



Power Optimization for Millimeter Wave MIMO System

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Abstract. Water filling algorithm can be used for mmWave (millimetre wave), MIMO (Multiple Input Multiple Output) systems. It offers channel optimality for AWGN and ISI (Additive White Gaussian Noise and Inter Symbol Interference) respectively. In this article we proposed the water filling algorithm for equal power allocation. In this we study the decomposition of MIMO wireless channel with the help of Singular Vector Decomposition (SVD). SVD is used for generating parallel channels and is a factorization of real and complex number. For optimality of MIMO power allocation constrained maximization problem occurs to solve this problem we use Lagrange multiplier. The performance of water filling algorithm may be approach the channel capacity. In this the result when SNR is high the suggested method with equal power allocation of water filling is studied.

Keywords: Water filing algorithm · mmWave · MIMO · SVD

1 Introduction

Millimeter wave MIMO is considered to be the key enablers for 5G wireless network. In 5G mobile network the a projected millimeter-wave (mmWave) with a bandwidth of 30–300 GHz [1]. MmWave technologies have great potential to satisfy the requirement of wireless communication systems such as high data rates, low cost and good user experience. In mmWave MIMO systems, there is precoding and combining for one user. An algorithmic solution was proposed utilising orthogonal matching pursuit for the MmWave precoder design problem of sparsity limited signal recovery. As a result, by using the SVD (Singular Value Decomposition) technique, it can effectively capture the governing Eigen modes, expand the number of channels, and remove interference. This allows mmWave systems to estimate channel capacitance [4]. For uplink broadcasts with the same power allotment, the AIA digital transmitter design method is advised. A low hardware complexity precoding approach was devised using a realistic spatial channel model [8]. Without real-time channel state information, spectral efficiency performance can get close to the best general transceiver design performance.

In this article we consider the millimeter wave MIMO systems. The space between different information transmitter and receiver antenna for multiplexing different information stream and hence it is called spatial multiplexing. It is based on MIMO technology, which carries several data streams concurrently within the same frequency range

by using multiple antennas at both the transmitter and receiver. We consider the SVD receiver for decomposition of MIMO systems. The SVD exist for any matrix even for non square matrix whereas other types of decomposition occur for square matrix e.g. Eigen values. In order to distribute the best amount of power among the various channels in multicarrier schemes, we thus take into account the water filling method, where the power distribution changes nonlinearly with the highest average transmit power to each channel.

1.1 The Main Contribution of this Article is:

- We suggested the water filling strategy for mmWave MIMO systems optimal power distribution.
- We model the singular vector decomposition because it exist any matrix even for square matrix.

2 Literature Review

Omar El Ayach et al. [3] in this paper is focusing on large millimeter wave MIMO systems using low complexity precoding. This is based on the water filling capacity. In this paper also include the large antenna array with one user beamforming and precoding in millimeter wave structure. In single user channel a layered transceiver design for millimeter wave systems are generated. The low hardware complexity precoding method is generated for the realistic spatial channel model.

Shiyu Zhou et al. [6] in this paper mmWave MIMO systems works on the Attitude information aided digital beamforming. For unequal power allocation case water filling power allocation is proposed. The UE (User Equipment) may assess its transmitter and choose to ignore the sequence impact rather than compensating for it, according to this paper. This research suggests an equal power distribution attitude information-aided transmitter design algorithm.

Renwang Li et al. [8] in this paper focusing in mmWave MIMO using Reconfigurable intelligent surfaces (RIS). To control the blockage case RIS algorithm is generated. Ergodic capacity is used for RIS aided model for mmWave MIMO systems under the Saleh Valenzuela channel model.

Wenbin Zhang et al. [2] This work uses combined beam and mmWave small cell system resource allocation for 5G. To solve the issue of resource allocation, the tiny cell base station's hybrid beamforming shape was presented. In this study, analogue, digital, and analogue precoding are accomplished using identity switched and matrix beam. To solve the problem of Lagrange duality it proposes the water filling algorithm for optimal power allocation.

