



The Effect of a Top Flexible Restraint on a Two-Bodies Vertical Spanning Wall

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Abstract. In unreinforced masonry structures, one of the most dangerous events that can occur during earthquakes is an out-of-plane mechanism. This type of response significantly changes if the wall is restrained by a horizontal element, like a floor, a roof or tie rods. The collapse, in this case, could take place for slipping/failure of the connection to the diaphragm or for overturning of the wall, following the formation of a crack at an intermediate height between the base and the top. Further, to evaluate the response of these kind of mechanisms, the assumption of a rigid top support can be too crude especially in case of a timber diaphragm or small diameter and large length tie rods.

In this context, in order to capture the complex dynamic behavior of the wall, formed by two stacked rigid bodies (free to rock) connected to a spring, a specific analytical model (updated to account for additional masses active on the wall only during the earthquake) is used.

For slender walls connected to a flexible restraint, the flexural out-of-plane mechanism is recurrent. These walls are common in the Emilia-Romagna region of Italy. For this reason, a building portfolio in Emilia is analyzed to derive mean and standard deviation of a log-normal distribution of the main parameters of the system.

The variation of relevant parameters is investigated, in order to evaluate the effect of the elastic restraint at the top. The results of the analysis highlighted that stiff diaphragm can significantly reduce the rotations. Additionally, the study on the effect of the wall size pointed out how the top spring causes a reverse scale effect.

Keywords: Multi-Rocking-Body Dynamic · Rocking · Top Diaphragm · Tie rods · Unreinforced Masonry · Scale Effect

1 Introduction

Paulay and Priestley [1] stated that out-of-plane failure for masonry structures is “one of the most complex and ill-understood areas of seismic analysis”. Unreinforced masonry (URM) structures are particularly vulnerable to out-of-plane mechanisms during earthquakes (Fig. 1) if the connections are inadequate [2, 3], provided that no masonry disintegration takes place [4]. However, if the walls are supported by horizontal elements like

floors, roofs or tie rods, the response to seismic ground motion can be very different. In these cases, collapse may occur as a result of slipping or failure of the diaphragm, or due to the overturning of the wall following the formation of a crack at an intermediate height between the base and the top [5–7]. While rigid floors are usually beneficial [8–10] for the earthquake performance of URM constructions, assuming a rigid top support when assessing the response of out-of-plane mechanisms may not be accurate, particularly in the case of timber diaphragms or small diameter and large length tie rods [11]. The height, at which the wall breaks, depends on the ratio between the weight due to the diaphragm and that of the wall. The larger the weight acting at the top, compared to that of the wall, the more the crack moves downwards [12].

Experimental tests by Baggio and Masiani [13] and Doherty et al. [14] assumed a rigid top restraint and developed analytical models accordingly. When the horizontal diaphragm is not rigid enough, an elastic top restraint must be introduced [15]. This boundary condition delivers a system with two DOFs, similar to the one observed in a stack of two bodies that are free at the top, as studied by Psycharis [16] and Spanos et al. [17]. Therefore, the complexity of the problem increases as four patterns (or rocking modes) are possible. Single-story and two-story one-way spanning walls that are connected to flexible diaphragms were studied by Derakhshan et al. [18], who formulated a model disregarding the thickness of the façade. This assumption was also made by Gabellieri et al. [19] in their study of a single-story URM wall. Penner and Elwood [20] conducted full-scale shaking table tests on five masonry wall specimens, which were connected to a steel frame by elastic springs. The inertia forces on the wall and spring reactions initiate the rocking motion as two semi-rigid bodies, causing a crack to form at an intermediate height. Derakhshan et al. [21] developed a three DOFs model to consider both the wall thickness and the deformation of the base diaphragm, emphasizing the strong influence of diaphragm stiffness on the response of the wall.

Prajapati et al. [22] highlighted the highly non-linear behavior of the multi-rocking-body dynamic (MRBD) model that describes the vertical spanning strip wall (formed by two stacked rigid bodies) connected to a flexible diaphragm. The complexity of the model lies in the different patterns that the system can assume during the motion, and in the transition between one and another. The MRBD model, proposed in the aforementioned study, requires an update to account for a diaphragm mass not supported by the investigated wall but that will transfer part of its seismic inertia force to the wall due to diaphragm deformability.

