



Forecasting the Value of Vibration-Based Monitoring Information in Structural Integrity Management

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Abstract. Structural deterioration and increasing load demand are two main factors that compromise the serviceability and functioning of civil constructions. The vastity of the bridge portfolio and the few resources available require maintenance optimization to provide the required user safety. In this context, vibration-based monitoring may provide information about the structural performance and support decisions in structural integrity management. In this paper, a novel definition of global and local information from a multi-sensor vibration-based system is provided and implemented for the cases of a parallel ductile Daniels system and a serial system. Furthermore, local and global integrity management actions are modeled and analyzed. Vibration-based information is used to optimize the maintenance strategy in terms of optimal action implementation. Decision and value of predicted information analyses are used to drive maintenance optimization. Indeed, each outcome of the monitoring system and maintenance strategy is associated with an expected utility and cost. Optimization is performed by determining the lowest expected cost corresponding to a maintenance strategy.

Keywords: Value of information · Vibration-based monitoring · Integrity management · Structural health monitoring

1 Introduction

The management of national bridge portfolios is a growing concern in modern societies. Adequate structural performance should be ensured to common users despite the budget constraints and portfolio vastity. In this context, inspections, destructive and non-destructive testing, and continuous monitoring system, e.g., vibration-based monitoring (VBM), can provide information support for the integrity management and the prioritization of interventions for structural systems. Collecting information may entail a cost that may not be worth the benefit achieved by its use for the improvement of integrity management and risk reduction. In these terms, a quantification of such benefit has been proposed by Raiffa and Schlaifer [1] who in 1961 formulated the value of information (VoI) concept based on the Bayesian decision theory. VoI has been used in several fields,

such as economics, medicine, environment, ecology, and structural integrity management. Comprehensive reviews are presented in [2, 3]. Furthermore, authors in [4, 5] have described an alignment of the decision value analysis (DVA) and the technology readiness levels (TRLs), commonly used to assess the maturity of developing innovative technology. Specifically, value forecasting (i.e., value quantification based on prior-information models), value analysis (i.e., value prediction based on information models developed based on laboratory results), and value quantification (i.e., value prediction based on information models based on information collected on operating structures) are identified as the three main phases of DVA in the context of new technology development. In the field of VBM, several authors formulated approaches for the assessment of VBM value in integrity management performing value forecasting, analysis, or quantification depending on the information models they assumed in their studies, see e.g., Giordano et al. [6], Erduran et al. [7], Kamariotis et al. [8], Thöns [9], Long et al. [10].

This paper aims to extend value forecasting by outlining a novel approach to model global, local, and joint information that may be obtained from VBM systems and used at different steps of the decision process. The value of global information (i.e., modal frequencies) and local information (i.e., mode shapes, curvature, etc.) provided by VBM is forecasted for two generic structural configurations, namely, a redundant (e.g., a parallel ductile Daniels system [11]) or non-redundant (i.e., serial) system. Furthermore, global and local actions are distinguished, accounting for the numerous intervention options in integrity management.

The paper is organized as follows: Sect. 2 reports the decision scenario and the analytical formulation from the Bayesian decision theory, which is used to assess the value of VBM technology. Section 3 includes the models for the system state, action, and utility models. Section 4 describes the novel information model. Section 5 reports two case studies, namely a ductile Daniels system and a serial system for which the value of local and global information from VBM systems is forecasted. Thus, conclusions are drawn in Sect. 6.

2 Decision Scenario

The decision scenario reflects the use of VBM in the integrity management of a generic structural system. VBM can provide both natural frequencies, i.e., global information, and mode shapes, i.e., local information, depending on the arrangement of the deployed sensors. The two approaches entail different costs. Indeed, in the case of a VBM system, one sensor provides a global indication of the structural state and entails a relatively low cost. Local information is proportionally more expensive depending on the number of sensors deployed and the relevant processing of data. Data collection and processing can be limited in normal conditions and can be widened whenever a critical structural performance is measured. In this paper, two operating modes of a multi-sensors system are considered.

Operating mode 1 consists in continuously processing data from a single sensor to identify global parameters, for example, modal frequencies. Data from all the other sensors are processed in case an anomaly is detected.