Woo-Hee Lim et al. [9] in this paper is focusing ZF (Zero Forcing) Hybrid downlink-based precoding and aggregation is provided by the mmWave multi-user MIMO system. Optimising the minimal use rate while allocating electricity to each user using the gradient descent method. This paper ZF precoding is used for transmission into download under the perfect CSI (Channel State Information). The MMSE (Minimum Mean Square Error) based precoding and combining are used to balance the hybrid precoder and combiner.

Jianwei Zhao et al. [7] focused on millimetre wave MIMO communication for tracking channels with unmanned aerial vehicle flight control system. This research also suggests the use of the Kalman filter to track the movement of unmanned aerial vehicles. The spatial channel was proposed to be represented by a 3D geometry-based total channel model, where the channel is chosen based on information about UAV movement status and remaining channel advantage.

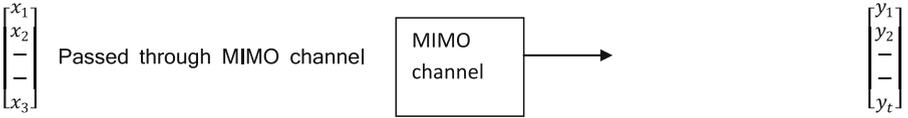
3 Methodology

- We use the decomposition of MIMO wireless channel with help of SVD to generate parallel channel.
- SVD (Singular Vector Decomposition) receiver is used, now let us some manipulation at the transmitter. With SVD we can show that a MIMO channel with m antenna at each side can be treated as m separate single antenna channel.
- We proposed the optimally MIMO power allocation to maximize the capacity or information rate of different transmission streams. The subject to restrictions problem arises while attempting to maximise overall MIMO capacity.
- The constraints maximization problem occurs to solve this problem we use the Lagrange multiplier. Now all the power depends upon the $1/\lambda$.
- We developed the water filling method to allocate the greatest amount of electricity among the several channels in multicarrier systems.

4 Proposed Work

Figure 1 depicts our system model. We consider a transmitter and receiver. In MIMO can increase the data rates by transmitting several information streams in parallel at same transmit power. This is as if i am utilizing this space between different transmitter and receiver antenna for multiplexing different information streams hence in context with MIMO is called spatial multiplexing.

In Fig. 1 shows that



\bar{x} = called transmit symbol vector (t dimension)

\bar{y} = called received symbol vector (r dimension)

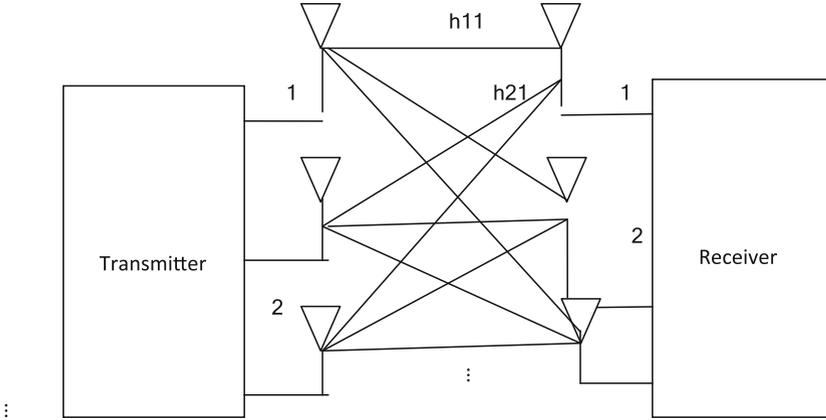


Fig. 1. System model for mmWave MIMO system

$$\begin{bmatrix} y_1 \\ y_2 \\ - \\ - \\ y_t \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} & - & h_{1t} \\ h_{21} & h_{22} & - & h_{2t} \\ - & - & - & - \\ h_{r1} & h_{r2} & - & h_{rt} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ - \\ - \\ x_3 \end{bmatrix} + \begin{bmatrix} n_1 \\ n_2 \\ - \\ - \\ n_r \end{bmatrix}$$