Then, the updated MRBD model is used to carry out a parametric analysis, to understand the influence of the different factors describing the system. For this reason, a building portfolio in Emilia (Italian region where the flexural out-of-plane mechanisms are more frequent) is analyzed to derive mean and standard deviation of a log-normal distribution of the main system parameters. Based on these data, the role of the slenderness and the size of the wall is finally investigated.



Fig. 1. Example of out-of-plane failure of URM wall connected to timber diaphragm during the 2012 Emilia, Italy earthquake.

2 Description of the Model

The MRBD model proposed in Prajapati et al. [22] is used as a starting point for the following study (Fig. 2a). The wall is assumed as an assembly of two rigid bodies and elastically restrained at the top, consequently, the system has two DOFs. The wall has finite thickness and, hence, it can take four different configurations or patterns. The heights of the lower and upper bodies are $2 h_1$ and $2 h_2$, respectively, while h_{tot} is the total height of the wall. The thickness of the wall is b but the thicknesses of the interfaces at the bottom of the lower and upper bodies are in general different and equal to $2 b_1$ and $2 b_2$, respectively, due to masonry crushing or mortar recess. The lumped mass of the diaphragm is m_d and its stiffness is k_d . Diaphragms with beams parallel to the wall may contribute with $m_d = 0$ to gravity load. However, they may develop an $m_{d,p}$ mass acting for horizontal accelerations (Fig. 2b).

As previously mentioned, the MRBD model developed in Prajapati et al. [22] does not account for this translational mass $m_{d,p}$, hence it must be modified in order to consider this additional element. The formulation of the updated model follows that presented in Prajapati et al. [22], and the equations of motion are formulated within a Lagrangian approach for each of the four patterns that characterized the system. Also in this case, the motion of the system (therefore of each point) is described by the rotations of the two bodies, namely θ_1 for the lower body and that θ_2 for the upper body. The scalar parameters of kinetic energy T and potential energy V , as well as non-conservative generalized forces Q_i are computed, to assemble the Lagrangian equation of motion:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}_i} \right) - \frac{\partial L}{\partial \theta_i} = Q_i \quad i = 1, 2 \quad (1)$$

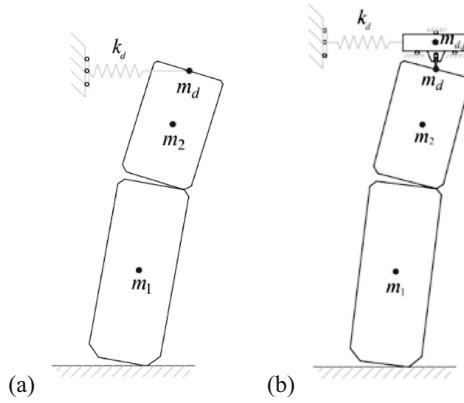


Fig. 2. Model of the vertical spanning wall elastically restrained at the top (pattern 1b): a) model by Prajapati et al. [22]; b) updated model.

where t is the time, $L = T - V$, $\dot{\theta}_i$ is the angular velocity, θ_i is the angular displacement, $i = 1$ or 2 refers to the lower or the upper body, respectively.

In this case the kinetic energy is updated to consider the top additional mass, and it takes the form:

$$T = \sum_{i=1}^2 \left[\frac{1}{2} (m_i |\mathbf{v}_{Gi}|^2 + I_{Gi} \dot{\theta}_i^2) \right] + \frac{1}{2} m_d |\mathbf{v}_C|^2 + \frac{1}{2} m_{d,p} v_{C,x}^2 \quad \text{where } |\mathbf{v}| = \sqrt{v_x^2 + v_y^2} \tag{2}$$

where m_i is the mass of the i -th body, I_{Gi} is the polar inertia moment of the i -th body about its center of mass G_i , \mathbf{v}_{Gi} and \mathbf{v}_C are respectively the velocity vectors of the i -th center of mass and of point C where the diaphragm mass is applied. The additional mass acts only in the horizontal direction; hence the potential energy is not updated and it remains in the same form of that proposed by Prajapati et al. [22]. Differently, the non-conservative forces, must consider the additional mass $m_{d,p}$; to this purpose the virtual work W is modified similarly to the kinetic energy.

During the motion of the system a pattern change can occur for two reasons: a) sudden accelerations; b) impacts. In the first case the horizontal translational mass has a strong influence; hence the detection of this pattern change must be properly updated. To detect this type of pattern change, it is necessary to determine a threshold acceleration. To this purpose, it is necessary to compare the internal moment M_I , which typically stabilizes the bodies, with the external moment M_E , which tends to overturn the bodies. Also in this case, the procedure to determine the threshold acceleration follows that one proposed in Prajapati et al. [22] appropriately modified to consider the top additional mass.

