# A Robust and Automatic Algorithm for Structural Mode Tracking of Bridges Subjected to Operational Changes

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**Abstract.** Automated operational modal analysis of civil engineering structures and suspension bridges based on vibrational monitoring data can be performed at a reliable enough level to consider its outputs as correct interpretations of the modal dynamics in the measurement data. This work proposes a new robust and automatic algorithm for tracking the evolution of these detected modal properties over time. The algorithm requires only a few inputs that are easy to define and does not require prior knowledge of the target bridge. It can deal with imperfect modal detection data and distinguish between closely spaced modes. The algorithms functionality is illustrated using two numerical examples and one experimental example from the Hardanger Bridge monitoring project.

**Keywords:** Modal Tracking · Automatic OMA · Operational Modal Analysis · Long-Span Bridge

## 1 Introduction

Structural health monitoring (SHM) aims to identify, quantify, and qualify the state of infrastructure to help make operational, maintenance, and safety-related decisions. A research area within SHM is damage detection on bridges and although many different approaches to this topic exist, a common one is to use a damage indicator that, based on the input features from the bridge and prior knowledge of its behaviour pattern, can tell if the bridge has been damaged. Common input features for damage indicators are modal properties (frequencies, damping, mode shapes) of the structure [1–3]. This information is readily available from vibration measurements of the bridge; however, it is not easy to identify from one measurement series to the next if the new modal detections represent the same set of detections as from the previous dataset. This labelling of modal detections is crucial to compare one dataset to previous datasets. Labelling these modal detections consistently throughout the datasets is known as modal tracking.

For bridge monitoring, few explicit applications of modal tracking have been discussed. A vast majority of modal tracking is performed implicitly and is not explained (for example [4]). These cases need to be redefined for each specific application. They rely on no errors in the modal detections and maintaining the variations in modal properties to a minimum. He et al. [5] propose a more robust tracking method where each


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new mode realisation is combined with the nearest mode in terms of frequency and mode shape. This method tracks the evolution of modal parameters. Still, it requires that there are no false detections within the modal detections, an assumption which cannot be made in the case of AOMA. Favarelli and Giorgetti [6] suggest tracking the evolution of modal frequencies using Gaussian kernels fitted to the frequency detections of each structural mode. Although this method works with imperfect modal detection data, it cannot track closely spaced modes in terms of frequency.

This work proposes a new robust modal tracking algorithm which responds to all these problems, as it can be deployed on all structures with minor or no changes to the algorithm's parameters, can deal with imperfect and false modal detections, and can detect closely spaced and overlapping modes in terms of frequency.

## 2 Theory

Note: In cases where confusion may arise, a single mode (point) detected from a stabilisation diagram is referred to as a mode realisation, and the eigen-properties of a structure are referred to as structural modes.

The modes of a structure can be identified from vibration data recorded at different locations along the structure simultaneously, through a process known as operational modal analysis (OMA). Many OMA algorithms exist, in both the time and the frequency domain. For brevity's sake, and because these methods are well documented (see for example [7]), they are not explained here. Covariance-driven stochastic subspace identification (cov-SSI) is used in this work.

The output of operational modal analysis traditionally needs to be interpreted manually to select the set of real modes detected from the dataset. As this is a time-consuming, labour-intensive task, automating it has seen a lot of research since the early 2010s, leading to many automatic operational modal analysis (AOMA) algorithms. They vary in their degree of automation (some need more fine-tuning and setting up than others) and their output quality [8]. This work uses the AOMA algorithm Kvåle 2020 algorithm [9], which has been shown to work well and requires minimal setup [8]. The Kvåle 2020 algorithm starts by clearing the stabilisation diagram resulting from the cov-SSI of the vibration data, removing all poles with a too large relative change compared to their nearest neighbour at the next inferior order. Once this is performed, the algorithm called HDBSCAN is applied to the pairwise distance in frequency and mode shape between all poles remaining in the diagram. This density-based clustering algorithm removes outliers within the data and returns clusters representing the structural modes. The outcome of the Kvåle 2020 algorithm is the average features of each cluster.