$$y_1 = h_{11}x_1 + h_{12}x_2 + \dots + h_{1t}x_t \tag{1}$$

$$y_2 = h_{21}x_1 + h_{22}x_2 + \dots + h_{2t}x_t \tag{2}$$

Therefore

$$\bar{y} = H\bar{x} + \bar{n} \tag{3}$$

- \bar{y} = received signal power
- \bar{x} = transmit signal power
- \bar{n} = noise vector

- Assume variance (power) of each noise component at each receiver antenna.

$$E = \{ |n_i|^2 \} = \sigma_n^2$$

- Further assume that power of each antenna is uncorrelated i.e. independent (Gaussian).

$$E\{n_i n_j\} = 0$$

n_i = noise at i^{th} receiver antenna

n_j = noise at j^{th} receiver antenna

4.1 Decomposition of MIMO Wireless Channel

SVD (Singular Vector Decomposition) used to generate parallel channel.

SVD of any matrix H is given by

$$H = U \Sigma V^H \tag{4}$$

$$H = U \Sigma V^H$$

$$H = \begin{bmatrix} u_1 & u_2 & \cdots & u_t \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}_{r \times t} \begin{bmatrix} \sigma_1 & 0 & \cdots & 0 \\ \sigma_2 & \ddots & \cdots & 0 \\ \vdots & \cdots & \ddots & \vdots \\ 0 & \cdots & \cdots & \sigma_t \end{bmatrix} \begin{bmatrix} V_1^H \\ V_2^H \\ \vdots \\ V_T^H \end{bmatrix}$$

Properties of Matrix

These columns of 4 matrixes are orthogonal i.e.

$$\|u_i\|^2 = u_i^H u_j = 0; \quad \text{if } i \neq j \tag{5}$$

$$\|V_i\|^2 = v_i^H v_j = 0; \quad \text{if } i \neq j \tag{6}$$

$$V^H V = V_V^H = I \text{ (unitary matrix)}$$

$$u^H u = u_u^H = I \text{ (unitary matrix)}$$

$$u^H u = I \text{ for } r = t$$

$$u^H u \neq I \text{ for } r \neq t$$

Now the structure of singular matrix is $\sigma_1 \sigma_2 \cdots \sigma_t$ are known as the singular vectors such that $\sigma_1 \sigma_2 \cdots \sigma_t \geq 0$ they are non negative. $\sigma_1 \geq \sigma_2 \geq \sigma_t \geq 0$ [singular values are ordered (decreasing order). This SVD exist for any matrix even for non square matrix. Whereas other types of decomposition occurs for square matrices (e.g. Eigen value decomposition).

4.2 SVD Receiver

Recall

$$H = u\zeta V^H \tag{7}$$

$$\bar{y} = H\bar{x} + \bar{n} \tag{8}$$

Using equation (7) in (8)

$$\bar{y} = u\zeta V^H \bar{x} + \bar{n} \tag{9}$$

At receiver multiple \bar{y} with u^H

$$u^H \bar{y} = \tilde{y} = u^H (u\zeta V^H \bar{x} + \bar{n}) \tag{10}$$

$$\tilde{y} = u^H u \zeta V^H \bar{x} + u^H \tilde{n} \tag{11}$$

$$[u^H u = I] \quad [u^H \tilde{n} = \tilde{n}]$$

$$\tilde{y} = \zeta V^H \bar{x} + \tilde{n} \tag{12}$$

Now let us do some manipulation at transmitter also i.e. before transmission \bar{x} i.e called precoding.