Operating mode 2 consists in the continuous processing of the data from all the sensors in order to acquire continuous local information.

In Fig. 1, the three decision trees illustrate the two operating modes and the case of no data collection. The main decision steps in the integrity management of a structure using VBM information are outlined: namely, collecting global information, collecting local information, and implementing actions.

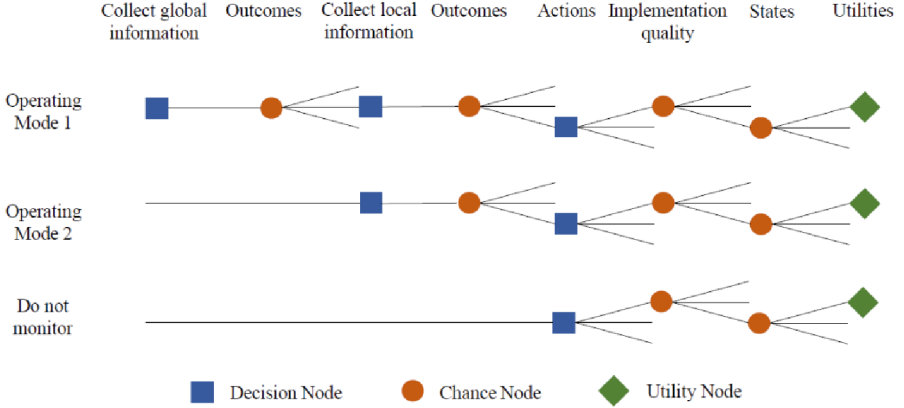


Fig. 1. Decision Trees

2.1 Analytical Formulation

An analytical formulation is necessary to evaluate the information strategies and assess the value of VBM in the integrity management of a structure. Table 1 reports the analytical formulation based on [1, 12] defining the objective functions for the predicted action (PA) decision analysis (DA) and the predicted information (by the m -th information strategy) and predicted action (PIPA) DA for the qualification of the expected and optimized utilities.

Table 1. Objective functions for PA and PIPA DA

	Objective function
Predicted Action DA	$U_{PA} = \max_{a_k} E_{X_I a_k, Y_k} [u(X_I, a_k, Y_k)] =$ $\max_{a_k} \sum_{X_I} P(X_I a_k, Y_k) \cdot u(X_I, a_k, Y_k) - c_{a_k}$
Predicted Information and Predicted Action DA	$U_{PIPA_m} = \max_{a_k} E_{X_I a_k, Y_k} [E_{O_{m,j} X_I} [u(X_I, a_k, Y_k)]] =$ $\max_{a_k} \sum_{X_I} P(X_I a_k, Y_k) \cdot \sum_{O_{m,j}} P(O_{m,j} X_I) \cdot u(X_I, a_k, Y_k) -$ $c_{a_k} - c_m$

where X_I are the system states, $O_{m,j}$ are the outcomes of the information system, i.e., the outcome j from the m -th information strategy, a_k indicates an action k that can be

implemented on the structure with an implementation uncertainty Y_k , c_{a_k} is the cost of the action k , c_m is the cost of collecting information on any component by the m -th information strategy. In order to compare the different information strategies, a normalized VoI, herein simply referred as VoI, is defined as follows:

$$VoI_m = \frac{U_{PIPA,m} - U_{PA}}{U_{PA}} \quad (1)$$

3 Models

This section encompasses the system state model, the action, and the utility models for the generic cases of parallel ductile Daniels systems and serial systems.

3.1 System State

Systems can be modelled by a number of components, which can be arranged both in series and in parallel. For the system component i , a limit state equation can be written as in Eq. (2):

$$g_{X_i} = M_{R_i} \cdot R_i(t) - M_{S_i} \cdot S_i \quad (2)$$

where M_{R_i} and M_{S_i} represent the resistance and load model uncertainties, $R_i(t)$ is the component current resistance and S_i is the load. At a specific time t_a of the service life, the component's resistance may be affected by damage and may be described as $R(t_a) = R_i(t_0) \cdot (1 - d_L(t_a) \cdot M_{d_i} \cdot D_i)$, where t_0 corresponds to the beginning of the structure service life, $d_L(t_a)$ is a capacity transformation factor at time t_a , later referred as d_L for simplicity of notation, M_{d_i} is the damage model uncertainty and D_i is the damage model. Two structural states are considered: X_{iF} for $g_{X_i} < 0$, i.e., failure state, and X_{iS} for $g_{X_i} \geq 0$, i.e., safe state. The probability of the component i to be in one of the two structural states can be written as $P(X_{iF}) = P(g_{X_i} < 0)$, i.e., probability of failure, and $P(X_{iS}) = P(g_{X_i} \geq 0)$, i.e., reliability.

