



# On the Use of Finite Element Method and LEFM to Assess Bearing Capacity of Historic Notched Timber Beams at Arbitrary Location

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**Abstract.** As notched beams are often and commonly found in historic structures, the assessment of potential bearing capacity is of utmost importance. In practice a few equations are found in the standards dealing with the problem, nevertheless, common use is only on tension side near the ends of the beams. To fill the gap and also allow for computing the critical force of arbitrary notched beam under arbitrary loads the authors use energy-based approach along with FEM. The idea is based on simple virtual crack closure technique and calculation of the work consumed by the crack to grow. Such a method is able to assess the bearing capacity of the whole beam. This procedure allows for a detailed analysis of the problem in real structure. The method described above is documented in the paper and a numerical model is constructed in ANSYS. The assessed values are compared to experimental work on real timber beams. Experimental work consisted of testing 9 beams (softwood; Norway spruce) with dimensions  $0.2 \times 0.25 \times 5.9$  m, a cut in the central part of the beam and 4-point bending load scenario. The numerical analysis is able to assess the critical load relatively well. Results are critically evaluated and drawbacks are discussed in-depth.

**Keywords:** LEFM · Bearing capacity · Notch · FEM

## 1 Introduction

Although the use of notches is not recommended in new structures and their use indirectly is so limited by the quite low bearing capacity according to the standard, they can be found very often in historic structures. The typology of carpenter like connections implies such kind of irregularities. In fact, the notches appear in nearly all historical structures and Fig. 1 demonstrates, what could be considered as a notch. Basically, all cuts made in wood including (and mainly) carpentry joints, but very often also the supports can form some kind of notch or notch-like problem. If the problem is seen from this perspective, the importance of the work is clear. During reconstructions of historical sites such an approach can be important.

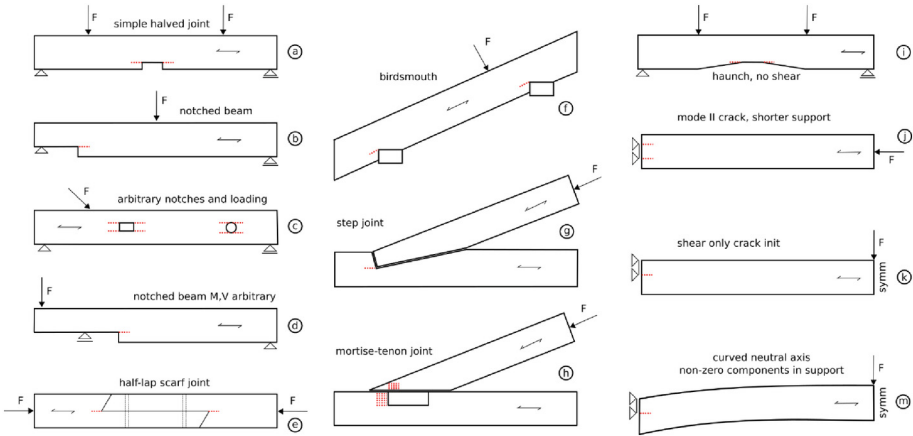


Fig. 1. What could be considered a notch?

Usually, when FEM is used for the analysis of the structural members, the analysis remains successful unless sharp edges are included in the problem. If such stress concentrations are evident, the stress-based analysis no longer becomes applicable in terms of utilization grades on stress level. This disadvantage can successfully be solved using energy-based solutions, which provide even in this case relatively stable solutions. According to Fig. 2, an educational and theoretical example of a 2D model is presented with applied bending momentum, equipped with a crack and variation of the mesh density (plain stress with thickness 1 mm, C24, 0.2 m height). As demonstrated, stress-based results vary and depend strongly on the mesh density, while energy-based method is stable over the whole range of element sizes. The level of verification is now shifted to  $M_f$  as theoretical critical momentum.

