



Physics-Based and Machine-Learning Models for Braking Impact Factors

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Abstract. The effect of vehicle braking can significantly amplify a bridge deflection compared to that induced by a vehicle moving at a constant speed. However, the magnitude of this amplification depends on vehicle bridge interaction (VBI) phenomena activated by the road roughness. The road roughness triggers the vehicle dynamics, thus magnifying the interaction between the vehicle and the bridge. This paper proposes a probabilistic model for the amplification factor. The amplification factor is associated with the vehicle's hard braking by the mid-span of the bridge under different road roughness classes. The amplification factor, defined as the ratio between the maximum deflections corresponding to a vehicle braking and moving at a constant speed, is estimated as a function of the mass, velocity, natural frequency and damping of the vehicle. The VBI model is obtained by discretizing the coupled governing equations using the finite difference method. The vehicle is modelled as a two-degrees of freedom system corresponding to the bouncing and pitching motions. The computational efficiency of this model supported an expensive set of analyses, where the parameter values were selected using the Latin Hypercube sampling scheme. The model outputs have been validated against a middle-span bridge's measured experimental displacement response under different scenarios.

Keywords: Bouncing · Braking · Bridge · Fragility curve · Genetic programming · Machine learning · Moving load · Neural network · Pitching · Roughness · Surrogate model · Vehicle-bridge interaction

1 Introduction

The displacement impact factor IF is commonly considered in order to quantify the effects of braking on bridge dynamics under traffic loads. It is defined as

follows:

$$\text{IF} = \frac{w_{b,braking}}{w_{b,constant}}, \quad (1)$$

where $w_{b,braking}$ and $w_{b,constant}$ are the maximum bridge deflection under braking conditions and the value corresponding to a vehicle that moves at a constant speed, respectively.

Although the realistic analysis of the vehicle-bridge dynamic interaction should account for all these factors, several studies are based on fairly similar, simplified models. The lack of comparative studies among different modeling approaches, in turn, prevents to understand to what extent all assumptions and parameters involved in braking are important for estimating its effects on bridge dynamics. The critical examination of alternative modeling approaches taking into account their complexity and accuracy as well as the associated computational cost is also important towards practical applications, such as sensitivity and reliability analyses. Hence, two classes of modeling approaches are herein considered to investigate the dynamics of the vehicle-bridge interaction in braking conditions. The first class of approaches encompasses physics-based models (i.e., models ruled by the dynamic laws). Specifically, six models characterized by different levels of complexity and accuracy are considered. They are listed in Table 1 together with the corresponding assumptions. Since these physics-based models are ruled by differential equations, their use can require a large computational effort if they must be solved a large number of times. Surrogate models (also known as metamodels) can provide a convenient way to cope with the computational burden required by these applications. In this context, the basic idea is to develop a data-driven approximation of the vehicle-bridge interaction in braking conditions that is more efficient to execute, so as to obtain a direct estimate of the desired output (i.e., the bridge response in terms of IF or $w_{b,braking}$) starting from a set of relevant parameters. The development of surrogate models requires both the discovery of the correct function that fits the data and the appropriate numeric coefficients of the function. Machine learning techniques have proven especially suitable to deal with this task in several scientific fields. Therefore, their feasibility in approximating the dynamics of the vehicle-bridge interaction under braking conditions is here investigated. Specifically, the following two supervised machine learning techniques are considered.

Table 1. Considered physics-based models of the vehicle-bridge interaction in braking conditions.

Model label	IF label	Model assumptions			
		Roughness	Vehicle model	Inertial effects	Braking friction
1dof-hb-st	$\text{IF}_{1,hb,st}$	Stationary	1 degree-of-freedom	No	No
1dof-hb-st	$\text{IF}_{1,bl,st}$	Stationary	1 degree-of-freedom	No	Yes
1dof-sb-nst	$\text{IF}_{1,bl,nst}$	Nonstationary	1 degree-of-freedom	No	Yes
2dof-hb-st	$\text{IF}_{2,hb,st}$	Stationary	2 degrees-of-freedom	Yes	No
2dof-sb-st	$\text{IF}_{2,bl,st}$	Stationary	2 degrees-of-freedom	Yes	Yes
2dof-sb-nst	$\text{IF}_{2,bl,nst}$	Nonstationary	2 degrees-of-freedom	Yes	Yes

- Neural network. The elaboration of the data-driven model for the vehicle-bridge dynamic interaction leverages on the well known capability of neural networks as universal function approximators.
- Genetic programming. Evolutionary computing is exploited to look for a proper model function, in symbolic form, which approximates as best as possible the desired output.

