Efficient Protective Jamming in 2D SINR Networks

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Abstract. This paper studies how intentional jamming can be used for selective hiding communication in the 2D Signal-to-Interference-plus-Noise-Ratio (SINR) model. We aim to place a set of additional jamming stations to generate interference that blocks all the signals in a specified restricted area, i.e., by making the SINR value of the genuine stations' signal below a pre-defined threshold. We aim to optimize the accuracy of the jamming strategy by minimizing the impact of the jamming stations on the area of desired genuine communication while jamming the signals in the given restricted zone. We present solutions in various network settings for uniform and non-uniform networks. As a secondary aim, we try to minimize the total energy of the jamming stations. Among others, we show that, surprisingly, it is possible to jam arbitrarily large areas by jammers using total energy arbitrarily close to zero. Our contribution is an extension of recent results for the same problem in the 1D SINR network. Let us stress, however, that a 2D environment is closer to real-life settings. Still, the 2D model turned out to be much more complex in analysis (even for the most uncomplicated cases) and required a different approach to constructing algorithms.

Keywords: SINR · information hiding · jamming

1 Introduction

This paper considers limiting genuine communication in two-dimensional SINR to protect it from eavesdropping selectively. We assume that there are some *restricted areas* where we expect that any entity should not successfully receive the genuine wireless communication signal. On the other hand, communication outside the restricted areas should be untouched. As a motivation, we can point to many scenarios, including military communication, preventing industrial espionage, privacy protection by hiding personal communication, or providing wireless services in selected workspaces without being overheard in other ones. Such

an approach is essential if it is not possible to use cryptographic mechanisms. A good example is an ad hoc network of computationally restricted devices without the possibility of pre-deployment of any cryptographic material. Finally, in some cases, one needs to hide not only the content of the message but also the fact that communication takes place (in systems providing anonymous communication).

Our paper assumes a standard SINR model (Signal to Interference plus Noise Ratio; model formulated in [1]). In the SINR model, it is assumed that the signal's power is fading with distance from the transmitting station and is impacted by interference from other network devices. It makes a model close to reality and acceptable from the technology perspective. On the other hand, analysis in this model can be challenging.

We consider two configurations of SINR networks - uniform and non-uniform in the 2D space. We construct algorithms for positioning the jamming stations under these configurations, drawing out the chosen restricted areas while reducing the unnecessary impact on the original reception zones outside the restricted areas. Below we recall the most important related work. We introduce the communication model in Sect. 2 and formalize the addressed Zone-restriction with the Max-coverage problem. Section 3 presents the algorithm for jamming network configuration for stations that can be heard only inside some area delimited by 2D convex geometric shapes in the uniform network model. Section 4 focuses on the non-uniform network and presents the 2D variant of *noisy dust* from [16]. It utilizes jamming stations with small power levels to cover arbitrary fragments of a 2D plane with interference high enough to block chosen station's signal. Notably, this approach allows the reduction of overall energy with the increase of jamming stations number, reducing its impact on protected station reception zone as well. Section 5 presents conclusions and the most important future directions.

Related Work. This contribution can be seen as an extension of [16], wherein a similar problem is considered in the 1D SINR model. The current paper uses the same notation, describing the problem statement similarly. Note, however, that the transition analysis of the 2D case is much more difficult. The class of topological regions in 2D Euclidean space is substantially richer than on the 1D line. Therefore, the presented analysis required a much more complex approach and could not be reduced to re-using the methods from [16], which relied on the interval-based representation of reception zones.

The approach taken in this paper, using jamming stations as a protective security mechanism (called *friendly jamming*, has been considered in [3,4,13,17] in the context of non-SINR models. Some similar approaches for other models were proved to be practically feasible [14]. Due to the complexity of the SINR model, our approach and the analysis needs to be completely different. Regarding the SINR, [3,15] consider a model similar to the one used in this paper, but with the additional assumption that some regions are restricted from positioning jamming stations (so-called *buffers*). In contrast, our solutions are designed to provide protection of arbitrary configurations without prior restrictions on their construction and target the optimization of the energy cost of the additional jamming stations. The directional antennas are also considered there, while this

paper focuses solely on the omnidirectional antennas, what makes fitting the noising substantially more challenging. Compared to another similar model in [17], we target the reduction of the jamming network energy consumption rather than limit the number of jamming stations. Moreover, a scheme for positioning stations in a grid relies on the combined interference of adjacent stations, which do not scale well with some of the network parameters we assume. Let us also stress that the approach change (primarily focusing on energy usage rather than limiting the number of stations) led to entirely different jamming strategies. Some other proactive approaches to securing communication in similar models can be found in [12].

Our paper can be seen as a continuation of a long list of results about the SINR model motivated by many real-life wireless networks, including 5G [5]. Note that in [7] authors consider SINR in D-dimensional space for some $D > 3$. Although such an assumption seems unjustified in the physical sense, the analysis of such a case turned out to be beneficial in analyzing algorithms of lower-dimensional spaces. Geometrical properties of the SINR model were studied by Avin et al. [2], who analyzed the properties of reception zones under the uniform SINR model, showing, among others, their convexity. Non-uniform network properties were analyzed in [7], along with a new point location algorithm, and in [8], where non-uniform SINR network model, combined with Voronoi Diagrams, proved to retain some of the valuable properties of the uniform setting. There is also a large amount of work considering the fundamental problems under the SINR model, such as *broadcasting* [9], *link scheduling* [10] or *power control* [11].

