each case, novel techniques of axiomatization or prung rhythm, operationalism or nonrepresentational art, were first introduced in order to deal with artistic or intellectual problems left over from the late nineteenth century – so having the status of interesting and legitimate new *means* – only to acquire after a few years the status of *ends*, through becoming the stock in trade of a newly professionalized school of modern poets, abstract artists or philosophical analysts. (p. 254)

Not *Zeitgeist*-like arguments or superficial analogies, then, as part of the explanation, but a common ground to all these processes, namely, "a consistent attempt to evade the social and political problems of Austria by the debasement of language."

Can the insights of Janik and Toulmin be imported into the historiography of mathematics fruitfully? It is curious that the passage quoted mentioned pure mathematics as having been affected by the same circumstances as other cultural manifestations. They do not give details about what they may have had in mind when saying this. Indeed, at a different place, they did state that "in a very few self-contained theoretical disciplines—for example, the purest parts of mathematics—one can perhaps detach concepts and arguments from the historic-cultural milieus in which they were introduced and used and consider their merits or defects in isolation from that milieu" (p. 27). But my point is not what was done in the book in relation to mathematics, but what *could be done* by analyzing Viennese mathematics at the turn of the century from the perspective afforded by the book.

I briefly indicate here specific parameters that might be considered in an attempted answer. In the relevant period, Vienna did have an interesting, original, and very productive mathematical community. Its more prominent names included Wilhelm Wirtinger (1865–1945), Philipp Furtwängler (1869–1940), Eduard Helly (1884–1943), Kurt Gödel (1906–1978), Kurt Reidemeister (1893–1971), Witold Hurewicz (1904–1956), Walther Mayer (1887–1948), Johann Radon (1887–1946), Alfred Tauber (1866–1942), Olga Taussky (1906–1995), Heinrich Tietze (1880–1964) and Leopold Vietoris (1891–2002). Each of these mathematicians arrived in Vienna from different places at different times, bringing their baggage drawn from the mathematical traditions from which they stemmed. Of course, even before we start to consider the question that occupies us here, one should be able to come forward with a more articulate understanding of the Vienna mathematical community than we now have what the main mathematical fields were pursued, what kinds of interactions existed with the local scientific communities and with neighboring mathematical institutions, what were the internal mechanisms

[17] (Janik 2001, 147–69) discusses the somewhat different relation between Hertz's famous Introduction and the late Wittgenstein.

of production, training, and transmission of mathematical knowledge, etc. Such questions have been pursued, sometimes in detail, for Göttingen and Berlin, for the various USA centers of mathematics, and some British and Italian contexts, but, unfortunately, much less so for Vienna.

One existing work of relevant historical research does indicate, however, that there might be some room for pursuing this question along the broader conception of modernism, as suggested above. Moritz Epple has investigated the mathematical contributions of Kurt Reidemeister on Knot Theory in the 1920s while comparing it with work conducted simultaneously in the same field at Yale, on the one hand, and at Princeton, on the other hand. Epple discussed the intellectual atmosphere of the city as part of the relevant intellectual background to Reidemeister, and his discussion on the rise of modern topology is framed in the broader context of modernism in mathematics (Epple 1999, 299–322; 2004).

As part of his account, Epple stressed the existence of different paths into modernity that led to other varieties of mathematical modernism, even within the same institutional context, i.e., that of mathematics at Vienna, where Reidemeister worked alongside Wilhelm Wirtinger, producing different brands of modern mathematics (Epple 1999, 236–26). Reidemeister had strong intellectual interactions with Hans Hahn, Otto Neurath, Otto Schreier, and Karl Menger, all of them engaged in the activities of the Vienna Circle, which is of obvious relevance for any discussion on modernism. In addition, the most prominent members of the literary milieu in the city at the time had formal training in mathematics and strong connections with the local mathematicians. The three most famous examples of this are Robert Musil (1880–1942), Herman Broch (1886–1951) and Leo Perutz (1882–1957). The latter continued to be actively involved in mathematics throughout his life (Sigmund 1999; Engelhardt 2018, 2021).

What kinds of mathematics were done at the time in Vienna? To what extent can such kinds of mathematics be properly called modernist? Is this somehow connected with the work and the person of their Viennese neighbor Boltzmann? Inspired by Mehrtens' kind of analysis, Mitchell Ash has recently discussed the linkages between "modern ways of thinking about science" and the radical development of the visual arts in Vienna at the time. In his view, the significance of technological modernism "presupposes a concept of knowledge-based less on self-referential abstraction than on what can be done with, or to, nature as well as other human beings" (Ash 2018, 27). In an attempt to bring out basic features that link science and the arts in that specific cultural context, he illustrated the plurality of modernisms manifest in the sciences and culture of 'Vienna 1900′ by discussing the work of Ernst Mach and Ludwig Boltzmann, on the one hand, and the music of Arnold Schoenberg, on the other. For all the merits of Ash's analysis, the question remains open, whether the kind of mathematics practiced in Vienna was peculiar and different to what preceded it, and, more importantly, if the processes leading to the changes that brought about this possibly new conception are similar or similarly motivated as all the other complex processes described in Janik and Toulmin's book concerning Viennese culture in general.