2 Methods

2.1 Road Roughness Model

The randomness of the road surface roughness can be represented by means of a periodic modulated random process. Within the ISO 8608 specifications [1], the road surface roughness depends on the vehicle speed through a formulation that relates the velocity power spectral density (PSD) and the displacement PSD. The general expression for the displacement PSD of the road surface roughness S_d is the following:

$$S_d(n) = S_d(n_0) \cdot \left(\frac{n}{n_0}\right)^{-a}, \quad (2)$$

where n is the spatial frequency (cycles/m), $n_0 = 0.1$ cycles/m is the reference value of the spatial frequency, and a is a suitable exponent. Equation 2 gives an estimate of the actual road roughness level $S_d(n)$ as function of the corresponding reference value $S_d(n_0)$. Note that the classification of the road roughness according to the ISO 8608 specifications is based on a constant vehicle velocity PSD by taking $a = 2$.

The road roughness r can be generated as function of the spatial coordinate x through the following equation:

$$r(x) = \sum_{p=1}^Q d_p \cos(n_p x + \varphi_p), \quad (3)$$

where n_p is the p th spatial frequency and φ_p is the p th random phase angle, being Q the total number of harmonics assumed to generate the road roughness profile. Moreover, d_p is the p th amplitude, which is computed as follows:

$$d_p = \sqrt{2S_d(n)\Delta n}, \quad (4)$$

where Δn denotes the sampling interval of the spatial frequency.

The nonstationary road roughness has been defined based on the paper by Yin et al. [2].

2.2 Vehicle-Bridge Interaction Model

Translation and rotational dynamic equilibrium equations of the vehicle mass can be written as follows:

$$m_v \left[\ddot{w}_v + \left(\frac{\ddot{r}(x_r) + \ddot{r}(x_f)}{2} \right) + \left(\frac{\ddot{w}_b(x_r, t) + \ddot{w}_b(x_f, t)}{2} \right) \right] + 2c_v \dot{w}_v + 2k_v w_v = 0, \quad (5)$$

$$I_v \left[\ddot{\theta}_v + \left(\frac{\ddot{r}(x_r) - \ddot{r}(x_f)}{B} \right) + \left(\frac{\ddot{w}_b(x_r, t) - \ddot{w}_b(x_f, t)}{B} \right) \right] + m_v h \ddot{x}_v + \frac{B^2}{2} c_v \dot{\theta}_v + \frac{B^2}{2} k_v \theta_v = 0, \quad (6)$$

where m_v , I_v , c_v , k_v are mass, inertia, damping coefficient and sprung stiffness of the vehicle, respectively. Moreover, h is the height of the vehicle centre of mass distance from the road surface and B is the distance between the contact points of the two wheels. The absolute vertical displacement of the vehicle mass is given by three terms, namely w_v (i.e., relative displacement between the road surface and the center of gravity of the vehicle mass), r (i.e., road roughness, that is the distance between the road surface and the averaged road surface), and w_b (i.e., deflection measured from the line axis of the bridge). Similarly, the absolute rotation of the vehicle is given by three contributions, namely θ_v (i.e., relative rotation between the vehicle and road surface), r , and w_b . The coupling between the bridge and the vehicle is due to the inertial terms in Eq. 5 and Eq. 6, whereas damping and elastic term are only affected by the relative displacement between the vehicle and the road surface.

The contact force of the rear wheel $f_{c,r}$ can be derived from the equilibrium equation:

$$m_w [\ddot{r}(x_r) + \ddot{w}_b(x_r, t)] + f_{c,r} - c_v \left(\dot{w}_v + \dot{\theta}_v \frac{B}{2} \right) - k_v \left(w_v + \theta_v \frac{B}{2} \right) = 0, \quad (7)$$

where m_w is the mass of the wheel. So doing, it is obtained:

$$f_{c,r} = -m_w [\ddot{r}(x_r) + \ddot{w}_b(x_r, t)] + c_v \left(\dot{w}_v + \dot{\theta}_v \frac{B}{2} \right) + k_v \left(w_v + \theta_v \frac{B}{2} \right). \quad (8)$$

Similarly, the contact force for the front wheel $f_{c,f}$ is:

$$f_{c,f} = -m_w [\ddot{r}(x_f) + \ddot{w}_b(x_f, t)] + c_v \left(\dot{w}_v - \dot{\theta}_v \frac{B}{2} \right) + k_v \left(w_v - \theta_v \frac{B}{2} \right). \quad (9)$$

The contact force due to the vehicle oscillation does not include the weight of the sprung-mass system $m_w g + m_v g$.