Highlighting and quantifying the evolution of the outcome of the (automatic) operational modal analysis (A)OMA – the automatization step is not necessarily required – is known as modal tracking. The outcome of AOMA is not always perfect. There can be real modes which are not identified in the dataset, modes which are detected multiple times, or false detections which do not represent a real mode. An ideal modal tracking algorithm should be able to deal with these issues. In the following section, a modal tracking algorithm is put forth capable of dealing with all these potential issues.

### 2.1 The Proposed Modal Tracking Algorithm

We introduce a novel, near-automatic, probabilistic two-step algorithm for modal tracking. The first step is used to setup the modal tracking procedure, and the second step is concerned with iteratively attributing new realisations to the existing modes. The algorithm is near-automatic, as it only relies on a few intuitive and readily available input parameters. It does not require prior modal information which can be difficult and resource-consuming to obtain.

The modal tracking algorithm can be initiated once a threshold number of baseline datasets containing modal information become available.

**Step 1: Defining the Structural Modes.** Once a threshold number of processed datasets are available, the first step of the modal tracking algorithm is to detect which structural modes are present and are supposed to be tracked.

Frequency, damping, and mode shape are the most common and naturally interpretable attributes of a modal detection. Of these attributes, the mode shape is the most likely to be a unique classifier. Two modes could have the same frequency or damping values, but not likely the same mode shape. For this reason, the mode shape is used to identify the number of structural modes. The MAC number is used to quantify the similarity of two mode shapes; the inverse MAC value, define as $(1 - \mathrm{mac}(\phi_1, \phi_2))$ is calculated for each mode realisation pair. The pair-wise similarity information is fed into a hierarchical clustering algorithm which iteratively groups the two most similar mode realisations into a single cluster, continuing until all the distances between the resulting clusters are above a set limit. The mode realisations belonging to the same structural mode are clustered together due to the high mode shape similarity, and simultaneously they will be separated from all other mode realisations not belonging to the same structural mode, because of their mode shape dissimilarity exceeding the clustering limit. The large clusters resulting from the hierarchical clustering process are selected as the structural modes to be used in the second step.

Single linkage hierarchical clustering is used because when clusters are merged at each step and the distance matrix is updated, the smallest values are retained, making the subsequent cut-off limit selection a more data-related task, facilitating its selection for the operator.

Once all the clusters are final, only the largest ones are retained. Large clusters are determined as any cluster containing more mode realisations than a predetermined percentage of the total number of datasets used in this step. To account for the potentially high variability of the mode realisations present in each dataset, due to different excitations states of the structure when the data was recorded and the imperfect nature of (A)OMA, the initial number of files is suggested to be around 100, the cut-off limit in the hierarchical process to be chosen to match a MAC value of 0.85, and the minimum number of mode realisations in a cluster for it to be considered large to be 15% of the number of initial files.

**Step 2: Iterative Modal Tracking.** In the second step, new datasets containing mode realisations are considered individually and one after the other as they become available. This second step is decomposed into three sub-steps. A flow chart of the full modal tracking algorithm is shown in Fig. 1. The zero-th, set-up, sub-step is to quantify the properties of each structural mode. To do so, the latest $m \in \mathbb{N}^+$ datasets are selected to determine the mean $\mu_f^{(i)}$ and variance $\sigma_f^{2,(i)}$ of the frequency of the structural mode. $m$ is the memory of the tracking algorithm and is an operator-selected value, influencing the adaptability speed to changes in the structural modes. A tally $n_m^{(i)}$ of the number of mode realisations present for each structural mode $i$ within the defined memory period is constantly maintained. It is possible that some datasets do not contain a mode realisation for each structural mode, hence: $n_m^{(i)} \leq m \forall i$.