In the case of a serial system, the global probability of failure $P(X_{GF})$ can be calculated equivalently as in Eqs. (3), (4).

$$P(X_{GF}) = P(X_{1F} \cup X_{2F} \cup X_{3F} \dots) \quad (3)$$

$$P(X_{GF}) = P(\min_{i=1:n} (M_{R_i} \cdot R_i(1 - d_L \cdot M_{d_i} \cdot D_i) - M_{S_i} \cdot S_i) \leq 0) \quad (4)$$

where $P(X_{1F})$ is the probability of failure of the first component, $P(X_{2F})$ is the probability of failure of the second component, etc. The system state reliability can be written as $P(X_{GS}) = 1 - P(X_{GF})$.

In the case of a parallel Daniels system, the system probability of failure $P(X_{GF})$ can be written as in Eqs. (5), (6).

$$P(X_{GF}) = P(X_{1F} \cap X_{2F} \cap X_{3F} \dots) \quad (5)$$

$$P(X_{GF}) = P\left(\sum_{i=1}^n M_{R_i} \cdot R_i(1 - d_L \cdot M_{d_i} \cdot D_i) - \sum_{i=1}^n M_{S_i} \cdot S_i \leq 0\right) \quad (6)$$

3.2 Action and Utility Models

Several actions can be implemented for the integrity management of a structure. Herein, four different actions are defined and reported in Table 2. Their effects, cost, and implementation uncertainties are taken exemplarily from the literature, see [13–15].

Table 2. Action options

Action type	Description	Action effect	Cost	Implementation uncertainty
a_0	Do nothing	-	-	-
a_1	Strengthening	$1.2 \cdot R$	0.5%	$Y_1 = Tr(0.95, 1.05, 1.10)$
a_2	Load reduction	$S/1.2$	2%	$Y_2 = N(1, 0.1)$
a_3	Consequence reduction	$c_{F,red} = 0,75 \cdot c_F$	0.3%	$Y_3 = Tr(0.85, 0.95, 1.05)$

Action a_0 is the “do-nothing action” having no effect on the system or costs. Action a_1 , i.e., “strengthening” and action a_2 , i.e., “load reduction” have an effect on the system by reducing the probability of a failure event. Action a_3 , i.e., “Consequence reduction”, does not affect the structural performance but reduces the consequences in the case of a failure event. Along with [13], a_1 and a_2 are referred to as “system state actions” whereas a_3 is referred to as “utility action”. A further distinction is proposed in this paper when dealing with actions on a multi-component system. Actions are distinguished into local and global actions. Local actions relate to single components, e.g., the strengthening action a_1 can be implemented on a single component rather than on the whole system. Nevertheless, the strengthening of a component contributes to enhancing the system’s performance. On the other hand, global actions affect more than one component and cannot be implemented on a single component. For example, limiting the loads (a_2) is likely to affect more than one component, modifying their structural performance and the system’s reliability. Similarly, utility actions refer to the whole system and are, therefore, defined as global actions.

A utility model is exemplarily defined to consider the cost of the failure event c_F , i.e., the cost associated with the system being in the state X_{G_F} , the zero cost of the system being in state X_{G_S} , the action costs c_{a_n} (defined per component in the case of local actions, see Table 2) For the sake of simplicity, all costs and utilities are defined as percentage of the cost failure c_F , which is set equal to 1. The information costs are defined as $c_{i_global} = 0.001 \cdot c_F$, and $c_{i_local} = 0.001 \cdot c_F$, per component.