Horizontal friction forces arise between the tyres and the bridge surface during braking. If the width of the contact area between the tyre and the bridge is T , then the bending moment induced by the friction forces is:

$$\begin{aligned} m(x) &= \frac{\mu m_v y_s}{T} \left[H \left(x_r - \dot{x}_v t - \frac{T}{2} \right) - H \left(x_r - \dot{x}_v t + \frac{T}{2} \right) \right] + \\ &+ \frac{\mu m_v y_s}{T} \left[H \left(x_f - \dot{x}_v t - \frac{T}{2} \right) - H \left(x_f - \dot{x}_v t + \frac{T}{2} \right) \right] \approx \\ &\approx \frac{\mu m_v y_s}{B} \left[H \left(x_v - \dot{x}_v t - \frac{B}{2} \right) - H \left(x_v - \dot{x}_v t + \frac{B}{2} \right) \right], \end{aligned} \quad (10)$$

where y_s is the distance between the beam center of mass and the road surface while H is the Heaviside step function. Equation 10 approximates the two distributed bending moments induced by friction forces corresponding to the two tyres with a single distributed bending moment that extends over the wheelbase.

The partial derivative of the approximated expression for the distributed bending moment can be written as:

$$\begin{aligned} \frac{\partial m(x)}{\partial x} &= \frac{\mu m_v y_s}{B} \left[\delta_r \left(x_v - \dot{x}_v t - \frac{B}{2} \right) - \delta_f \left(x_v - \dot{x}_v t + \frac{B}{2} \right) \right] = \\ &= F \left[\delta_r \left(x_v - \dot{x}_v t - \frac{B}{2} \right) - \delta_f \left(x_v - \dot{x}_v t + \frac{B}{2} \right) \right], \end{aligned} \quad (11)$$

where δ_r and δ_f are the Dirac delta corresponding to the rear and front wheels, respectively.

The bridge is modeled as linear elastic Euler-Bernoulli beam with constant mass per unit length ρA (where ρ is the specific mass and A is the cross-section area of the beam) and constant bending stiffness EJ (where E is the elastic modulus and J the cross-section inertia of the beam). The continuous vertical displacement $w_b(x, t)$ of the bridge is governed by the following partial differential equation [3]:

$$\begin{aligned} \rho A \ddot{w}_b(x, t) + EJ w_{r,xxxx}(x, t) &= \delta_r(x_r - \dot{x}_v t) \left(f_{c,r} + m_w g + \frac{1}{2} m_v g \right) + \\ &\delta_f(x_f - \dot{x}_v t) \left(f_{c,f} + m_w g + \frac{1}{2} m_v g \right) + \frac{\partial m(x)}{\partial x}, \end{aligned} \quad (12)$$

The following boundary conditions apply on the left and on the right for a pinned-pinned beam:

$$w_b(0, t) = 0, \quad w_{b,xx}(0, t) = 0, \quad (13)$$

$$w_b(L, t) = 0, \quad w_{b,xx}(L, t) = 0, \quad (14)$$

respectively.

2.3 Machine-Learning

A multi-layer feed-forward artificial neural network is used in this paper. The optimization of the artificial neural network layout is critical in maximizing its accuracy level as well as its predictive capability against new data. Among the different strategies that have been proposed to optimize the artificial neural network layout [4], a genetic-algorithm-based approach has been adopted in the present work [5]. A computer program is a predictive model that provides (in symbolic form) the instructions for combining relevant variables, constants and operators. Starting from an initial (random) collection of candidates, these computer programs are then manipulated through a sequence of genetic operators until a stopping criterion is not fulfilled. Out of the available approaches to cope with bloat in genetic programming [4], the strategy implemented in the present work assumes model accuracy and model complexity as conflicting criteria following Ekart and Nemeth [6].

3 Results

The considered case study is a simply supported pre-stressed concrete girder bridge belonging to the Italian motorway network [7–9]. The cross-section has trapezoidal shape and is 2.3 m high, with two cantilevered wings 3.85 m wide, which are pre-stressed by bonded post-tensioned tendons. A pair of piers, whose centre distance is about 40 m, sustains each bridge span. The Monte Carlo simulation shows that the effect of braking mostly depends on the road roughness and initial velocity of the vehicle [10]. Figure 1 plots the IF as a function of the initial velocity, based on the experimental data, physics-based and surrogate models.

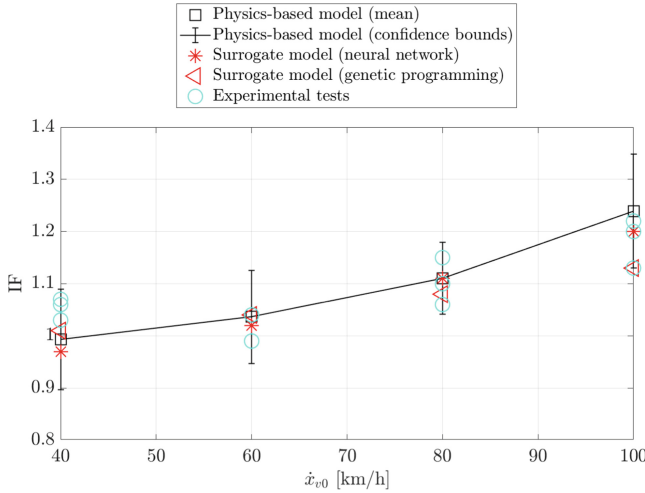


Fig. 1. Variation of the IF as a function of the vehicle velocity: confidence bounds estimated using the physics-based model, nominal values carried out through surrogate modelling, and experimental data.