2   Model and Problem Statement

Notation. In the following paper, we use the notation presented in [16] extended and adapted to the 2D model. Let us stress that the rest of the technical part of this contribution is completely different. Indeed, we failed to re-ruse the techniques from the previous paper, possibly because the topology of $2D$ case is much richer, and from the algorithmic point of view, one needs to use subtler methods to limit communication even in regular-shaped regions.

We consider D-dimensional Euclidean spaces. Since D is always initially fixed, we indicate a metric simply by d. We denote points as $p = (p_1, \ldots, p_D)$, vectors as $\overrightarrow{v} = \overrightarrow{(v_1, \ldots, v_D)}$ and line segments between points p_0 and p_1 as $\overline{(p_0, p_1)}$. For some polygon $\mathcal{P}$, we will denote the set of its edges as $F_\mathcal{P} = \{\overline{(x, y)} : x, y \in \mathbb{R}\}$, where x, y for each edge will be consecutive vertices of the polygon $\mathcal{P}$. Moreover, for $n \in \mathbb{N}$, we use the notation $[n] = \{1, \ldots, n\}$ and a D-*ball* of radius r is denoted as $\mathcal{B}(r, p) = \{x \in \mathbb{R}^D : d(x, p) \leqslant r\}$.

Model of SINR *network*
The SINR **network** is a tuple $\mathcal{A} = \langle D, S, N, \beta, P, \alpha \rangle$, where:

- $D \in \mathbb{N}^+$ is the dimension of the network,
- $S = \{s_1, \ldots, s_n\}$ is a set of positions of stations in $\mathbb{R}^D$,
- $N > 0$ is an ambient background noise (fixed real number),

- $\beta \geqslant 1$ is the reception threshold (fixed real number),
- $P : S \rightarrow \mathbb{R}$ is a stations' power function; by $P_i = P(s_i)$ we denote the power of station s_i,
- $\alpha \geqslant 2$ is a path-loss parameter (fixed real number).

For a network $\mathcal{A}$, we define the SINR function for station $s_i \in S$ and a point $x \in \mathbb{R}^D \backslash S$ as:

$$\text{SINR}_{\mathcal{A}}(s_i, x) = \frac{P_i \cdot d(s_i, x)^{-\alpha}}{N + \sum\limits_{s_j \in S \backslash \{s_i\}} P_j \cdot d(s_j, x)^{-\alpha}} \ .$$

If a network $\mathcal{A}$ is known from a context, we simplify the notation to $\text{SINR}(s, x)$ for any station s. For $x \in S \backslash \{s_i\}$ we put $\text{SINR}(s, x) = 0$ and it is not defined for $x = s_i$. The model in which $N = 0$ is called SIR. Therefore we replace the SINR function/model with SIR whenever it is admissible.

We define a *reception zone* of some station s in a network $\mathcal{A}$ as the space where communication of the station s can be correctly received and we denote it as $H_s^{\mathcal{A}} = \{x \in \mathbb{R}^D : \text{SINR}_{\mathcal{A}}(s, x) \geqslant \beta\}$. $H_i^{\mathcal{A}}$ will be equivalent to $H_{s_i}^{\mathcal{A}}$. Finally, we define a *range* of station s for a network with positive noise value ($N > 0$) as $\text{range}(s) = \left(P(N\beta)^{-1}\right)^{\frac{1}{\alpha}}$, which maximizes the radius of reception zone of s in the network consisting of the single station s. This value is also an upper bound for the possible range of s while other stations are present in the network. Due to the lack of the noise component in the SIR model, the range definition does not apply.

Formulation of the Zone-restriction with Max-coverage Problem. For a network $\mathcal{A}$, there is given a *restricted area* $\mathcal{R}$: a subset of the space, wherein no station should be heard. In other words, in all points in $\mathcal{R}$, the SINR function of all stations in the set S has to be lowered below the threshold β. It can be done using two techniques. The first is to modify the network parameters – one can increase the threshold value β, decrease the stations' powers, or increase the path-loss parameter α. Second, we can add special *jamming stations* to the network to generate interference and change the shapes of the reception zones of the original set of stations in the network $\mathcal{A}$. An illustration of such approaches for a single broadcasting station is presented in Fig. 1.

Assume that there is a network $\mathcal{A} = \langle D, S, N, \beta, P, \alpha \rangle$ and some subspace $\mathcal{R} \subset \mathbb{R}^D$ representing a *restricted area* to be excluded from any communication involving stations from S. The problem of Zone-restriction with Max-coverage is to find a set of *jamming stations* $\mathcal{J} = (S^{(J)}, P^{(J)})$ with positions in $S^{(J)}$ and powers defined by the function $P^{(J)}$ in such a way that the resulting network $\mathcal{A}^{\mathcal{J}} = \langle D, S^{(J)} \cup S, N, \beta, P \cup P^{(J)}, \alpha \rangle$ satisfies the following two conditions (1 and 2).

Condition 1. $S^{(J)}$ *correctly protects* $\mathcal{R}$, i.e. $(\forall \, s \in S)(\forall \, x \in \mathcal{R}) \, \text{SINR}(s, x) < \beta$.

