Mathematical and Computational Thinking in Children's Problem Solving with Robots

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Introduction

In this paper, we investigate two young (3–4 years old) children's interactions with a programmable floor robot. With the integration of computational thinking (CT) into mathematics in the Norwegian school curriculum (Kunnskapsdepartementet, 2019), there is a need to investigate whether there are overlaps between CT and mathematical ideas in how young children engage in programming tasks, in barnehage, Norwegian early childhood centres. Although young children's programming of robots has been researched for at least a decade (see, for example, Highfield, 2010), the focus has mostly been on intervention studies to do with CT (see, for example, Bakala et al., 2021). In their systematic review of previous research, Jung and Won (2018) found only one article that focused on how preschool children engaged with mathematical ideas and this was Highfield's (2010) intervention study. Although Palmér (2017) stated in her study which linked programming to mathematics, "there is a lack of studies on programming conducted in everyday preschool practices" (p. 76), hers was also an intervention study. There is, therefore, a need for research that investigates "what already is" as well as "what ought to be", as Palmér (2017) described the distinction in research types between naturally occurring situations and intervention studies. "What already is" research is important for understanding the children's point of view, which can then inform intervention studies.

Therefore, the research question is "what CT and mathematical understandings do children use when engaging in problem solving with robots at barnehage?" To answer this research question, we analyse the problems two children identified when working with robots, to determine potential relationships between

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mathematics and CT. To determine the potential intersections between, we first describe previous research on CT and mathematics in early childhood education.

The Intersection of Mathematics and Computational Thinking From Using Robots

We begin by briefly describing Bishop's (1988) six mathematical activities used in early childhood education and care (ECEC) research and how they related to research to do with robots, before describing aspects of computational thinking used by young children. Then we consider how computational thinking and mathematics have been related.

Bishop's (1988) six mathematical activities form the basis for mathematics in the Norwegian barnehage curriculum (Reikerås, 2008) and have been used extensively in research on ECEC in Scandinavia (see, for example, Fosse et al., 2020; Helenius et al., 2015). The six mathematical activities are: Playing Explaining; Designing; Locating; Measuring; and Counting. In early childhood mathematics, Playing has been connected to playing games through rule following and rule negotiation and also to problem solving (Helenius et al., 2016). Explaining is to do with how children show and describe their understanding (Fosse et al., 2020). Designing is about using mental images of shapes to design an artefact (Helenius et al., 2015). Locating is about how young children explore and describe themselves and other objects in space, through words, actions and drawings, including maps (Helenius et al., 2015). Measuring for young children is often associated with comparing attributes, either directly or indirectly, such as by using pencils to determine the length of something (Helenius et al., 2015). Counting involves understandings about discrete amounts and the relationship between those amounts, through, for example, one-to-one correspondence, dividing and combining objects into different groups, and using basic arithmetic (Helenius et al., 2015).

In studies related to the use of robots in ECEC, Locating was the most common of Bishop's (1988) six activities. For example, Highfield (2010) identified spatial concepts, including positional language and angle rotation, both aspects of Locating as they were to do with locating objects in space. In research from the first years of school in Panama, Muñoz et al. (2020) showed a similar use on location concepts when working with robots. In another intervention study, Angeli and Valanides (2020) investigated the computational thinking of five-to-six-year olds in ECEC. They hypothesised that children would not have difficulties with the commands to move forward and backward, but may have had difficulties with turning right and left. However, their pre-test results showed that only the command to move backwards was unfamiliar to children. The intervention provided experiences with the commands which seemed to lead to higher post-test results. Similarly, Di Lieto et al. (2017) found improvements in preschool children retaining visual-spatial knowledge in their working memory. Palmér's (2017) study also focused on improving children's spatial thinking and showed that engagement in programming activities with a robot likely resulted in changes in the post-test results of the eight preschool children in the study. Nevertheless, Clarke-Midura et al. (2021) posited that young children's developing coordination system, connected to difficulties matching their own movements to that of a robot, could be similar to the impreciseness shown in young children's early number sense understandings, suggesting there was a developmental progression that children moved through.