It is relevant to stress that Epple's methodological proposals include reliance on Weber's idea of "patterns of rationality" as a way to contextualize the mathematical practice of a specific culture. But at the same time, his comparative analysis is based on the idea of an "epistemic thing", originally introduced by historians of experimental sciences (Rheinberger 1997). Epple uses this concept to explain in what senses Reidemeister's topological research differed from other, contemporary ones. In so doing, he suggested the plausibility that the specificity of Reidemeister's work was tied to Viennese intellectual modernism, even though, the nature of the tie remains to be explained.[18] In an ideal study of the mutual relationship between modernism and mathematics one might also be led to go the opposite direction, namely by understanding the specifics of Reidemeister's (and Hahn's and Menger's), and uncover new historical mechanisms behind the development of Viennese modernism.

A different, and perhaps highly relevant direction in which the analysis of Janik and Toulmin can provide illuminating hints to historians of mathematics has to do with the development of the modern axiomatic approach. I have devoted considerable attention in my research to the work of David Hilbert, to the centrality of the axiomatic approach for his work, and to the significant impact that this aspect of his work had on mathematics and physics in the early twentieth century, precisely at the time under discussion here. In my analysis, I have shown how the work of Hertz and Boltzmann had a direct influence on Hilbert and on the consolidation of the axiomatic approach and its application to both geometry and physics. I have also stressed the pervasive presence of Mach's ideas and his empiricist-oriented criticism in the background of Hilbert's work (Corry 2004, Chps. 2–3). Considering this, it is remarkable that in the three-stage model of Janik and Toulmin, precisely this thread, leading from Mach to Hertz and Boltzmann, which the authors single out as highly important, is not completed with its third stage. My suggestion here is that one might look at the process leading to, and at the consolidation of, Hilbert's new axiomatic approach precisely as that third stage. A historical analysis of the kind provided for Hilbert may plausibly be complemented with an eye on the types of processes described by Janik and Toulmin. In this way, the mathematics embodied in and promoted by Hilbert's approach could be seen as an aspect of mathematical modernism, not just because of a series of characteristic features associated with it, but rather because it might be seen as the outcome of a process with specific historical-cultural roots that gave rise to modernism in so many fields of culture at the time.

[18] Epple's description of the intellectual background to *fin-de-siècle* Vienna also strongly relies on the classical study (Schorske 1980).

16.6 Prospective Remarks

An emphasis on the formal, as opposed to thematic values; a rite of passage through avant-garde; a radical break with tradition (or even a "desire to offend tradition"); the wish to explore subjective experience as opposed to representing "outward experience"; a high degree of self-consciousness; a criticism of the basic principles of the discipline and its limits using the tools of the discipline. These are some characteristic features typically associated with modernism in its various cultural manifestations at the turn of the twentieth century. Some of them are mentioned and analyzed by the authors referred to above, and I take them to be an illustrative rather than an exhaustive sample of scholarly discussions on the topic of modernism. We may find such basic attitudes also in the mathematics of the period in question. Historians of modern mathematics might debate the degree to which such traits are central and pervasive and hence the extent to which it may be appropriate to describe, based on our current historical knowledge, modern mathematics as a modernist endeavor. My tentative proposal in this survey is, in contrast, that rather than exploring the topic in this straightforward way, we should ask ourselves if the perspective of modernism may lead us to look for new insights into making sense of the history of modern mathematics.

Considering, for instance, Jeremy Gray's emphasis on the sense of anxiety that arose at the end of the nineteenth century side-by-side with the enormous successes of the discipline. Talk about this success is standard in any historical account concerning this period, but the concomitant anxiety indicated by Gray has been much less discussed (if at all). By situating it in a modernist context, Gray draws our attention to the possibility that this is a more significant issue than we have realized thus far. He gives the example of an inaugural lecture delivered in 1910 at Tübingen by Oskar Perron (1880–1975). Perron was a proficient mathematician with acknowledged contributions to various fields, but his prominence was far from the high profile of a Hilbert or a Noether. Thus, one will not find his name often mentioned in discussions about mathematical modernism. But as Gray indicates, it is essential to hear what a mathematician like him had to share about his discipline at the turn of the twentieth century. In his lecture, Perron addressed mainly questions related to the gap between the public perception of mathematics and the actual practices in the discipline, particularly concerning the question of the certainty and exactness of its methods (Perron 1911). One should then ask whether this is an isolated phenomenon or a manifestation of a more generalized concern of the practitioners of mathematics at the time and whether, by looking at the kind of considerations discussed in the preceding sections, we can gain some innovative historical types of insights on this question.