Note that Condition 1 itself could be trivially solved by adding single stations with appropriately high transmission powers in every connected component of
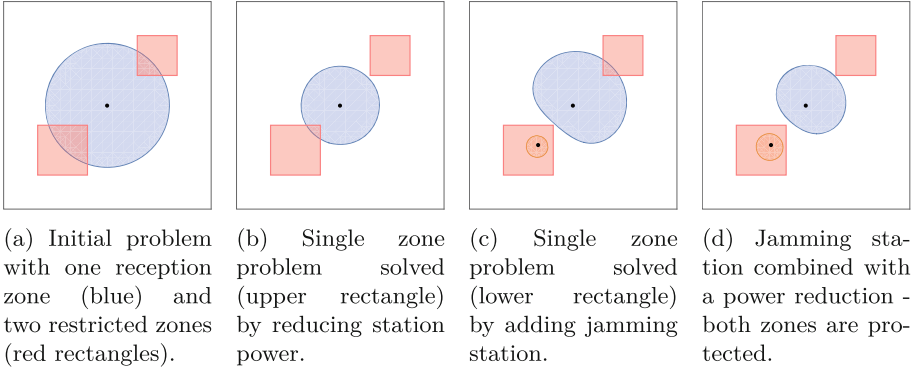
(a) Initial problem with one reception zone (blue) and two restricted zones (red rectangles).

(b) Single zone problem solved (upper rectangle) by reducing station power.

(c) Single zone problem solved (lower rectangle) by adding jamming station.

(d) Jamming station combined with a power reduction - both zones are protected.

Fig. 1. Sample problem for a single broadcasting station.

the restricted area within the ranges of broadcasting stations. It would, however, significantly suppress the desired communication in the reception zones of the genuine network. In order to control the above-undesired issue, we define a yardstick called a *coverage* – specifying how new reception areas correspond to their original sizes, excluding the restricted area.

Condition 2. $S^{(J)}$ *maximizes the following coverage (ratio) formula:*

$$\mathrm{Cover}(\mathcal{J}, \mathcal{A}) = \left| \bigcup_{s_i \in S} \left(H_i^{\mathcal{A}^{\mathcal{J}}} \cap (H_i^{\mathcal{A}} \setminus \mathcal{R}) \right) \right| \cdot \left| \bigcup_{s_i \in S} H_i^{\mathcal{A}} \setminus \mathcal{R} \right|^{-1} ,$$

where $|A|$ denotes the measure (volume) of a set A. The inverted part is the size of the maximal area in which the station's signal can be received, excluding the restricted areas. The first part is the size of the real reception area with jamming. Namely, for each station, we consider $H_i^{\mathcal{A}^{\mathcal{J}}}$, which is cropped to the maximal area where s_i can be heard i.e. $(H_i^{\mathcal{A}} \setminus \mathcal{R})$. Note that $\mathrm{Cover}(\mathcal{J}, \mathcal{A})$ is always properly defined, as long as $N > 0$. Moreover $0 \leqslant \mathrm{Cover}(\mathcal{J}, \mathcal{A}) \leqslant 1$. To summarize, the problem considered in this paper is specified as follows:

> **Zone-restriction with Max-coverage problem:** *For a given network $\mathcal{A}$ and a restricted area $\mathcal{R}$, find a set of jamming stations and their powers, $\mathcal{J} = (S^{(J)}, P^{(J)})$, correctly protecting $\mathcal{R}$ and maximizing $\mathrm{Cover}(\mathcal{J}, \mathcal{A})$.*

We also would like to minimize a *total (jamming) power*, defined as

$$\mathrm{Cost}(\mathcal{J}) = \sum_{s \in S^{(J)}} P^{(J)}(s) .$$

3   Uniform Networks Jamming

In this section, we consider networks of the form $\mathcal{A} = \langle D = 2, S, N, \beta, P \equiv 1, \alpha \rangle$, i.e., *uniform networks*, for which every station will have identical power. Without a loss of generality, this can be reduced to $((\forall s \in S)(P(s) = 1))$. Such networks

have nice properties as described in [2], and some of the calculations simplify as we can remove power parameters. We will start by describing the two stations' mutual impact when positioned next to each other in this model in Subsect. 3.1, and in the following sections, we will present different jamming approaches for more specific network configurations.

3.1   Two Stations in the Uniform Model

In the following lemma, we describe how a single jamming station can split the plane into two half-planes, such that one is jammed.

Lemma 1. *For a network $\mathcal{A} = \langle D = 2, S = \{s_0, s_1\}, N, \beta, P \equiv 1, \alpha \rangle$ and some point $b = (b_x, 0)$, where $s_0 = (0,0)$ and $s_1 = \left(b_x \left(1 + \beta^{\frac{1}{\alpha}} \right), 0 \right)$, for any point $p \in \{(a,b) \in \mathbb{R}^2 : a \geqslant b_x\}$:*

- *$\mathrm{SIR}(s_0, p) \leqslant \beta$,*
- *$\mathrm{SINR}(s_0, p) < \beta$, for $N > 0$.*

Proof. At first, we are trying to find the distance $x = d(s_1, b)$, such that $\mathrm{SIR}(s_0, b) = d(s_0, b)^{-\alpha} d(s_1, b)^{\alpha} = b_x^{-\alpha} x^{\alpha} = \beta$. This will give us $x = b_x \beta^{\frac{1}{\alpha}}$. Now examine the point $b^* = (b_x, h)$, located on the line perpendicular to the segment $\overline{s_0 s_1}$ and crossing the point b. The distances from b^* to stations s_0 and s_1 are equal to $d(s_0, b^*) = \sqrt{b_x^2 + h^2}$ and $d(s_1, b^*) = \sqrt{x^2 + h^2}$ respectively, for $h = d(b, b^*)$. A value of SIR for s_0 and such points take the form of:

$$\mathrm{SIR}(s_0, b^*) = \frac{d(s_0, b^*)^{-\alpha}}{d(s_1, b^*)^{-\alpha}} = \left(\frac{x^2 + h^2}{b_x^2 + h^2} \right)^{\frac{\alpha}{2}} = \left(\frac{b_x^2 \beta^{\frac{2}{\alpha}} + h^2}{b_x^2 + h^2} \right)^{\frac{\alpha}{2}}.$$