The first sub-step marks the first iterative step. A new dataset is analysed, and each mode realisation within is compared to each structural mode to identify to which structural mode it belongs. If a mode realisation is too different from all structural modes, it is regarded as a false detection of the (A)OMA and not attributed to any structural mode. If two or more mode realisations are identified as belonging to the same structural mode, only the mode realisation with the highest probability of belonging to the structural mode is retained. The likelihood of a mode realisation $R$ (with frequency $r_f$ and mode shape $r_\phi$) belonging to a structural mode is calculated as the product of two probabilities from two fitted distributions. One distribution is a student-t distribution describing the frequency fit and the second is an exponential distribution describing the mode shape fit.

$$p(R) = p(r_f) \cdot p(r_\phi) = T\left(r_f | \mu_f^{(i)}, \sigma_f^{2,(i)}, r_m^{(i)}\right) \cdot Exp\left(\overline{\text{mac}}(r_\phi) | \lambda\right) \tag{1}$$

The student-t $\mathcal{T}(\cdot)$, distribution of each structural mode is set based on the mean and the variance of the last $m$ realisations to be attributed to the structural mode, and the degree of freedom is $n_m^{(i)}$. The exponential distribution Exp$(\cdot)$ used to determine the matching probability of the mode realisation shape to the structural mode is set using:

$$\lambda = \frac{1}{m^2} \sum_{a \in M^{(i)}} \sum_{b \in M^{(i)}} (1 - \text{mac}(\phi_a, \phi_b)) \tag{2}$$

The mode shape comparison of the new mode realisation and the structural mode is calculated in a similar fashion:

$$\overline{\text{mac}}(r_\phi) = \frac{1}{m} \sum_{a \in M^{(i)}} \left(1 - \text{mac}(r_\phi, \phi_a)\right) \tag{3}$$

If the new mode realisation is outside of the 98$^{\text{th}}$ percentile of any of the two probability distributions, it is considered too different from all structural modes.

The second sub-step is to update the structural modes' properties, known as the short-term memory, before a new dataset is considered. A structural modal which saw a new mode realisation associated with it, has its frequency and damping statistics updated to include the latest mode realisation and exclude the oldest term in the memory. A structural mode which sees no new association keeps the same properties as previously, except that the tally $n_m^{(i)}$ is decreased by one.

Steps 1 and 2 are then repeated for each new dataset made available. During this process, a ledger, a long-term memory, of all the associations is kept, as the short-term memory does not maintain information dating further back than $m$ iterations.