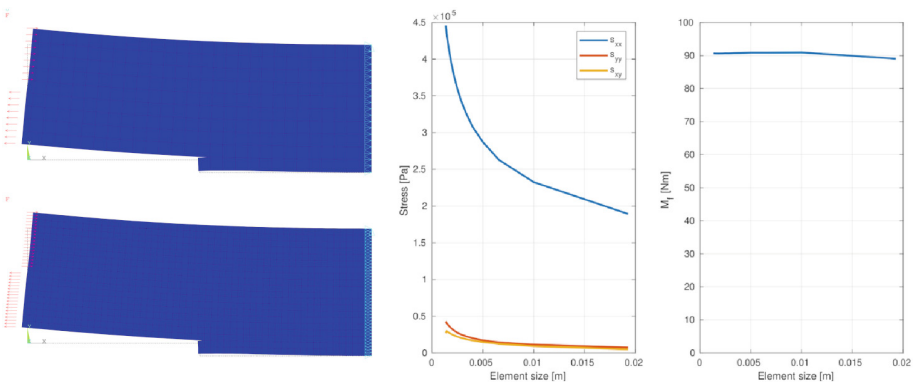


Fig. 2. Stability of the solution when looking for the critical momentum and varying the mesh density: Numerical models with different mesh densities (left), results from stress-based analysis (center) versus LFM-based ones (right)

As the method itself is old and uses basically the classic Irwin-Kies energy equation, the idea was mostly used for research purposes, as reviewed in Krueger et al. [1]. Nevertheless, the development continues and today, the method could be applied widely by professionals using FE solvers widely available to the public. Of course, to get some reasonable solution, we need to understand boundary conditions of the problem, know well the appropriate material properties of the timber for structural modelling and the decisive parameter in terms of a value for the critical strain energy release rate fitting to the expected failure mode or combinations of them. In this article, for sake of simplicity, the mode is limited only to mode I, representing tensile failure perp to grain. Such values have been researched and gathered in recent 30 years and are well summarized in the book by de Moura et al. [2]. All the publications [3–6] deal with the same subject. As reference value for strain energy release rate (in mode I) a value of  $G_c = 200\text{N/m}$  was chosen.

## 2 Methods

### 2.1 Virtual Crack Closure

The basic idea of virtual crack closure [1] is the Irwin-Kies equation:

$$G_c = \frac{F^2}{2B} \frac{dC}{da} \quad (1)$$

where  $G_c$  is the critical strain energy release rate,  $F$  is the acting load,  $B$  is the width of the sample,  $C = \frac{u}{F}$  is the compliance of the structure and  $a$  is the length of the crack. This concept is true, when we are interested in material testing for assessment of the value of strain energy release rate.

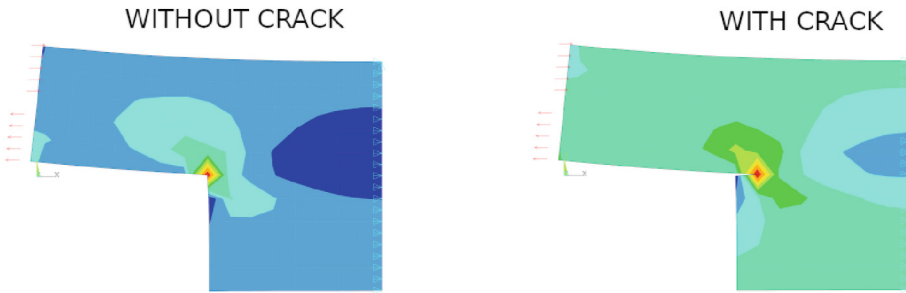
Nevertheless, if the value of strain energy release rate is considered as a material constant and the value is known *a priori*, the approach can be repeated with a FE model, which is capable of simulating the crack propagation. In this case, the FE model, which is able to simulate a crack with finite length  $\Delta a$ , is subjected to a unit load and the change of overall compliance due to crack formation is computed. Based on this, we can change the Eq. (1) with simple algebraic operation and rewrite in discrete form by changing derivation into difference:

$$F_{crit} = \sqrt{\frac{2BG_c}{\frac{\Delta C}{\Delta a}}} \quad (2)$$

This is the way the article computes the critical load based on an FE model. For clarity, the two model without and with crack is depicted in Fig. 3, where is outlined a 2D model not directly related to our problem modeled here, however, very easy to understand.