3 Parametric Study

Here, the MRBD model is used to explore the response of a vertical spanning wall elastically restrained at the top when subject to earthquakes. The mechanical and geometrical characteristics used in the following analysis are derived from existing buildings to investigate the response of “real life” façades.

For a slender façade connected to a diaphragm, the flexural out-of-plane mechanism is recurrent. These façades are more common in Emilia-Romagna compared to other Italian regions, because in the former brickwork is used due to the prevalence of alluvium soil and the lack of natural stone. A building portfolio located in Emilia (52 façades) is analyzed to derive mean and standard deviation of a log-normal distribution of the geometrical parameters. As input, seven natural accelerograms compatible with the code spectrum of Mirandola, located in Emilia, are used (Fig. 3). Further, a return period of 475 years and a soil type C are assumed. The code REXEL was used for record selection [23].

Floor orientation substantially influences the stiffness of the diaphragm (hence the response of the system), which was estimated based on the literature collection in Giresini et al. [24]. Therefore two configurations are considered: 1) the floor is parallel to the façade, and therefore only a conventional 10% of its mass develops a vertical force on the façade, while the rest will act only as a seismic mass (i.e. it will only be able to translate in the horizontal direction); 2) the floor is perpendicular to the façade, in this case the beams rest on the façade, and therefore 50% of the mass of the floor acts both as a gravity mass and a seismic mass.

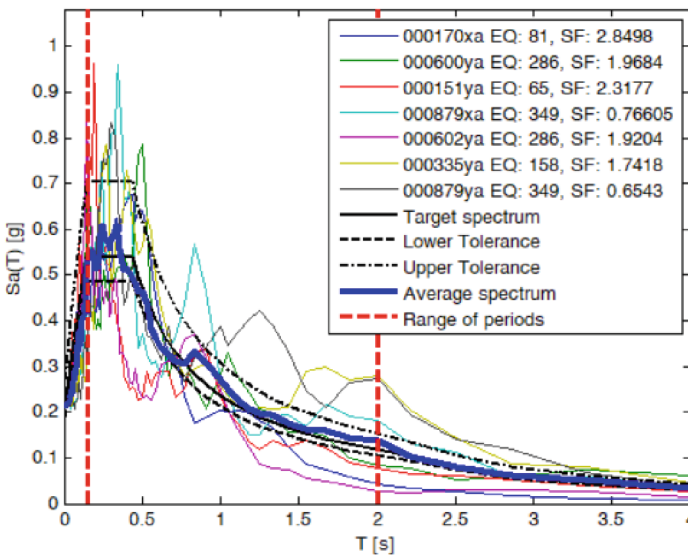


Fig. 3. Elastic response spectra compatible with the code spectrum of Mirandola (Emilia-Romagna, Italy)

3.1 Description of the Parametric Analysis

An analysis was carried out with the goal of determining the impact of each system parameter on the dynamic response. The parametric analysis is carried out by defining a reference façade for both configurations (parallel floor and perpendicular floor). Then, for each investigated parameter (Table 1), the response of the reference façade is evaluated for three cases: 1) Reference façade with the parameter of interest equal to the value corresponding to the lognormal mean, me , minus the lognormal standard deviation, σ ; 2) Reference façade with the parameter of interest equal to the value corresponding to the mean value; 3) Reference façade with the parameter of interest equal to the value corresponding to the mean value plus the standard deviation. For each case, the response of the façade is evaluated as the slenderness (h_{tot} / b) varies. As will be shown in the following, slenderness mildly affects the results unless it becomes rather large (> 12 or more). The response to the seven input records mentioned above is summarized by the median maximum absolute normalized rotation, reported in the plots of the parametric analysis. Hence, for each reference case, 189 dynamic analyses are performed.

Table 1. Values of the investigated parameters corresponding to lognormal mean, me , and standard deviation, σ .

	b	h_{tot} / b	Floor parallel	Floor perpendicular
			k_d	k_d
	m	-	kN/m	kN/m
$(me - \sigma)$	0.35	9	500	400
(me)	0.50	12	1200	1400
$(me + \sigma)$	0.70	15	3100	4600

3.2 Results of the Parametric Analysis

The parametric study on the thickness b of the wall (Fig. 4), pointed out that, especially for slender walls, the contribute of the flexible diaphragm produces a reverse scale effect: the bigger the wall the less relevant the role of the diaphragm because the inertia force is much larger than the elastic top force. In the configuration where the floor is parallel, this phenomenon is amplified.