For $h = 0$ we get $b^* = b$ and $\mathrm{SIR}(s_0, b^*) = \beta$. On the other hand, for $h > 0$, we get:

$$\frac{\mathrm{SIR}(s_0, b^*)}{\beta} = \left(\frac{b_x^2 \beta^{\frac{2}{\alpha}} + h^2}{b_x^2 \beta^{\frac{2}{\alpha}} + h^2 \beta^{\frac{2}{\alpha}}} \right)^{\frac{\alpha}{2}} \leqslant 1,$$

as $\beta \geqslant 1$; and strict inequality for $\beta > 1$. Replacing SIR with SINR, where $N > 0$, also gives us strict inequality. Realize, that any point (x^*, y^*), such that $x^* > b_x$, will be closer to s_1 and further away from s_0 than some point $b^* = (b_x, y^*)$, meaning that $\mathrm{SINR}(s_0, (x^*, y^*)) < \mathrm{SIR}(s_0, (x^*, y^*)) < \mathrm{SIR}(s_0, (b_x, y^*)) \leqslant \beta$.   □

From Lemma 1, we immediately conclude that one can configure the position of jamming station s_1 for an arbitrary line and a given station s_0 in such a way that it guarantees the limitation of s_0's reception zone to one side of this line.

3.2   Jamming the Enclosing Area

Let us define a class of *enclosing* restricted areas, which will surround one or more jamming stations. In this class, let us define two subclasses - *polygonal*, denoted as $\mathcal{R}_{\mathcal{P}}^{\mathrm{ep}} = \mathbb{R}^2 \setminus \mathcal{P}$, where $\mathcal{P}$ is a convex polygon and *circular*, denoted as $\mathcal{R}_{(x,y),r}^{\mathrm{ec}} = \mathbb{R}^2 \setminus \mathcal{B}(r, (x, y))$.

Algorithm 1: Assign 2D uniform jamming stations

Algorithm `AssignUniformJammingStations`$(\mathcal{P}, s)$

    $S^{(J)} \leftarrow \{\}$

    for $\overline{(x_j, y_j)} \leftarrow F_{\mathcal{P}}$ **do**

        $l_j^d \leftarrow$ `GetLine`(x_j, y_j)

        $l_j^p \leftarrow$ `GetPerpendicularLine`(l_j^d, s)

        $b_j \leftarrow$ `GetLinesCrossingPoint`(l_j^d, l_j^p)

        $s_j \leftarrow s + \overrightarrow{(b_j - s)} \cdot \left(1 + \beta^{\frac{1}{\alpha}}\right)$

        $S^{(J)} \leftarrow S^{(J)} \cup \{s_j\}$

    return $S^{(J)}$

Starting with the *enclosing polygonal* area, we will focus on the problem of a single station s inside some polygon $\mathcal{P}$, and we want to block the station's signal outside the polygon's boundaries. The following functions are used in the algorithm:

- `GetLine`(x, y) creates a line, which includes the segment $\overline{(x, y)}$,
- `GetPerpendicularLine`(l, s) generates a line passing through the point s and being perpendicular to the line l,
- `GetLinesCrossingPoint`(l, l') calculates the position of the crossing point for the lines l and l'.

The algorithm uses Lemma 1 on each of the polygon edges to position one station on the opposite side of the edge from the s position and within the distance, which will provide enough interference along the edge to block a signal of s.

Theorem 1. *For a network* $\mathcal{A} = \langle D = 2, S = \{s\}, N, \beta, P \equiv 1, \alpha \rangle$, *a station* $s \in \mathcal{P}$ *and some restricted area* $\mathcal{R}_{\mathcal{P}}^{ep} = \mathbb{R}^2 \setminus \mathcal{P}$, *where* $\mathcal{P}$ *is a convex polygon, which encloses* s, *Algorithm 1 returns a set of jamming stations' positions* $S^{(J)}$ *such that the set of jamming stations* $\mathcal{J} = \{S^{(J)}, P \equiv 1\}$ *correctly protects restricted area* $\mathcal{R}_{\mathcal{P}}^{ep}$.

Proof. The algorithm constructs a straight line for each polygon segment, splitting space into two half-planes. Then the positioning of jamming station s_j for such a segment is done according to the scheme presented in Lemma 1, which guarantees that all points on the half-plane at the opposite side of the line to station s, are outside its reception zone. Since we operate for all segments of the convex polygon, all of these half-planes could be united into the restricted area $\mathcal{R}_{\mathcal{P}}^{ep}$. An additional interference introduced from other stations can only reduce the reception zone, so the restricted area will be correctly protected. □

This approach works well for the areas given as the convex polygon, but we cannot apply it directly when the restricted areas contain some curvy or circular fragments. Nevertheless, if we assume that some station s is in the center of some circular enclosing area, it can be solved by applying the method from Fact 1.