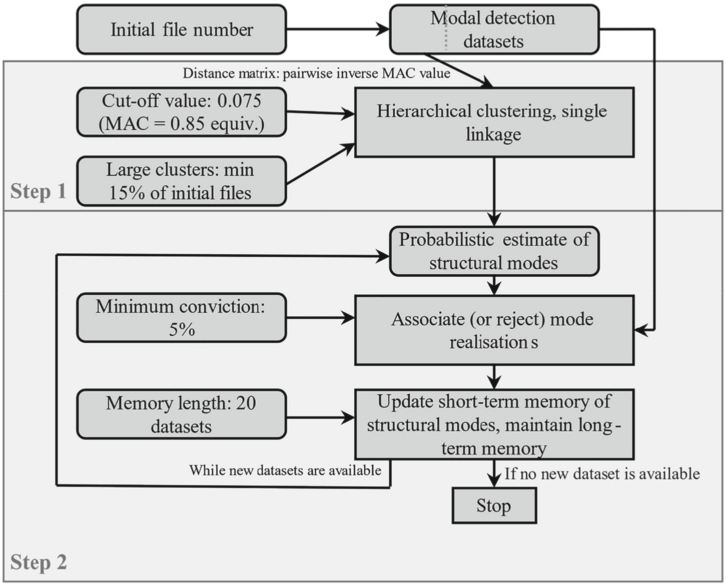


**Fig. 1.** Flowchart illustrating the proposed modal tracking algorithm.

## 3 Examples and Test Results

The modal tracking algorithm's performance is illustrated using three different cases. Two are based on numerical data simulated from an 8-storey 2-D shear frame excited by a white noise process, and a third is using real experimental data recorded at the Hardanger bridge in South-Western Norway [10]. All three cases are composed of 450 datasets from which the mode realisations have been extracted using the Kvåle 2020 automatic operational modal analysis algorithm. The modal properties are obtained using cov-SSI. In the first case, no alterations are made to the shear frame during the initial 100 datasets, to provide a clear baseline for the first step of the modal tracking algorithm. During the remaining 350 datasets, the one of the columns' stiffness is gradually reduced. The reduction in stiffness leads to a change in the natural frequencies of up to 1.5%. In the second case, all the shear frames' columns' stiffnesses are reduced by 40% over the last 350 datasets, leading to large changes in the structural modes' frequencies. Furthermore, the reduction in stiffness is largely concentrated over 50 datasets. Like in the first case, the first 100 datasets do not feature any variations to the shear frame to provide a clear estimate of the initial modes. Both numerical simulation cases have added measurement noise, are sampled at 200 Hz, and use a 2-min signal for the mode identification. For the third case, all datasets are from chronologically ordered vibration recordings. All datasets are subject to the natural changes in modal characteristics of the structure exposed to variable loading conditions, providing a more significant challenge for the first step of the modal tracking algorithm to estimate the structural modes. Each vibration recording is 10 min long and sampled at 10 Hz.

### 3.1 Case 1: Shear Frame with Minor Variations

The shear frame with small gradual changes to one of its columns' stiffness is designed to be an easy reference case to highlight how the modal tracking algorithm deals with the variations present in the outcome of (A). The memory length used in this case is $m = 20$. Figure 2 and Fig. 3 illustrate all the mode realisations for all datasets (the inputs into the modal tracking algorithm) (black points) and the result of the tracking process (coloured points). As can be seen, all false detections made by the (A)OMA algorithm are rejected by the modal tracking algorithm leaving clean and well-defined structural modes.