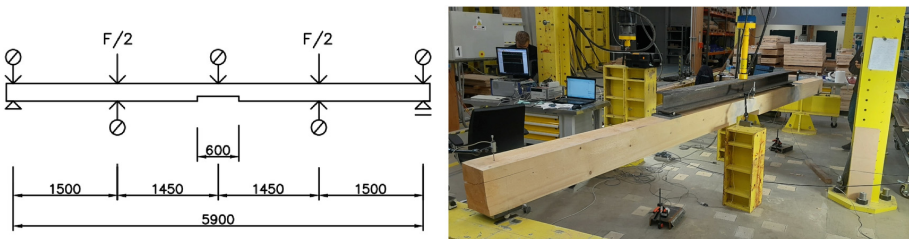
### 2.2 Experimental Work

To prove the validity of the method in structural scale, a series of experiments ( $n = 9$ ) has been prepared using Norway spruce timber with dimensions of the testing device



**Fig. 3.** Principle of virtual crack closure: A 2D model made in ANSYS without a crack (left) and with one already present (right). The overall energy of external forces acting on the deformed body has to be equal to the energy of crack growth.

depicted in Fig. 4. Width of the beams was 20 cm, height in average 25 cm. The cut had depth of 70 mm and was made using hand tools and a high precision Japanese saw. Four-point bending was used to induce only mode I. Loading speed was set to 1 mm/min. Average MC was 14,3% abs. Average density was 500 kg/m<sup>3</sup>. The experiment was stopped after all cracks on both sides of the test specimen at the location of the two notches had been initiated. According to Table 2, the experiments have shown relative consistent results.



**Fig. 4.** Scheme of the experiment (left) and experiment in the testing hall (right)

It is worth noting, that the initiation of all four cracks was not instant. Rather it appeared consecutively. All experiments were tracked by cameras according to the concept of DIC, although no precise evaluation has been made for the work presented here in the paper. There was no strong peak found in the loading diagram of the experiment. The clarification of this fact is probably smoothing by the transmission of the bending momentum mainly by the weakened cross section without big help of the cut parts (SRR). However, the test setup was sufficient for proving the validity of the computation.

### 2.3 Numerical Model

Numerical model has been realized in the ANSYS environment. A 2D model meshed with quadratic planar PLANE183 elements was constructed using APDL language. The structure was modeled as plane stress. Node releasing (or also called crack closure) was

made using constraint equations prescribing in the ‘locked’ model the nodes in the crack region to have the same displacements. On demand, the nodes were released in order to simulate the crack formation. Since the model in Fig. 6 is not very detailed, a detail of the cracked region is illustrated in Fig. 5. Mesh size was set to be 1 cm. The material model was linear orthotropic with constants shown in Table 1. Symmetry was used to lower the complexity of the problem. Buckling out of plane was restricted. Linear elastic problem without any CZM or other nonlinearities was used.

**Table 1.** Orthotropic material model used in the analysis

Quantity	Value [GPa]	Quantity	Value [GPa]	Quantity	Value []
$E_L$	11,0	$G_{LT}$	0,7	$\nu_{LT}$	0,4
$E_T$	0,4	$G_{TR}$	0,1	$\nu_{TR}$	0,05
$E_R$	0,4	$G_{LR}$	0,7	$\nu_{LR}$	0,4

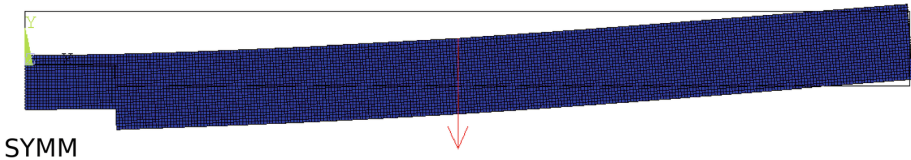


**Fig. 5.** The region of interest (left) in cracked mode after the experiment with cracks evident on both sides of the notch. The same domain (right) as detail of the numerical model (zoom in) also in cracked mode.

## 3 Results

### 3.1 Experimental Work

Results according to Table 2 represent the value of force, when all of the cracks are already visible. As mentioned earlier, the results are not simultaneous and it is not easy to record them. Nevertheless, the presence of all cracks in all four corners (two from each side) can be measured. Sometimes happened, that three were already present and the last not. However, this order is not easy to explain in detail without further investigations in local material behavior and it is also not important, since the values were not absolutely different and appeared relatively soon after each other. The scatter of values is relatively big, nevertheless, that is the nature of wood. As a fact of interest, the drying crack played a significant role, since the cracks always appeared in the side, where there is higher stiffness and no drying cracks.