Let

$$\bar{x} = v\tilde{x} \tag{13}$$

(Transmit vector \bar{x})

$$\tilde{y} = \zeta V^H v\tilde{x} + \tilde{n} \quad [V^H v = I]$$

$$\tilde{y} = \zeta \tilde{x} + \tilde{n} \tag{14}$$

where,

$$\begin{bmatrix} \tilde{y}_1 \\ y_2 \\ - \\ - \\ \tilde{y}_t \end{bmatrix} = \begin{bmatrix} \sigma_1 & 0 & \dots & 0 \\ \sigma_2 & \dots & \dots & 0 \\ \vdots & \ddots & \dots & \vdots \\ 0 & \dots & \ddots & \sigma_t \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ x_2 \\ - \\ - \\ \tilde{x}_t \end{bmatrix} + \begin{bmatrix} \tilde{n}_1 \\ \tilde{n}_2 \\ - \\ - \\ \tilde{n}_t \end{bmatrix}$$

Decoupling of MIMO channel (or parallelization of MIMO systems).

Now observed

$$\begin{aligned}\tilde{y}_1 &= \sigma_1 \tilde{x}_1 + \tilde{n}_1 \\ \tilde{y}_2 &= \sigma_2 \tilde{x}_2 + \tilde{n}_2 \\ &\vdots \\ \tilde{y}_t &= \sigma_t \tilde{x}_t + \tilde{n}_t\end{aligned}\quad (15)$$

Now SNR of i^{th} parallel channel = $\frac{\sigma_i^2 P_i}{\sigma_n^2}$

Maximum rate is given by Shannon's capacity. Therefore

Maximum rate (capacity of channel) = $\log_2(1 + \text{SNR})$

Therefore maximum rate of i^{th} parallel channel (or capacity of i^{th} parallel channel)

$$c_i = \log_2\left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right)$$

Total MIMO capacity,

$$c_i = c_1 + c_2 + \dots + c_t$$

$$\sum_{i=1}^t c_i = \sum_{i=1}^t \log_2\left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right)\quad (16)$$

4.3 Optimally MIMO Power Allocation

This is to maximize the capacity i.e.

$$\text{Maximize (capacity or rate)} = \max \sum_{i=1}^t \log_2\left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right)$$

Subject to constraints,

$$\sum_{i=1}^t P_i = P\quad (17)$$

This is called constraints maximization problem to solve this, I have to consider Lagrange multiplier.

Let,

$$f = \sum_{i=1}^t \log_2\left(1 + \frac{P_i \sigma_i^2}{\sigma_n^2}\right) + \lambda(P - \sum P_i)\quad (18)$$

(λ = Lagrange multiplier)

Differentiate w.r.t P_i

$$\begin{aligned} \frac{df}{dP_i} &= 0 \\ \frac{\sigma_i^2/\sigma_n^2}{\left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right)} + \lambda(-1) &= 0 \\ \frac{\sigma_i^2/\sigma_n^2}{\left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right)} &= \lambda \\ \frac{\sigma_i^2}{\sigma_n^2} \left(\frac{1}{\lambda}\right) &= \left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right) \\ \frac{1}{\lambda} &= \frac{\sigma_n^2}{\sigma_i^2} \left(1 + \frac{P_i\sigma_i^2}{\sigma_n^2}\right) \\ \frac{1}{\lambda} &= \frac{\sigma_n^2}{\sigma_i^2} + P_i \\ P_i &= \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}\right) \end{aligned} \tag{19}$$

Considering $i = 1, 2, 3, 4 \dots \dots \dots t$

$$\begin{aligned} P_1 &= \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_1^2}\right)^+ \\ P_2 &= \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_2^2}\right)^+ \\ &\vdots \\ P_t &= \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_t^2}\right)^+ \end{aligned}$$

Now here all the power depends upon $\frac{1}{\lambda}$ so how to find $\frac{1}{\lambda}$ or λ
 Using the condition of total power allocation to find more about the λ

$$P = \sum_{i=1}^t \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}\right)^+ \tag{20}$$

Now optimally power allocation P_i is given by

$$P_i = \left(\frac{1}{\lambda} - \frac{\sigma_n^2}{\sigma_i^2}\right)^+ \tag{21}$$

To solve the problem of capacity a water filling algorithm of ideal power allocation. We use the water filling algorithm to fill the area with water below $1/\lambda$.