Other mathematical activities were present in some studies. In her description of spatial concepts, Highfield (2010) included transformational geometry, such as rotation, which is part of Bishop's (1988) mathematical activity of Designing. Highfield (2010) also identified concepts and processes that were similar to the mathematical activities of Measuring and Counting (Helenius et al., 2015). Palmér (2017) noted that to programme young children needed to have number understandings, in particular one-to-one correspondence, to relate the number of presses on the robot to the number of squares it was expected to move.

Young children's problem solving was also mentioned in most of the earlier studies about using robots in early childhood centres. Problem solving has been linked to Bishop's (1988) mathematical activity Playing, because problem solving often requires imagining "what if" scenarios (Helenius et al., 2016). Problem solving in programming robots has been highlighted as important (Fessakis et al., 2013). For example, Di Lieto et al. (2017) stated "educators claim that robotic 'hands-on' experimentation facilitates the transformation of abstract concepts into concrete and verifiable operations, promoting new perspectives for thinking and developing problem-solving skills" (p. 17). Given the emphasis on children engaging in problem solving in the barnehage curriculum (Fosse et al., 2020), it is valuable to consider the connections to CT.

Although definitions of CT are still debated, Bakala et al. (2021) stated that in research on robots in early childhood education, the most frequently included components of CT were, "algorithmic thinking, abstraction, decomposition, sequencing, generalization, and debugging" (p. 2).

Algorithmic thinking is often described as the ordering of actions for completing the whole task and so are linked to sequencing. Palmér (2017) considered that there was a relationship between the sequencing of actions and mathematics, "the children showed an ability to sequence, which includes, planning and putting objects (commands) in the correct order, which is important in both literacy and mathematics" (p. 83). Muñoz et al. (2020) found that at least half of the 4-to-5-year-old children could provide an appropriate sequence of actions for moving a robot, before their intervention began.

Decomposition is the ability to identify the parts of a program. Angeli and Valanides (2020) found that most of 5-to-6-year-old children "decomposed the task in a number of subtasks equal to the number of commands in the task and chose to execute one subtask at a time" (p. 10). They considered that this showed that children had the capability to break tasks down into small, more manageable steps. Palmér (2017) also noted that the children in her study decomposed the tasks into different sets of sub-tasks.

Debugging involves identifying issues in the running of the program and fixing them (Bakala et al., 2021). In earlier research, Palmér (2017) noted that it was often conflated with problem solving because debugging is usually described in relationship to fixing problems in the programs. Over half of the children in Muñoz et al.'s (2020) study were able to debug problems in programming a Bee-Bot at the start of the intervention. When preschool children could identify the problem, Lavigne et al. (2020) found that they were more able to fix it. In both Lavigne et al.'s (2020) study and Bakala et al.'s (2021) literature review, children were noted as successfully debugging or using more sophisticated debugging strategies, with the help of the teacher. These studies worked with older ECEC children, "children as young as 5 years old are able to debug through trial-and-error practices but could achieve more sophisticated debugging strategies if provided with the necessary scaffolding and learning opportunities" (Bakala et al., 2021, p. 9). Younger children, or children without the help of adults, may struggle with debugging, perhaps because they could not identify the problem or because they did not have the strategies to fix these problems.

Although most earlier research about young children programming robots illustrated links between mathematics and CT, these connections were rarely discussed. By starting with children's own problems with programming robots, our aim is to describe where the connections between mathematical understandings and CT understandings were important in their problem solving.

Methodology

As a part of a wider study about the use of digital apps in a barnehage, four, short video recording were captured serendipitously of a Blue-Bot robot being programme. A Blue-Bot can be programmed to move around a mat (see Fig. 1), by pressing buttons that represent the actions of going forward (Forward), going

Fig. 1 Task's layout and children

backwards (Backward), turning left 90° (Left Turn), and turning right 90° (Right Turn). A Start button when pushed starts the Blue-Bot moving through the programmed sequence of actions and a button which clears the program from the Blue-Bot's memory (Clear).

In the videos, two children (C1 in red dress, 4 years old, and C2 in grey, 3 years old) were attempting to programme the robot, with a teacher (T). The barnehage had a focus on using digital tools, but field notes indicated that the robot was a recent addition to the barnehage and the participants had limited previous experience with them. As the videos, showed children in a nutralistic setting, it provided an opportunity to explore a "What already is" situation.