4 Global and Local Information

One of the main contributions of this paper stays in the modeling of global and local information in the context of the integrity management of multi-component systems. This distinction is crucial when dealing with VBM as it can provide both global (system-related)

and local (component-related) information. For example, in the case of a bridge, VBM can be used to identify natural frequencies which are system-related information, and mode shapes which contain component-related information. Mode shapes may indicate the presence of local damage. Damage-related information may be obtained directly, e.g., by measuring a crack size, or indirectly, e.g., from a variation of the modal parameters identified through VBM [8].

Measurements may lead to an improved probabilistic model of a random variable, in accordance with the measurements' temporal and spatial boundaries. One approach to account for measurable and non-measurable parts of a random variable is to use model uncertainties as they are determined with a measurement system [16]. With this approach, measurements can be modeled as realizations of the model uncertainty, normalized to the model predictions, and subjected to measurement uncertainty. Local information can be modeled directly on the components' damage model uncertainties, and subjected to a local measurement uncertainty, i.e., $M_{U_{local}}$. To the best knowledge of the authors, global outcomes have not yet been modeled with model uncertainties. The approach to model global outcomes is to convert the individual components' model uncertainties and their correlation to a multivariate Gaussian distribution. The joint model uncertainty distribution is subjected to the global measurement uncertainty $M_{U_{global}}$. Different outcomes of the VBM system correspond to diverse system states that are usually discretized defining threshold values of the outcomes. Herein, the thresholds between the VBM outcomes are calculated through two different methods, namely a reliability-based method for the global information in operating mode 1 and a risk-based method for local information in both operating modes 1 and 2. In the former case, the outcome domain Ω_{G_1} is defined such that $P(X_{GF}|\mathbf{O}_G \ni \Omega_{G_1}) < P_t$, where X_{GF} indicates the system failure state, \mathbf{O}_G is the set of components' normalized measurements collected on the several components $\{M_{d_1}, M_{d_2}, \dots\}$, all affected by the same measurement uncertainty $M_{U_{global}}$, and P_t is a target probability of failure, e.g., $P_t = 10^{-3}$ for serviceability [17]. Thus, two global outcomes \mathbf{O}_{G_1} and \mathbf{O}_{G_2} are defined: $\mathbf{O}_{G_1} = \mathbf{O}_G \ni \Omega_{G_1}$ includes the combinations of the components' model uncertainties realizations such that the system probability of failure is lower than the target value whereas $\mathbf{O}_{G_2} = \mathbf{O}_G \not\ni \Omega_{G_1}$ includes the combinations of the realizations such that the system probability of failure is higher. In operating mode 1, a safe outcome \mathbf{O}_{G_1} would induce action a_0 – do nothing. A failure outcome \mathbf{O}_{G_2} would induce the collection of local information on the system. Local information is discretized through a risk-based approach allowing for the definition of several outcomes and relative optimized integrity management actions. Indeed, the risk-based approach is adopted for each component i to identify the thresholds $\eta_{i,j}$ indicating $j = 1, \dots, N$ states of the system in which the minimum risk corresponds to the same management action.

The joint probability of the component i to be in the failure state and the local indication $O_{i,j}$ to be collected, can be written as in Eq. (7):

$$P(X_{iF}, O_{i,j}) = M_{R_i} \cdot R_i \cdot (1 - d_L \cdot M_{UL_i} \cdot M_{Di}|_{-\infty}^{\eta_{i,j}, Trunc} \cdot D_i) - M_{S_i} \cdot S_i \leq 0 \quad (7)$$

where $M_{UL_i} \cdot M_{Di}|_{-\infty}^{\eta_{i,1}, Trunc}$ corresponds to outcome $O_{i,1}$.

The joint probability of failure of the system and the collection of the set of local measurements $O_{i_1} = M_{UL_i} \cdot M_{Di}|_{-\infty}^{\eta_{i,1}}$, can be written as in Eq. (8) for the case of a Ductile Daniels system and as in Eq. (9) for the case of a serial system.

$$P(X_{GF}, O_{11}, O_{21} \dots O_{n1}) = P\left(\sum_{i=1}^n M_{R_i} \cdot R_i \cdot (1 - d_L \cdot M_{U_{L,i}} \cdot M_{D_i})|_{-\infty}^{\eta_{i,1}, Trunc} \cdot D_i - \sum_{i=1}^n M_{S_i} \cdot S_i \leq 0\right) \quad (8)$$

$$P(X_{GF}, O_{11}, O_{21} \dots O_{n1}) = P(\min_{i=1:n} (M_{R_i} \cdot R_i \cdot (1 - d_L \cdot M_{U_{L,i}} \cdot M_{D_i})|_{-\infty}^{\eta_{i,1}, Trunc} \cdot D_i) - M_{S_i} \cdot S_i \leq 0) \quad (9)$$