Well, if we follow the lead opened by Gray, we do find instances that give us further food for thought. Thus, for instance, an interesting text by Alfred Pringsheim (1850–1941), who, like Perron, was a well-known mathematician and not one whose work is typically discussed in relation to modernism. In addition to his mathematical activities, Pringsheim was deeply immersed in the broad cultural trends of his time, and that to an unusual degree. He came from a wealthy Jewish family in Berlin who used his wealth to support art, and Alfred became a well-known art collector. He had a strong, well-cultivated musical background and became one of the earliest Wagner supporters. His daughter Katia, one of the first active women university students at Munich, married Thomas Mann. The family house in Berlin and his one in Munich (both known as "Palais Pringsheim") were prominent architectural icons (though they were far from any clue of modernist taste) (Perron 1952).[19] In 1904, on the occasion of the 145th anniversary of the Munich Academy of Sciences, Pringsheim gave a lecture entitled "On the Value and Alleged Lack of Value of Mathematics" (Pringsheim 1904). Without going into the details of the talk, I will say that it reflects the kinds of concerns addressed by Perron very closely. Incidentally, Pringsheim had been one of Perron's most influential teachers in Munich (Frank 1982).

One may mention additional texts that go in the same direction (von Mises 1922), and, more importantly, one is motivated now to look for more. Still, the question remains open whether we can *find* not just additional texts that serve as evidence for these kinds of concerns (which is quite likely), but rather if we can *understand their roots* and the processes leading to their rise and consolidation. And more specifically: whether these roots may be found to be directly connected, or at least closely related, to those found at the basis of modernism as a broad cultural phenomenon (and this, I think, is less likely, though still plausible).

Think, for instance, of Greenberg's explanation of the rise of modern painting in terms of the need existing in each art by the late nineteenth century to determine—purely with the help of its means—what was unique and exclusive to itself, according to the nature of its medium. The intense foundational activity, one of the acknowledged characteristic features of mathematics at the turn of the twentieth century, can be easily seen as a similar manifestation—purely from within the discipline—of the phenomenon indicated by Greenberg in the case of painting. Indeed, this is a point typically stressed in the debates about modernism in mathematics. But can we, in addition, explain the timing and the main thrusts of this foundational activity on the same grounds that Greenberg adduced for the arts? For Greenberg, in the wake of the Enlightenment, the arts were gradually assimilated into entertainment, pure and simple. This was a primary trigger that led to the kind of internally pursued self-criticism laying at the basis of modernism. Can we come up with a similar explanation in the case of mathematics? Dan Albright, to take another example, sees the roots of modernism as related to the new and odd situation for art, plagued with insecurity, as opposed to the confidence in the validity of the delight and edification it had provided to their audiences in previous times (Albright 2000). Perhaps this could be a fruitful lead to follow in connection with the topic of anxiety just mentioned above. Can we trace a direct relationship

[19] In this context it is natural to stress that also Wittgenstein was born to a privileged and immensely wealthy Viennese family, who generously supported the likes of Gustav Klimt and Alfred Loos as well as the poets Georg Trakl and Rainer Maria Rilke. The circle of friends of the Wittgenstein family included many distinguished figures of the Viennese musical milieu, such as Johannes Brahms (Monk 1991).

between the changes of status in the arts, with related changes of position in mathematics, with questions about certainty and the unity of mathematics, and with the increasing trend of foundational research at the turn of the twentieth century? Can the explanations of Janik and Toulmin about the centrality of the problem of language in the modernist culture of Vienna, and its social and ideological roots, be of any help in consolidating such an explanation?

Answering these questions would require, I believe, additional historical research taking into consideration that modernism is a historical phenomenon with an internal evolution and geographical specificities that are often overlooked. Thus, with the Great War in Europe precisely in the middle of the period that frames our discussion (1890–1930) and its profound social and cultural impact it is obvious that a single idea of "modernism" is too coarse to account for all the developments typically related with the term without further historicizing it. What is less obvious, but no less significant, are the differences among modernist cultures across the continent and in the USA. As mathematics is the quintessential universal endeavor, these geographical differences would seem irrelevant for the discussion. Still, I suggest that they are not and that the right way to consider modernism in mathematics would be, if at all, at the local level: modernist Paris mathematics, modernist Viennese mathematics, etc. Moreover, this approach would inherently emphasize the need to analyze not just the pronouncements of the Hilberts and the Weyls but also the pronouncements and the mathematical deeds of the Pringsheims the Perrons, and the Reidemeisters.

How useful is, then, the term 'modernism' for understanding the history of early twentieth-century mathematics? I hope to have shown that while the answer to this question may potentially be positive, there is a long way to go before this potentiality can be translated into reality. In particular, a plain characterization via *checklists of putatively defining features of what modernism is* (which is a much-debated question anyway) will not suffice. What may be of use for gaining new insights into the history of mathematics from the perspective of this question is a deeper understanding of the *historical processes leading to modernism* in its various cultural manifestations.

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