To focus on how the children made sense of the programming of the robot in the naturalistic setting, we decided to identify when the children were unable to solve problems immediately. To do this, we looked for signs of uncertainty in the children's spoken utterances and in body language. It was decided to focus on young children's body language as it was likely to provide more information than their spoken utterances alone (Johansson et al., 2014). As a group, we watched the videos several times together to gain agreement on when the children showed uncertainty. We identified particular body actions that appeared in three of the four videos, which we agreed showed the children's uncertainty. These included gestures, like an open mouth or a finger in the mouth (see C1 in Fig. 2), averting the child's gaze from the adult and moving themselves away from the mat. Once uncertainty was identified, we considered what occurred before and after to determine what the problem was which had caused the uncertainty and if and how the problem was resolved in the interaction. If the tracing back indicated that the problem was not related to mathematics or CT, it was not analysed any further. Five problems were identified as concerning CT and mathematics.

Although some examples of other mathematical activities were apparent in the data, the problems that caused the children's undertainty were mostly about Bishop's (1988) mathematical activities of Locating and Counting, with all the problem solving situations being considered to be about Playing. If the problem was about identifying the route, positioning Blue-Bot on the map, or orientating it in the situation, it was classified as being about Locating (Bishop, 1988). When the child's problem was about the number of squares the Blue-Bot had to move, it was considered to be about the mathematical activity of Counting (Bishop, 1988).

In regard to CT, we deemed the problems to be about sequencing and decomposition, and to a lesser extent debugging. Sequencing was identified when the child struggled programming the Blue-Bot's actions in order. When the child focused on the individual actions of the programme, we classified this as decomposition. Debugging occurred when the children identified a problem with the programme, when the robot did not move to where they expected or wanted it to go, and tried to resolve it.

Fig. 2 Uncertainty shown by holding finger near the mouth

Results and Discussion

In this paper, we present three of the five problems, identified in the video recordings, which illustrated most clearly a potential relationship between understandings about sequencing and Locating, and between decomposition and debugging with Counting.

Problem 1

This problem occurred after C1 and C2 had already worked with the teacher to programe the Blue-Bot to move along a complex path to get to the castle square on the mat. After attempts to programme the whole sequence in one go, the teacher had supported the children to programme individual actions. In this episode, the teacher tried again to have C1 sequence a series of actions together, which would make the robot move four steps forward, turn right and then go another four steps forward. This program involved C1 engaging in algorithmic thinking through sequencing the set of actions and in decomposition, breaking the robot's path down into the different actions. For C1, integrating the turn into the program caused her uncertainty around how the two actions of going forward four squares were related to what she considered to be the robot's eight-square path.

The teacher began by asking where the robot would go next, starting from the castle square. C1 chose the green flower and counted to eight while pointing once to each square, to show the path (Figs. 3 and 4).

T asked, "How many do we have to count before it will turn?". C1 counted and pointed at the squares, "one, two, three, four {the square in the corner}" (See Fig. 5). T stated "Four!", while C1 continued, "five". T interrupted her, "Four! {T moved closer and pointed at the square} There are four {points again, looking into C1's eyes}, and then turn". C1 nodded twice slowly, then sat with her gaze on the mat, suggesting that she was confused about why she had to stop at four squares, when the whole path was eight squares.

Struggling with integrating the turn could be a problem about Locating (Bishop, 1988), although it was clear that C1 understood the proposed path for the robot. Therefore, it seems more likely that the confusion was over splitting the eight square path into two parts.

T then suggested programming the robot, C1 opened her mouth (see Fig. 6), but then slowly nodded, suggesting she remained uncertain. C1 began programming by pressing the Clear button. She then followed T's instructions to press the Forward button four times. T then told her to press the Right Turn button and asked her which direction the Blue-Bot had to turn. C1 looked at the corner and touched it with her right hand, before moving her finger over the Blue-Bot. Holding her hand over the corner square, T asked again about the direction. C1 touched the fifth square, saying, "This one", then she turned to the Blue-Bot and pressed the Right Turn button.

Fig. 3 Marking the starting point

Fig. 4 Marking the end point

Fig. 5 Coming to the turn

C1's actions and words reinforced that she was not confused about the direction of the robot's path (Figs. 7 and 8).