In the case only global information is collected, its joint probability of failure of the system of a ductile Daniels system and a serial system can be written as in Eqs. (10), (11).

$$P(X_{GF}, O_{G1}) = P\left(\sum_{i=1}^n M_{R_i} \cdot R_i \cdot (1 - d_L \cdot M_{U_G} \cdot M_{D_i})|_{\Omega_{G1,i}}^{Trunc} \cdot D_i - \sum_{i=1}^n M_{S_i} \cdot S_i \leq 0\right) \quad (10)$$

$$P(X_{GF}, O_{G1}) = P\left(\min_{i=1:n} M_{R_i} \cdot R_i \cdot (1 - d_L \cdot M_{U_G} \cdot M_{D_i})|_{\Omega_{G1,i}}^{Trunc} \cdot D_i - M_{S_i} \cdot S_i \leq 0\right) \quad (11)$$

When collecting both local and global information, the joint probability of failure becomes as in Eq. (9) for a ductile Daniel system and Eq. (10) for a serial system.

$$P(X_{GF}, O_{G1}, O_{11}, O_{21} \dots O_{n1}) = P\left(\sum_{i=1}^n M_{R_i} \cdot R_i \cdot (1 - d_L \cdot M_{U_G} \cdot (M_{U_{L,i}} \cdot M_{D_i})|_{-\infty}^{\eta_{i,1}, Trunc})|_{\Omega_{G1,i}}^{Trunc} \cdot D_i - \sum_{i=1}^n M_{S_i} \cdot S_i \leq 0\right) \quad (12)$$

$$P(X_{GF}, O_{G1}, O_{11}, O_{21} \dots O_{n1}) = P\left(\min_{i=1:n} M_{R_i} \cdot R_i \cdot (1 - d_L \cdot M_{U_G} \cdot (M_{U_{L,i}} \cdot M_{D_i})|_{-\infty}^{\eta_{i,1}, Trunc})|_{\Omega_{G1,i}}^{Trunc} \cdot D_i - M_{S_i} \cdot S_i \leq 0\right) \quad (13)$$

5 Case Studies

The cases of a serial system and a parallel ductile Daniels system are analyzed to forecast the value of global and local information in structural integrity management. Table 3 summarizes the probabilistic models, taken from the literature, see e.g., [16], and used in the analysis.

Structural components are designed such that their initial resistance $R_i(t_0) = \gamma \cdot A \cdot R_{mat}$, where γ is safety factor (equal to 1.5, [17]) and A represents the component geometrical properties and R_{mat} corresponds to the material resistance, contribute to a system structural reliability equal to $P(X_{S2}) = 10^{-5}$, see [17]. Due to common fabrication, load distribution and modeling, some considerations on the correlation between the distributions to be assumed for the several components are made. R_{mat} was considered as fully correlated as related to the common components' production. Damage is considered to be correlated, e.g., as in the case of corrosion. Structural model uncertainties as M_R and M_D , were assumed to be non-correlated the first, and fully correlated the second, as little reference has been found in the literature. Load characteristics, i.e., M_S and S , are considered fully correlated as the external load distributes equally to the system components. Local measurement uncertainty is considered non-correlated for simplicity reasons.

Table 3. Probabilistic Models

Variable	Distribution	Mean	Variance	Correlation
R_{mat}	Lognormal	1.0	0.05	1
M_R	Lognormal	1.0	0.05	0 to 1
D	Lognormal	1.0	0.02	1
M_D	Normal	1.0	0.1	0 to 1
S	Gumbel	1.0	0.2	1
M_S	Lognormal	1.0	0.1	1
MU_{local}	Normal	1.0	0.03	0
MU_{global}	Normal	1.0	0.05	-
d_L	-	0.4		

5.1 Serial System

A serial system of three components is considered, and the integrity management analysis is performed. Figure 2 summarizes the value of information of the two information strategies, i.e., collecting continuously global data widening to local data in case an anomaly is detected (operating mode 1), and collecting directly local data (operating mode 2). Model uncertainties M_R and M_D are considered under the two correlation conditions as reported in Table 3. For the same information strategy, a darker color indicates no correlation, and a lighter one full correlation.