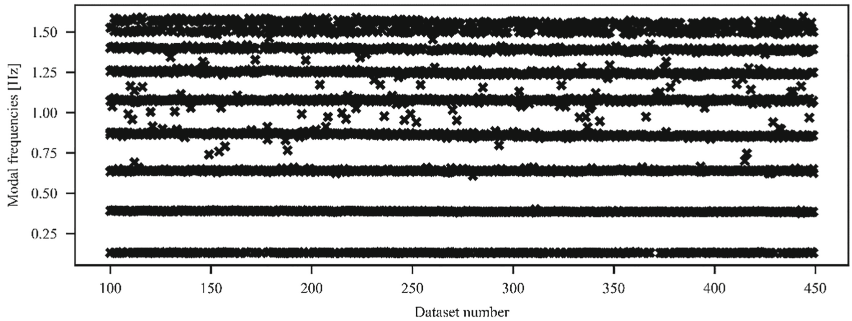


**Fig. 2.** Shear frame 1, frequency values of all mode realisations of all datasets.

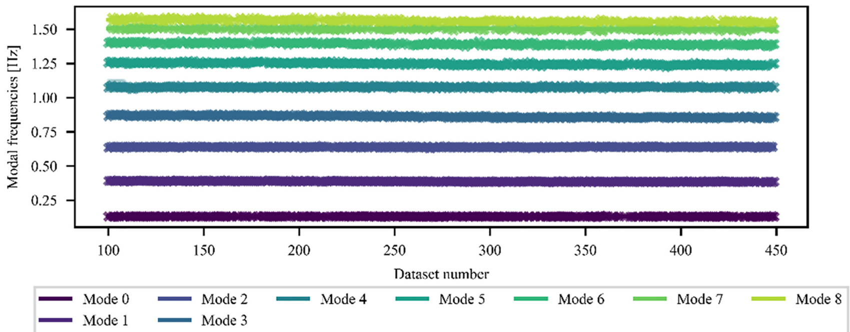


**Fig. 3.** Shear frame 1, subjected to the modal tracking algorithm.

### 3.2 Case 2: Shear Frame with Large Variations

The second shear frame case which is subject to a large and rapid decrease in all columns' stiffnesses, is used to illustrate the modal tracking algorithm's potential in tracking marked changes to the structure's modal properties. The best tracking results are achieved with a memory $m = 15$ because this allows a quicker response to changes and a more relaxed confidence interval for the modal frequency acceptance, due to the lower degrees of freedom of the student-t distribution. Figure 4 shows the outcome of the modal tracking results, which shows that all nine modes are tracked throughout the datasets. No false detection made by the (A)OMA algorithm is retained in the tracking process. Mode 8 changes by approximatively 0.1 Hz (7% relative change) in frequency within 20 datasets. This abrupt change is tracked without issue by the modal tracking algorithm.

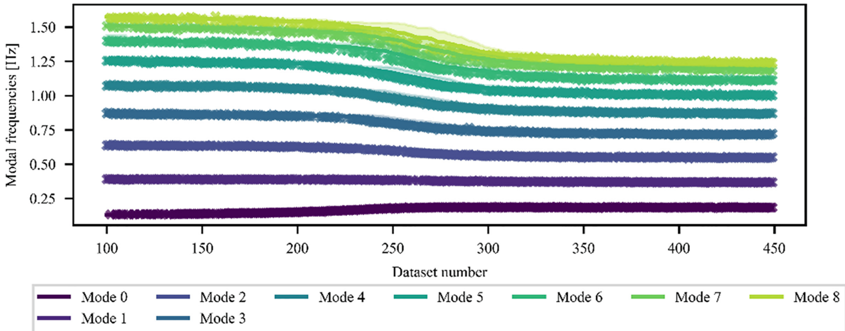


**Fig. 4.** Shear frame 2, result of the modal tracking process.

### 3.3 Case 3: Experimental Data from the Hardanger Bridge

The Hardanger bridge data contains natural variations in modal properties due to changes in wind excitation – the fluid-structure interaction leads to aerodynamic damping and stiffness [11]. For lack of space, the reader is referred to [10, 12] for a description of the bridge, the measurement system, and its dynamic properties. The variations are also present in the initial data used to identify the structural modes, rendering this task more complicated than idealised synthetic data. The results show that the modal tracking algorithm can identify all 13 modes known to be present and identifiable in the [0–0.425] Hz range [12]. Figure 5 shows how the modal tracking algorithm follows all the modes throughout all 350 tracking datasets. This includes structural modes which are closely spaced in frequency, such as modes 1 and 2, and 6 and 7, structural modes which are not regularly detected by the (A)OMA, such as mode 8 between datasets 200 and 300, and structural modes which exhibit considerable variations to their frequency content over few datasets, such as mode 2. Figure 6 is a focused view of modes 1 and 2 from the Hardanger bridge data and highlights the variations in frequency for mode 2. The mode varies between 0.11 Hz and 0.12 Hz over about 50 datasets, a change that the modal tracking algorithm can follow.