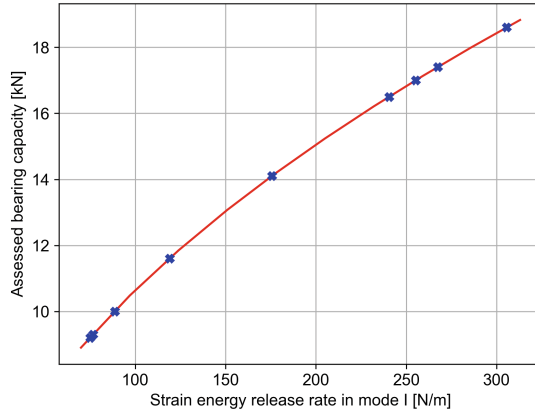
**Fig. 6.** Numerical model of the problem

**Table 2.** Experimental test results

Case	Initiation of all four cracks [kN]
1	9,2
2	9,3
3	16,5
4	14,1
5	10,0
6	17,0
7	17,4
8	18,6
9	11,6
average	<b>13,78</b>
stdev	3,77
var. Coef	0,27

### 3.2 Numerical Modeling

The output of the analysis was processed according to Eq. (2). The displacements of the structure before and after crack were used to evaluate the change of compliance related to the crack growth. The results are plotted in Fig. 7. In the middle of the graph, the value of 200 N/m is allocated and the rest is recomputed with different strain energy release rate interval to have an idea, which could be expected with other values of  $G_c$ . When we look at the experimental results, we can state that: 1) the numerical and experimental values show relatively big scatter due to scatter of decisive input parameters and 2) that the average value of bearing capacity is 13,78 kN and linked to the value of  $G_c = 168$  N/m.



**Fig. 7.** Results of the numerical analysis: Bearing capacity of the beam compared to different values of critical strain energy release rate, the experimental results are shown in blue, values along the x-axis contain the values of  $G_c$  for each experiment

## 4 Discussion

The article demonstrates solution only in the context of mode I. This restriction to one mode instead of considering several modes is not a restriction of the method itself. It was rather selected for publication in SAHC conference - this particular subject to discuss such a method in the community with a clear solution. Mode I is well documented both on the material scale and also is easy to perform and there are no complicated questions like mixed mode etc.

The verification of the results would need more configurations and more species, densities etc. This can be done in the future. This time, only an easy solution is presented. Also, discussion on the validity of the numerical model and reliability of the FE results is not covered here. It is so, because sensitivity analysis of parameter influence on the results like mesh density, elastic material constants is outside of scope of SAHC and would be interesting rather in a FEM-oriented conference. However, it should be noted here, that FPZ length in mode I in Norway spruce should be in the order of a few millimeters [10], so the LEFM should be applicable.

Another, rather more general discussion could be started about the LEFM approach. Isn't it too old, outdated and for research purpose dead theme? The authors of the article thing it is not. In the end, the complicated nonlinear behavior in FPZ has to be linearized, since we are unable to use such an approach in daily engineering practice with complicated and often unstable solutions. The aspects of ease of use, simplicity, elegance and low computing time play major role. Another reason should be the ability to incorporate most of FE-solvers without big problems. Would anybody from practice would trust in results, if the calculation process would not work in some cases? In this context, we also can discuss the optional use of CZM elements, which have the capability not only to simulate the crack formation, but can be theoretically used everywhere and the program itself chooses based on the parameters of CZM, if the crack growth is initiated. To our best knowledge, this approach is rather research-focused and not usable

in commercial structural code is, also because of high computation cost. In LEFM, the pre-crack or the crack growth direction has to be known. Luckily, in timber we know that the growth will be triggered by the direction parallel to the grain. That is why we can easily focus only on places in the model, where there is high probability of stress concentration. LEFM primarily looks for initiation of crack growth, which is, however, very important, because in real structures with load-controlled processes further unstable crack growth is dangerous. Therefore, the initiation can be in practice equal to growth and a synonym for failure of the affected member.

Of course, there are locations in structures either prone to cracks or not so sensible for destabilization. This issue has to be solved by the structural engineer and the adequate methods can provide valuable information for decision making.

In the field of LEFM and timber, the most important and crucial challenge is to decide, which values should be used for assessment, since there is a big difference within published values. Dourado et al. [3] recommends  $G_c = 120 \text{ N/m}$  while in [5], we can find  $200 \text{ N/m}$ .

## 5 Conclusion

This work illustrated, how to simply apply numerical methods for the assessment of bearing capacity of historic timber beams. Of course, the material properties of timber have to be clearly specified for each wood specie. In standards, only the lowest value may be implemented for beams, which still are unknown in the stage of modelling and assessment. In the context of already existing structures, better values could be applied based on in situ and nondestructive investigations. However, the calculation method itself works well and provides values, which would never be possible when using EC5 equations for a beam with a notch close to the supports and allocated at the tensile bending side.

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