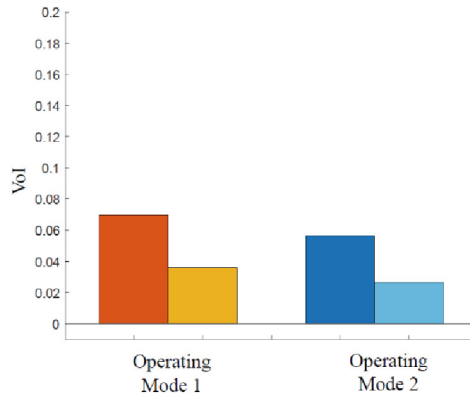


Fig. 2. VoI for operating mode 1 (global data collection and local data collection in case of damage detection) and for operating mode 2 (local data) for non-correlated (darker color) and fully correlated (lighter color) model uncertainties for a serial system

Figure 2 illustrates that operating mode 1 provides the most valuable information for the integrity management. Nevertheless, operating mode 2 provides a relevant benefit in

the integrity management of a serial system, despite entailing a higher cost. Collecting local data only when an anomaly is detected provides budget savings despite entailing the choice of an anomaly target reliability, herein taken from [17]. Hence, the calibration of such a parameter may be crucial in an optimization perspective.

A lower VoI is found when considering full correlated model uncertainties. This relates to the fact that, in serial systems, correlated structural characteristics contribute to a higher structural reliability. The failure of the system occurs whenever one component fails. Therefore, information is less valuable in integrity management of a structure where structural characteristics are assumed correlated than in case of no correlation.

5.2 Parallel Ductile Daniels System

In this second analysis, a Daniels system composed of three parallel components is considered. The system is modeled to have the same initial structural reliability of the serial system analyzed in Sect. 5.1, allowing for a comparison of their integrity management. The value of the two information strategies is represented in Fig. 3.

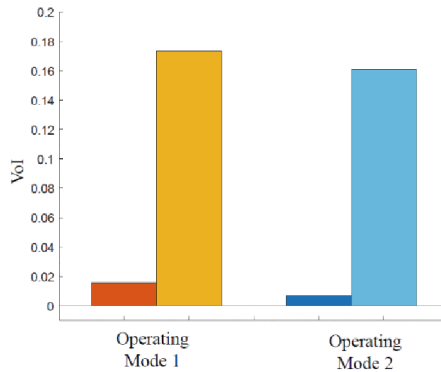


Fig. 3. VoI for operating mode 1 (global data collection and local data collection in case of damage detection) and for operating mode 2 (local data) for non-correlated (darker color) and fully correlated (lighter color) model uncertainties for a ductile Daniels system

In the study of the ductile Daniels system, operating mode 1 is found to be the most advantageous information strategy in integrity management. As in the previous case, the difference between operating mode 1 and 2 consists mainly in the information costs.

A lower VoI is found when considering non-correlated model uncertainties. In the case of ductile Daniels systems, correlated model uncertainties lead to a lower structural reliability and therefore, information is more valuable.

6 Conclusion

This paper presents the integrity management analysis of a serial and a parallel ductile Daniels system with local and global actions supported by global and local information. A novel modeling of global and local information from VBM is developed and

implemented in the study. Furthermore, the adopted action model distinguishes local and global actions in integrity management and accounts for both enhancing the structural performance and reducing the consequences of a failure event. The optimal action is identified by performing a minimization of the total expected cost and the relevant value of VBM to support this choice is investigated. Results show that operating mode 1, i.e., using global and local VBM, leads to a higher information value than the operating mode 2 (continuous local VBM) despite a fixed and not optimized target reliability value for anomaly detection. Furthermore, the calculated VoI is found strongly dependent on the correlation of the structural characteristics and the system reliability. In the analyzed cases, it is found that VBM information is more valuable for the integrity management of a serial system with non-correlated structural characteristics and of a ductile Daniels system with correlated structural characteristics.

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