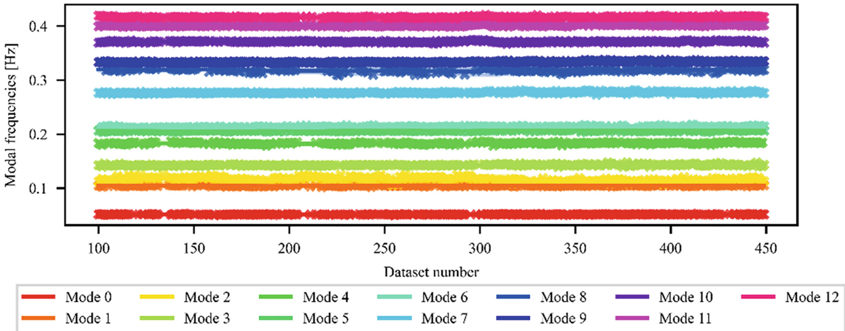


**Fig. 5.** Result of the modal tracking for the Hardanger bridge data.

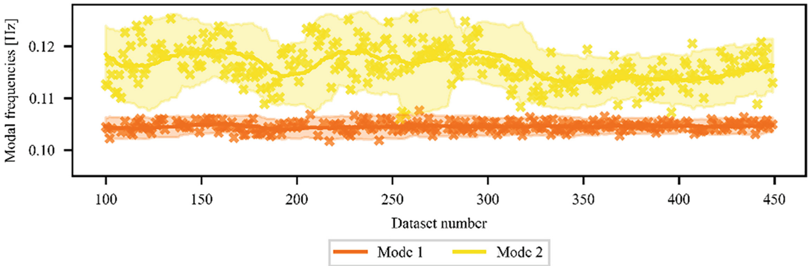


**Fig. 6.** Result of the tracking of the Hardanger bridge data, with a zoom into modes 1 and 2. The shaded areas around each mode mark the 95% confidence range used to accept or reject mode realisations based on frequency for the given dataset while the tracking was performed. The solid line indicates the mean value of the frequency of the short-term memory.

## 4 Discussion

The modal tracking algorithm can follow the structural modes' evolution in a satisfactory fashion for the right input parameters. The proposed algorithm is near-automatic, requiring few inputs from the user. The required inputs are chosen to be as intuitive to select as possible. Five parameters are required as inputs in this algorithm. Firstly, the initial number of files is used to determine which structural modes are present in the data. Ideally, this value should be as large as possible, as more files will lead to more chance of detecting all the structural modes present in the data. An operator can select this value as the largest reasonable amount of dataset they are willing to sacrifice from what is tracked. For example, in an intense measurement campaign of one month with measurements made around the clock every thirty minutes, 100 datasets would be equivalent to two of the 30 days of data. This is a reasonable portion of the data. The minimal number of mode realisations in each hierarchical cluster is more difficult to determine. A low percentage increases the chance of detecting all the structural modes, but if too low, false modes will also to be included as structural modes.

The MAC value threshold for the hierarchical clustering relates to the subjective question of what value represents a good mode shape fit. This question does not have a definitive answer, but a value around 0.85 is considered a decent estimate of a good mode shape fit.

Defining the confidence range of the student-t and exponential distributions intended to catch false detections made by the (A)OMA is like well-known outlier detection problems in statistics where 95% or 99% certainty is required to claim there is a statistical difference. In this case, the higher the percentage, the more modes will be kept as part of the modal tracking, at the risk of including a false detection. In a case where the modal properties are not expected to change rapidly, a lower confidence bound can be used to reduce the chance of including outliers. Conversely, if rapid changes in modal properties are expected, it may be necessary to sacrifice some precision to keep up with the changes by introducing a higher confidence bound.

The short-term memory length also affects the reactivity to change of the modal tracking algorithm. A shorter memory will have a higher reactivity, but a memory length too short can negate these gains, as the variance of the frequency values becomes more sensitive with less points to define it. This leads to a smaller variance, which in turn reduces the confidence range in which mode realisations are considered to match a structural mode.

## 5 Conclusion

The proposed modal tracking algorithm can follow the modal properties of a structure over chronologically ordered datasets which are pre-processed with (A)OMA to extract their modal features. It is nearly fully automated with only a few easy-to-select parameters to be chosen. The algorithm uses a two-step approach. The first step is used to identify the structural modes and the second step consists of matching and attributing new modal detections to the pre-defined structural modes. It is shown to capture the evolution of all the modes present in two synthetic test cases, with two different levels of modal property changes, as well as for experimental data from the Hardanger bridge.

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