The effect of the diaphragm stiffness is also investigated. For the configuration where the floor is assumed perpendicular to the façade, the parametric analysis (Fig. 5) highlighted that to an increase in stiffness is associated a decrease in terms of rotations. This trend is usually true also for the configuration where the floor is assumed parallel to the façade, although for extremely slender façades the benefic effect of this stiffness decreases.

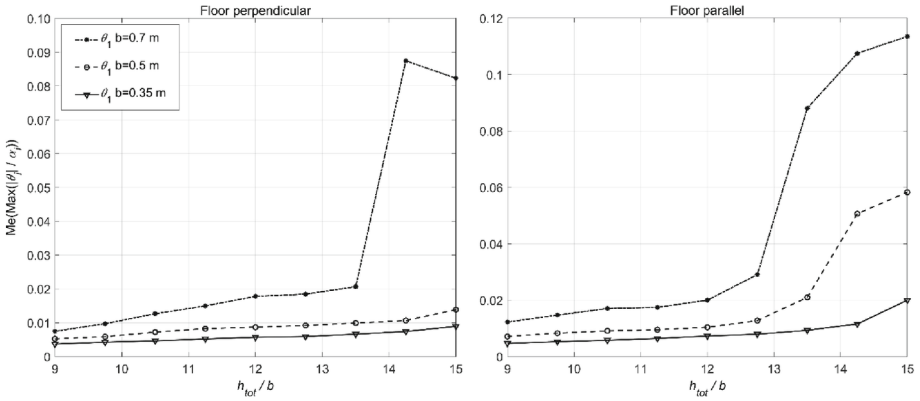


Fig. 4. Median, over seven accelerograms, of the maximum absolute non-dimensional rotations for the bottom body, varying floor orientation with respect to façade, slenderness, wall size.

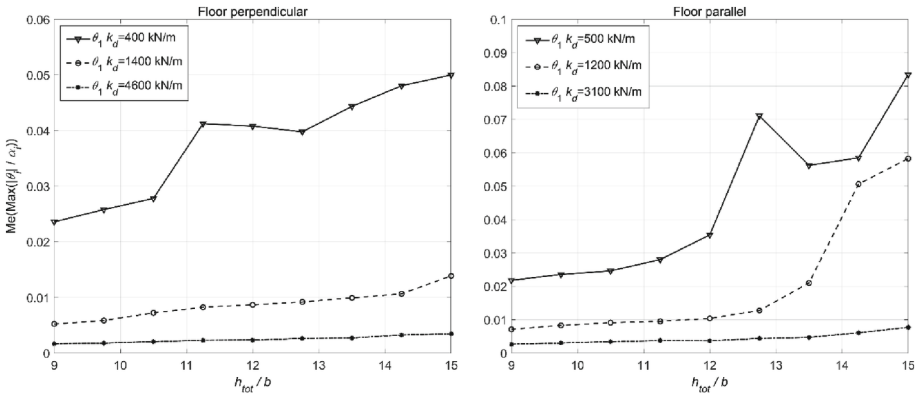


Fig. 5. Median, over seven accelerograms, of the maximum absolute non-dimensional rotations for the bottom body, varying floor orientation with respect to façade, slenderness, floor stiffness.

4 Conclusions

The out-of-plane response of an unreinforced masonry strip wall elastically supported at the top was studied in this paper, accounting for additional masses active on the wall only during the earthquake. Hence, two configurations of the floor orientation were investigated: a) floor parallel to the façade; b) floor perpendicular to the façade. A numerical analysis is carried out to understand the effect of the different parameters that characterize the system on the global response. Therefore, the preliminary investigation of a building portfolio in Emilia-Romagna (region of Italy) was conducted to calculate the mean and standard deviation of a log-normal distribution for the main parameters of the system. The nonlinear time history analyses were performed using seven natural accelerograms compatible with the code spectrum of the city of Mirandola, located in Emilia.

The analysis outcomes indicated that, for this location, the response of the system is deemed dangerous only when there is a considerable height-to-thickness ratio. Additionally, the investigation on the diaphragm influence revealed that the top spring significantly affect the response of the system. The diaphragm stiffness can notably decrease rotations and, as a result, lower the risk of overturning. Furthermore, when the wall size effect is explored, an inverse scale effect is observed, making the top restraint stiffness less relevant for large walls.

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