Fact 1. *For a network $\mathcal{A} = \langle D = 2, S = \{s\}, N, \beta, P \equiv 1, \alpha \rangle$, a station s and some restricted area $\mathcal{R}_{s,r}^{ec} = \mathbb{R}^2 \setminus \mathcal{B}(r, s)$, Algorithm 1 with a regular polygon $\mathcal{P}$, inscribed into the disk $\mathcal{B}(r, s)$, as an input, returns a set of jamming stations' positions $S^{(J)}$ such that a set of jamming stations $\mathcal{J} = \{S^{(J)}, P \equiv 1\}$ correctly protects restricted area $\mathcal{R}_{s,r}^{ec}$.*

By inscribing the polygon into the circular area, we can directly apply the Algorithm 1, and it will correctly block the signal outside the polygon. We can use different n-gons as the inscribed polygons. The choice of n impacts the cost (i.e., $\mathrm{Cost}(\mathcal{J}) = n$) and the coverage. In Fig. 2, we present numerical results for some of the regular polygons. The coverage of a chosen regular polygon can be bounded using Lemma 2.
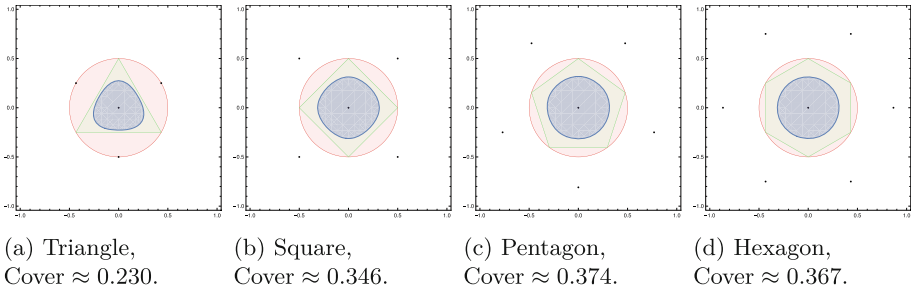

(a) Triangle, Cover ≈ 0.230.   (b) Square, Cover ≈ 0.346.   (c) Pentagon, Cover ≈ 0.374.   (d) Hexagon, Cover ≈ 0.367.

Fig. 2. Approximations of different circular shapes. Red spaces represent the initial disks, green spaces – the polygons – and blue spaces are the final reception zones. (Color figure online)

Lemma 2. *Let s be a single broadcasting station and $0 < r < \mathrm{range}(s)$. If a restricted area is given by $\mathcal{R}_{s,r}^{ec} = \mathbb{R}^2 \setminus \mathcal{B}(r, s)$ and a jamming network $\mathcal{J}$ is created by Algorithm 1 for some regular n-gon $\mathcal{P}$, then coverage of the returned network with a set of jamming stations $\mathcal{J}$ satisfies:*

$$\frac{(b(\beta b^\alpha N + n)^{-\frac{1}{\alpha}})^2}{r^2} \leqslant \mathrm{Cover}(\mathcal{J}, \mathcal{A}) \leqslant \frac{|\mathcal{P}|}{\pi r^2} \ ,$$

where b is the length of the polygon's apothem (the distance between s and sides of the polygon $\mathcal{P}$).

The upper bound is obvious from Fact 1 — we limit the maximal reception zone by some polygon $\mathcal{P}$. The lower bound can be calculated by approximating the maximal range of station s in a direction to one of the jamming stations s_j. It is realized by modifying the resulting network, which assumes that all jamming stations are placed in the same point s_j (this trick effectively increases the power of s_j n times). It allows us to calculate the station range in this scenario. We skip the details of this proof due to space limitations.

4   Noisy Dust for Non-uniform Networks

This section considers non-uniform networks, wherein the reception zones can be concave, increasing the analytic complexity. We apply the *noisy dust* approach from [16] to flood the restricted area with jamming stations having small power levels. Note that despite the similarity of the problem and jamming strategy, the 2D case technically significantly differs from considerations in [16].

4.1   Single Station Effective Jamming Range

Consider a single station $s = (0,0)$ with power $P(s) = 1$ and some *border point* $b = (b_x, 0)$ such that $0 < b_x < \text{range}(s)$. Let us place a jamming station $s_j = (b_x(1 + F_j), 0)$ where $F_j = (P_j \beta)^{\frac{1}{\alpha}}$ (from now on, we tacitly assume that $P(s_j) = P_j$) and $r = b_x F_j$ (see the arrangement in Fig. 3a). Note that we require $F_j < 1$, so we keep the $\alpha \geqslant 2$ and $P_j < \beta^{-1}$ (what also corresponds to the forementioned property $P_j \ll P$). Clearly the segment $\overline{(b_x, s_j)}$ is jammed. The disk $\mathcal{B}(s_j, r)$ could be used as an initial approximation of a space, where a single disturbing station can effectively jam the signal emitted by s – however, it would be imprecise if we would compare it with the real effective jamming space (see Fig. 3b - blue space denotes $\mathcal{B}(r, s_j)$ and a green curve represents a boundary of the maximal region, where s_j correctly jams s).
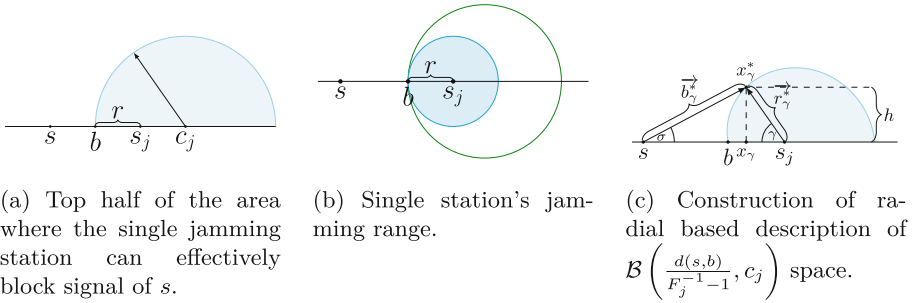

(a) Top half of the area where the single jamming station can effectively block signal of s.

(b) Single station's jamming range.