4.4 Water Filling Algorithm

In multicarrier systems, the water filling algorithm is a method for dividing up the best amount of power among the many channels.

It offers the channel's best performance when combined with AWGN and ISI (Inter Symbol Interference).

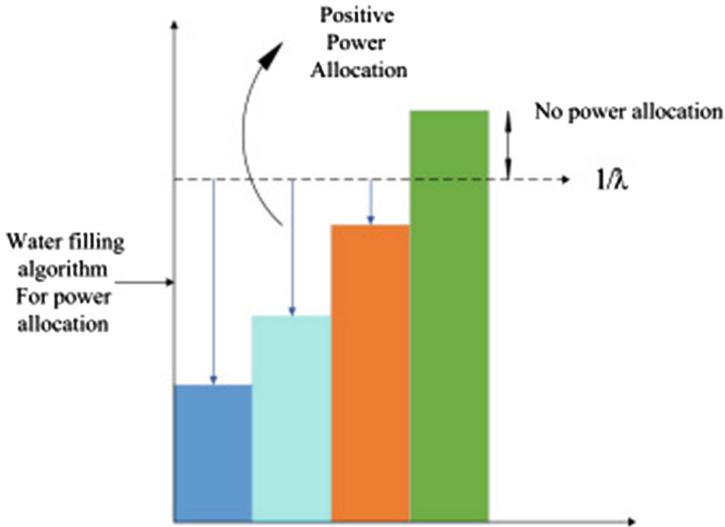


Fig. 2. Water filling algorithm (source: ScienceDirec.com)

Considering there are 4 channels here

$$\frac{1}{\lambda} > \frac{\sigma_n^2}{\sigma_1^2} \text{ i.e this power allocation is positive}$$

$$\frac{1}{\lambda} > \frac{\sigma_n^2}{\sigma_2^2} \text{ i.e this power allocation is positive}$$

Likewise similarly for 3rd but

$$\frac{1}{\lambda} > \frac{\sigma_n^2}{\sigma_4^2} \text{ i.e this power allocation is negative}$$

Now if i want to fill the area with water below $\frac{1}{\lambda}$ then first three will be filled with water is known as water filling algorithm. The water filling algorithm distributes the energy according to the channel circumstances. When the channel condition is good, more power is allocated, and when it is bad, less power is allocated.

In Fig 2 since the channel is too short, more power is given to this sub channel, but since channel 4 is too big, no power is sent to it. The water filling algorithm's output demonstrates the sub channel's features, which include a big power allocation to a significant channel gain in order to increase the sub channel's capacity or information rate. It guarantees that the effective channel can transport more data. If the circumstances

are bad enough, the transmit power cannot be given for that channel. The water filling algorithm uses useful channels and ignores useless ones.

5 Result and Discussion

The simulation results are obtained through MATLAB 2015a. We demonstrate the simulation results in this section to illustrate how well the provided water filling algorithm works. In the simulation result we consider 100 blocks make up the number of blocks, and there are 4 transmitters and receivers. The multipath channel's path count is set to 1: numBlocks. From the fig it shows that the capacity of the channel varies when SNR increases. The suggested approach consistently achieves a data stream capacity that is near to that of possible equal power allocation. When SNR approaches the 10dB the capacity may increase the 12.