The symbolic form of the final surrogate models for IF and $w_{b,braking}$ obtained via genetic programming is the following:

$$\begin{aligned} \text{IF} = & \theta_{IF1} + \theta_{IF2}f_{v,p}h^2 + \theta_{IF3}B\dot{x}_{v0}^2 + \theta_{IF4}Bf_{v,p}h \\ & + \frac{1}{m_v} (\theta_{IF5}\dot{x}_{v0}^2f_{v,p}h + \theta_{IF6}Bf_{v,p}h + \theta_{IF7}B^2h), \end{aligned} \quad (15)$$

$$\begin{aligned} w_{b,braking} = & \theta_{w_b1} + \theta_{w_b2}B + \theta_{w_b3}\dot{x}_{v0} + \frac{\theta_{w_b4}h + \theta_{w_b5}m_v}{f_{v,b}} \\ & + \frac{\theta_{w_b6}f_{v,p} + \theta_{w_b7}h + \theta_{w_b8}m_v}{B^{\theta_{w_b9}}} \quad (\text{in [m]}), \end{aligned} \quad (16)$$

respectively. The involved sets of coefficients, Θ_{IF} and Θ_{w_b} , are listed in Table 2. These surrogate models for IF and $w_{b,braking}$ involve the same parameters except

Table 2. Coefficients of the genetic programming-derived surrogate models for IF and $w_{braking}$ in Eq. 15 and Eq. 16, respectively.

IF (Θ_{IF})	$w_{braking}$ (Θ_{w_b})
$\theta_{IF1} = 1.009$	$\theta_{w_b1} = 2.8268E-04$
$\theta_{IF2} = 0.0024$	$\theta_{w_b2} = 9.9309E-06$
$\theta_{IF3} = -1.6712E-07$	$\theta_{w_b3} = -2.4731E-06$
$\theta_{IF4} = -0.0013$	$\theta_{w_b4} = -4.9481E-07$
$\theta_{IF5} = 2.0845E-05$	$\theta_{w_b5} = -7.2693E-06$
$\theta_{IF6} = -0.0042$	$\theta_{w_b6} = -8.9722E-05$
$\theta_{IF7} = -6.7429E-04$	$\theta_{w_b7} = -6.7173E-05$
	$\theta_{w_b8} = -2.1517E-04$
	$\theta_{w_b9} = 0.0833$

for $f_{v,b}$, which is missing within the symbolic expression for IF. The results demonstrate that genetic programming is also able to produce good surrogate models. Considering the compact form of the final symbolic expressions, the accuracy of the genetic programming is still deemed satisfactory. However, the surrogate model for the IF carried out via genetic programming is slightly less accurate than those produced via artificial neural networks, being R^2 between 80% and 85%. The surrogate models for $w_{b,braking}$ obtained by means of genetic programming and artificial neural networks display almost the same accuracy level, being R^2 around 99% for both machine learning techniques.

4 Conclusions

The main findings of the present paper are the followings:

- Hard braking is the most severe dynamic loading condition, since it yields IF values equal to or larger than one on average. The use of an accurate vehicle model (i.e., two degree-of-freedom vehicle model) is needed in order to obtain the most conservative estimates of the bridge response in such braking condition.
- While a two degrees-of-freedom vehicle model yields larger values of the IF in case of hard braking, an opposite trend is observed on average in case of soft braking. In this case, a single degree-of-freedom vehicle model turns out to produce more conservative estimates of the bridge displacements.
- The effects of a soft braking on the bridge response depend on the road roughness model. In this case, a nonstationary road roughness model yields IF values higher than those obtained from a stationary road roughness model on average. However, such influence of the road roughness model on the bridge response reduces when the road roughness level increases. If a hard braking condition is considered, then the higher the road roughness level, the lower the mean IF value.

- The dispersion of the IF values basically depends on braking conditions and vehicle model. The dispersion of the IF values is especially high in case of soft braking condition and two degrees-of-freedom vehicle model. The dispersion of the IF values also depends, to some extent, on the roughness level in case of hard braking condition: in such case, the lower is the road roughness level, the larger is the dispersion of the IF values.

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