(c) Construction of radial based description of $\mathcal{B}\left(\frac{d(s,b)}{F_j^{-1}-1}, c_j\right)$ space.

Fig. 3. Effective jamming range construction.

In the SIR model, the shape of the space, where s_j blocks the signal of s, is expected to form some oval, irregular shape. Surprisingly, it forms a circle, centered at $c_j = \left(b_x + \frac{d(s,b)}{F_j^{-1}-1}, 0\right)$.

Theorem 2. *Let $\mathcal{A} = \langle D = 2, S = \{s, s_j\}, N, \beta, P, \alpha \rangle$ be a network, then for any $x \in \mathcal{B}\left(\frac{d(s,b)}{F_j^{-1}-1}, c_j\right)$, the condition $\text{SINR}(s, x) \leqslant \beta$ is satisfied.*

Fix s_j and s. We are looking for such points x, that $\text{SIR}(s, x) = \beta$. These points form the border of the area where the signal is blocked (by continuity of SIR

with respect to the tested position). We are using the radial approach, i.e., we create a vector $\overrightarrow{r_\gamma^*}$ in some direction ($\gamma \in [0, \pi]$ is an angle between the segment $\overline{ss_j}$ and the vector), such that $x_\gamma^* = s_j + \overrightarrow{r_\gamma^*}$ and $\mathrm{SIR}(s, x_\gamma^*) = \beta$ (note that SIR is monotonous in the direction of the vector, so there is exactly one appropriate x_γ^*). This method is presented in Lemma 3 with construction depicted in Fig. 3c. We must analyze only half of the reception zone, as the other half is symmetrical.

Lemma 3. *Let $\mathcal{A} = \langle D = 2, S = \{s, s_j\}, N, \beta, P, \alpha \rangle$ be a network. For $\gamma \in [0, \pi]$, we define $r_\gamma^* = d(s, s_j)((F_j^{-2} - \sin^2 \gamma)^{\frac{1}{2}} + \cos \gamma)^{-1}$ and a point $x_\gamma^* = s_j + \overrightarrow{r_\gamma^*}$, where $\overrightarrow{r_\gamma^*} = \overline{(-r_\gamma^* \cos \gamma, r_\gamma^* \sin \gamma)}$. Then $\mathrm{SIR}(s, x_\gamma^*) = \beta$ and $\mathrm{SINR}(s, x_\gamma^*) \leqslant \beta$. Moreover, for any point $x \in \overline{s_j x_\gamma^*}$, we get $\mathrm{SINR}(s, x) \leqslant \beta$.*

Proof. Let us define a base vector $\overrightarrow{r} = \overline{(b - s_j)}$. The vector $\overrightarrow{r_\gamma^*}$ is acquired by rotating $\overrightarrow{r}$ by angle γ in clockwise direction. Obviously, if $r_\gamma^* = \|\overrightarrow{r_\gamma^*}\|$, then $\overrightarrow{r_\gamma^*} = \overline{(-r_\gamma^* \cos \gamma, r_\gamma^* \sin \gamma)}$. Let us define a new vector $\overrightarrow{b_\gamma^*} = \overrightarrow{x_\gamma^*} - s$ of length b_γ^* and the angle between $\overrightarrow{b_\gamma^*}$ and $\overrightarrow{s_j - s}$ as σ (see Fig. 3c). Note that $\sin \gamma = \frac{h}{r_\gamma^*}$, $\sin \sigma = \frac{h}{b_\gamma^*}$, $\frac{r_\gamma^*}{b_\gamma^*} = \frac{\sin \sigma}{\sin \gamma}$. Point x_γ^* has to keep the property $\mathrm{SIR}(s, x_\gamma^*) = \beta$, so $\frac{r_\gamma^*}{b_\gamma^*} = F_j = \frac{\sin \sigma}{\sin \gamma}$ and $\cos \sigma = \sqrt{1 - F_j^2 \sin^2 \gamma}$. By applying it to the $d(s, s_j) = b_\gamma^* \cos \sigma + r_\gamma^* \cos \gamma$, we get

$$d(s, s_j) = \frac{r_\gamma^* \sqrt{1 - F_j^2 \sin^2 \gamma}}{F_j} + r_\gamma^* \cos \gamma = r_\gamma^* \left(\sqrt{F_j^{-2} - \sin^2 \gamma} + \cos \gamma \right) .$$

Finally, we get: $r_\gamma^* = \frac{d(s, s_j)}{\sqrt{F_j^{-2} - \sin^2 \gamma} + \cos \gamma}$. By the properties of the construction it is guaranteed that $\mathrm{SIR}(s, x_\gamma^*) = \beta$ for any γ, so in particular $\mathrm{SINR}(s, x_\gamma^*) \leqslant \beta$. From monotonicity of s and s_j energy functions in the direction of $\overrightarrow{r_\gamma^*}$, for any point $p \in \overline{x_\gamma^* s_j}$, $\mathrm{SINR}(s, x_\delta^*) \geqslant \mathrm{SINR}(s, p)$, making all such p correctly jammed. $\square$

In the next step, we want to convert the vector representation of $\overrightarrow{r_\gamma^*}$ to a parametric one. In particular, we may specify h component of $\overrightarrow{r_\gamma^*}$, basing on the x_γ argument as $\overrightarrow{r_\gamma} = \overline{(x_\gamma, r^*(x))}$, via a function $r^*(x) = h$, where $x = d(b, x_\gamma) \in [0, d(b, x_\pi)]$. This transformation is presented in Lemma 4:

Lemma 4. *For every point x_γ^* ($\gamma \in [0, \pi]$), there exists x such that $x_\gamma^* = (b_x + x, r^*(x))$, where $r^*(x) = \left(-x^2 + \left(\frac{2d(s,b)}{F_j^{-1} - 1} \right) x \right)^{\frac{1}{2}}$ and $b = (b_x, 0)$. Moreover, $\{x_\gamma^* : \gamma \in [0, \pi]\}$ forms a half of a circle.*

Proof. Let $x_\gamma^* = (b_x + x, h)$, where $x \in [0, d(b, x_\pi)]$. We want to calculate h in this formula. It depends on angle γ as follows:

$$r_\gamma^* \cos \gamma = d(s_j, b) - x , \qquad r_\gamma^* \sin \gamma = \sqrt{(r_\gamma^*)^2 - (d(s_j, b) - x)^2} . \qquad (1)$$

Combining the previously calculated value of $d(s, s_j)$ with Eq. 1 brings

$$\left(r_\gamma^*\right)^2 = ((d(s, b) + x)^2 - (d(s_j, b) - x)^2)(F_j^{-2} - 1)^{-1} \ .$$

This equation might have two real solutions for $\left(r_\gamma^*\right)^2$. However, we consider only the positive one, which under assumptions $d(s, b) \geqslant d(s_j, b)$ and $F_j^{-2} - 1 > 0$, satisfies: $r_\gamma^* = (((d(s, b) + x)^2 - (d(s_j, b) - x)^2)(F_j^{-2} - 1)^{-1})^{\frac{1}{2}}$. Finally, we can use this result to calculate the parametrization $r^*(x) = r_\gamma^* \sin \gamma$:

$$r^*(x) = r_\gamma^* \sin \gamma = \sqrt{\left(r_\gamma^*\right)^2 - (d(s_j, b) - x)^2} = \sqrt{-x^2 + \left(\frac{2b_x}{F_j^{-1} - 1}\right) x} \ .$$

Moreover, the last formula is a geometric mean of x and $\left(\frac{2b_x}{F_j^{-1}-1} - x\right)$, hence $\{x_\gamma^* : \gamma \in [0, \pi]\}$ is a half of a circle of diameter $\frac{2b_x}{F_j^{-1}-1}$. Therefore, the considered region is in fact $\mathcal{B}\left(\frac{d(s,b)}{F_j^{-1}-1}, c_j\right)$. □

Lemmas 3 and 4 conclude the proof of Theorem 2. If we know the point b and the expected $r = \frac{d(s,b)}{F_j^{-1}-1}$, then we can calculate the power level of station s_j as: $P_j = \beta^{-1}\left(1 + \frac{d(s,b)}{r}\right)^{-\alpha} = \beta^{-1}r^\alpha d(s, c_j)^{-\alpha}$. We will use this equation in the following sections to calculate power levels for stations with fixed positions and for predefined values of r.

4.2   Noisy Dust Algorithm

Using the effective jamming range of a single station, represented by some disk, we can approximate such the disk by inscribing some hexagon inside. We may use this fact to tile the 2D regions requiring the jamming. Let us define such a hexagonal grid by $\mathcal{H} = \{h_0, h_1 \ldots\}$ where h_i are central points of equally sized regular hexagons, each with radius r and assume such grid fully covers the restricted zone inside the reception zone of some station s. Then the algorithm for positioning stations for each hexagon is defined in Algorithm 2.

The center of the hexagon can be treated as the c_j from the Theorem 2. The algorithm will position the jamming station somewhere on the line going through the $h = c_j$ and the s and assign enough power to cover the whole disk circumscribed on the hexagon with center h, providing correct protection. Correctness of the offset and power assignment comes directly from the Theorem 2 and related constructions.

One must create the hexagonal grid to use the algorithm - the process details are not part of this paper. For the algorithm to work, the grid must densely fill the restricted area region intersecting the reception zone of the protected station s (note that details of the algorithm can be aligned to protect more than one station).

Algorithm 2: Create noisy dust for $s = (0,0)$, restricted area $\mathcal{R}$ and hexagonal grid $\mathcal{H}$ with circumradius equal to r.

Algorithm `GenerateNoisyDust`$(s, \mathcal{H}, r)$

> $J \leftarrow \{\}$
> **for** $h \leftarrow \mathcal{H}$ **do**
>> $P_j \leftarrow \left(\frac{1}{\beta}\right)\left(\frac{d(s,h)-r}{r} + 1\right)^{-\alpha}$
>> $F_j \leftarrow (P_j\beta)^{1/\alpha}$
>> $s_j \leftarrow h + (r - F_j(d(s,h) - r))\left(\frac{\overrightarrow{s-h}}{d(s,h)}\right)$
>> $J \leftarrow J \cup \{s_j, P_j\}$
> **return** J

Let us consider the energy cost of Algorithm 2. We assume that the parts of the restricted area located outside of the range of s are excluded and that the required number of hexagons of circumradii r required to fill some restricted area $\mathcal{R}$ is defined as $n = (|\mathcal{R} \cap \mathcal{B}(\text{range}(s), s)| + o(A(r)))A(r)^{-1}$, where $A(r) = \frac{3\sqrt{3}r^2}{2}$ is the area of a hexagon with circumradius r (the assumption about the value of n is fulfilled in all realistic scenarios). The area of the effectively restricted region $|\mathcal{R} \cap \mathcal{B}(\text{range}(s), s)|$ is a constant ($\mathcal{R}$ and s are given a priori). It is naturally bounded by the area of the initial disk around the broadcasting station in SINR model: $|\mathcal{R} \cap \mathcal{B}(\text{range}(s), s)| \leqslant |\mathcal{B}(\text{range}(s), s)| \leqslant \pi \cdot \text{range}(s)^2$. Cumulative energy required to set up jamming stations for arbitrary $\mathcal{R}$ is given by $\sum_{i=0}^{n-1} \beta^{-1}r^\alpha d(s, c_i)^{-\alpha}$, where the circumradius of every single hexagon equals r, and each jamming station s_i is positioned in a unique hexagonal cell and vice versa. Each cell contains only one jamming station.