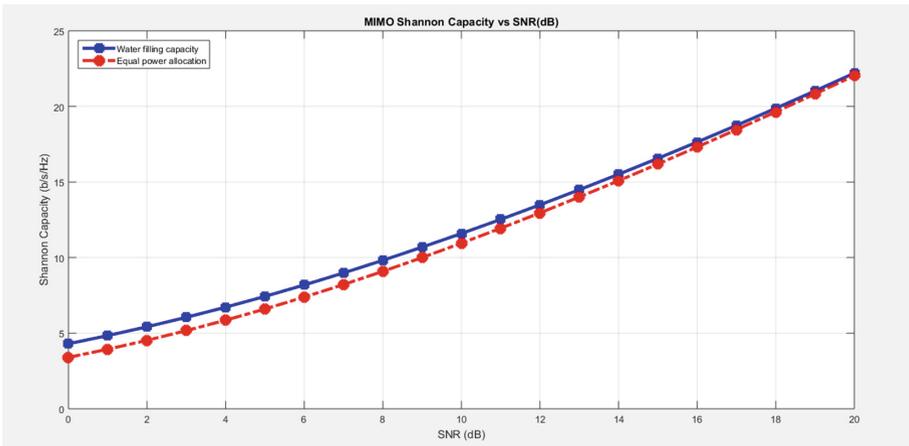


Fig. 3. Transceiver designs for a 4 × 4 mmWave system are shown to have varied Shannon capacities.

We can utilise a water filling model, determine the production of equitable power allocation. We take into account the 10db SNR for the 4 × 4 and 8 × 8 mmWave systems. The outcome in Fig. 3 demonstrates that the suggested water filling procedure outperforms earlier simulation. Additionally, when the SNR is large, the suggested equal power allocation technique resembles the water filling algorithm. When the SNR in Fig. 4 is 10 dB, the capacity gets close to 24. Similar to how the SNR increases, so does the channel's ability to hold water.

Therefore, the performance of the algorithm is well defined by the various numbers of data symbols. Additionally, the suggested method for allocating water filling power performs quite well.

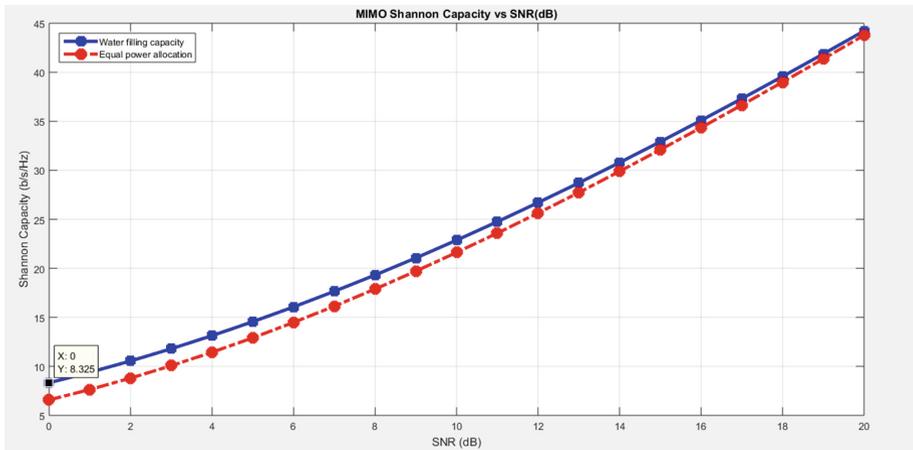


Fig. 4. Shannon capacity for an 8×8 mmWave system displaying several transceiver configurations

6 Conclusion

In this article we consider the water filling algorithm for mmWave MIMO systems. We establish the relationship between Shannon capacity and SNR. We develop water filling algorithm with proposed algorithm the decomposition of MIMO wireless channel we use the singular vector decomposition. This SVD exist for any matrix even for non square matrix where as other types of decomposition occurs for square matrices (e.g. Eigen value decomposition). For estimation of optimal power allocation in MIMO the constraint maximization problem occurs to solve this problem we use Lagrange multiplier.

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