Observe that one can limit the value of $d(s, c_i)$ by a distance between s and the closest single hex within the hexagonal grid — let us denote it by $d_s = \min\{d(s, c_j) : j = 1, 2, \ldots, n\}$. Since $d(s, c_i) \geqslant d_s$ for any hex cell:

$$\sum_{i=0}^{n-1} \beta^{-1}r^\alpha d(s, c_i)^{-\alpha} < \sum_{i=0}^{n-1} \beta^{-1}\left(\frac{r}{d_S}\right)^\alpha = \frac{nr^\alpha}{\beta d_S^\alpha} \approx \frac{2|\mathcal{R} \cap \mathcal{B}(\text{range}(s), s)|}{3\sqrt{3}\beta d_s^\alpha}r^{\alpha-2} \ .$$

Remark that for $\alpha = 2$, this upper bound is constant $- \frac{2|\mathcal{R}|}{3\sqrt{3}\beta d_s^\alpha}$ and one can similarly find a lower bound of cumulative energy required to set up jamming stations, by substitution of d_s by its antipodal counterpart $\max\{d(s, c_j) : j = 1, 2, \ldots, n\}$ (which is also bounded by $\text{range}(s) + r$) and realizing that $n \geqslant \frac{|\mathcal{R} \cap \mathcal{B}(\text{range}(s), s)|}{A(r)}$, what shows that in this case (of $\alpha = 2$), the cumulative energy is $O(1)$ as $r \to 0^+$. On the other hand, for $\alpha > 2$, the upper bound converges to 0 as $r \to 0^+$, which upholds the zero-energy property from the 1D version of the noisy dust algorithm. When $\alpha < 2$, both upper and lower bounds are $O(r^{2-\alpha})$, as $r \to 0^+$, so in this case, the total energy usually rises along with the number of jamming stations.

We are going to check the actual coverage numerically. We consider four different scenarios for initial network configuration of $\mathcal{A} = \langle D = 2, S = \{s\}, N =$

$1.0, \beta = 1.0, P, \alpha = 3.0 \rangle$. Each experiment is conducted for hexagons with radii $r \in \{0.125, 0.25, 0.5, 1\}$ and the coverage results, along with example visualization presented in Fig. 4. One can easily see that all cases hold the property that the coverage value increases as the sizes of hexagons decrease. We see that the method might not work very well for larger sizes of hexagons in some configurations (like, e.g., the one presented in Fig. 4a), but generally, the method is quite efficient in practice.
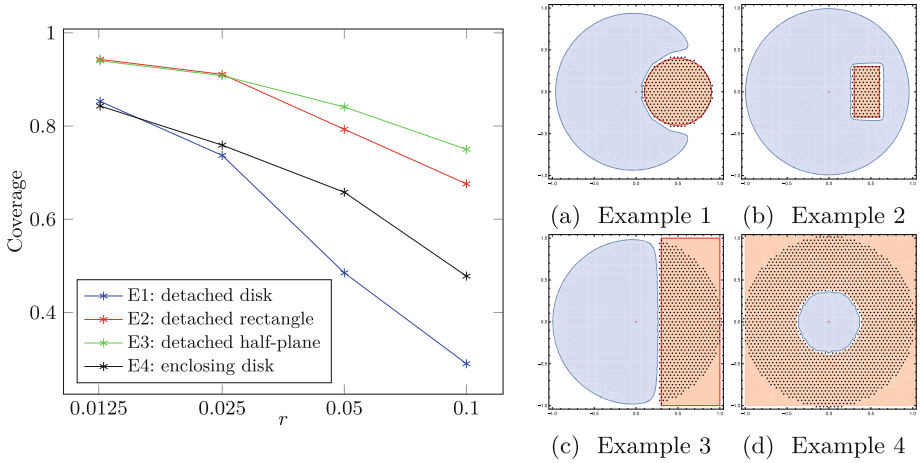

(a)   Example 1   (b)   Example 2

(c)   Example 3   (d)   Example 4

Fig. 4. Coverage obtained for four considered examples with respect to circumradius r of each hexagon in the grid and illustration of examples with $r = 0.025$.

5   Conclusions and Future Work

In our paper, we study the problem of protecting communication in the 2D SINR network. We introduced a formal, realistic model and presented algorithms usable for uniform and non-uniform network settings. The idea for designing these algorithms is to limit communication by introducing a carefully prepared noise generated collectively by a set of stations.

Even though presented solutions are introduced only for some chosen, limited scenarios, they should be capable of generalization for more complex ones since more complicated (but still realistic) cases can be represented as combinations of regular-shaped areas investigated here.

There are multiple directions in future research that can extend these results. One such is the idea of dynamic environments, where stations and restricted areas are not static space objects but can change locations and parameters with time, modeling real-world scenarios like cars or drones. Another direction would be extending the solutions to 3D or creating generic versions for any number of dimensions. Finding the energy bounds for generic configurations or tighter coverage bounds for presented solutions is also challenging. The model can also be

an object of modifications, e.g., assuming we have different receivers' sensitivity (e.g., like in [3], where adversary and legitimate receivers use different reception thresholds).

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