While pointing towards the remaining four squares, the teacher said, "Should we count how many times it has to go forward to the flower?" C1 nodded, then C1 and T pointed at two different positions on the mat (Fig. 9). This suggests that C1 did not understand that the path had to be split into two parts. T seemed to recognise that C1 was confused and so reinforced that the robot's path had to be split into two actions (decomposition). She said, "It stands here. {C1 moved closer to the corner}. It stands here and turns {T pointed to the next square (the same movement is shown in Fig. 10)}. Then you have to count from here {T pointed at the next square again.

Fig. 6 Showing uncertainty with open mouth

Fig. 7 T shows the turn direction

C1 nodded twice, with a slightly opened mouth} and onward" {T moved her index finger to indicate three moves towards the right}.

Tapping the next square, T continued, "This is one, {C1 held her hand on the corner square (see Fig. 10)} one. {T moved her finger to the right. C1 kept her hand on the corner.} Because it stands here {T pointed at the corner}." C1 moved her finger to the next square, saying "one" and proceeded to point and count "two, three". With C1, T pointed to the last square (Fig. 11). C1 said "four" and T agreed, "Four to the flower". As shown earlier, C1 did not show difficulties matching the number words to each of the squares as she moved along the path. However, it is unclear if she considered the final number to represent the total amount, or a position on the path.

C1 returned to the robot and T gave a direction, framed as a question "Will you press four times the Forward button?". C1 began to place her finger on the button, but then removed it, "I have already done it!". T replied, "Then we have turned.

Fig. 8 C1 shows the turn direction

Fig. 9 Two starting points for counting

First, we went four forward {T moved her hand along the route}, then turned right {T made a rotation gesture over the corner square}, then four more forward to come to the flower {T showed the rest of the route}. Now we are going to try how this will go. If you now press the Forward button four times". C1 pressed the button four times (see Fig. 12), but with her mouth slightly open, suggesting she remained uncertain. Although the uncertainty could be because the number of squares was the same for both parts of the path, it seemed more likely that what was unclear was each lot of four steps was related to the eight steps. This suggests it was decomposition, not algorithmic thinking, that C1 struggled with in the programming.

After pressing the start button (see Fig. 13), the robot began to move. C1 moved her hand to the end of the mat (Fig. 14), as she seemed to be uncertain that the robot would turn. When it did, C1 looked confused.

Fig. 10 T showing next square

Fig. 11 Joint counting

C1 could show the robot's proposed path, but she struggled with decomposing it into individual actions (four steps forward, turn right, four steps forward) and this impeded her programing the robot appropriately. As the Bee-Bot remained at the starting point, the relationship between the different parts of the path and the pressing of the buttons were hidden. Bakala et al. (2021) noted the high cognitive demands of programming on children as they had to remember the sequence of commands being put into the robot. This could explain some of the difficulties that

Fig. 12 Programming the robot

Fig. 13 C1 after pressing Start

C1 experienced with understanding how both forward actions were related to the eight squares she had counted.

In regard to her mathematical understandings, the child's uncertainty seemed only to some degree to be about Locating – how the turn affected where the robot went. Rather understandings about Counting seemed to more likely to be contributing to her uncertainty. Although she showed one-to-one correspondence between the counting words, the squares and the pressing of the buttons, C1 seemed not to recognise that the total amount of squares, eight, was the same as two groups of four. This requires understanding about addition to do with total amounts being composed of smaller amounts and how this relates to reciting counting words

Fig. 14 Blocking with her hand

(Baroody, 1987). It may be that the child used the counting words to mark the order of squares and as a result has an ordinal, rather than a cardinal understanding of number, which has been noted as typical for children of this age (Bruce & Threlfall, 2004). Nevertheless, by holding her hand at the end of the mat, C1 seemed to be predicting that the robot would not turn (Fig. 14), suggesting that she saw that her number of presses of the go-forward button would result in the robot moving further than the original four. This suggests that C1 did have some understandings of cardinality (Bruce & Threlfall, 2004). These results suggest that for this child the CT aspects of decomposition and algorithic thinking are connected to Counting, highlighting the need for children to have understandings about cardinality and early addition.

Problem 2

In this episode, C2's problem seemed to be about the Blue-Bot not stopping on the boat square, her chosen end point, which was three squares up from her starting point in the bottom left-hand corner. C2 had no difficulty locating the straight path of the robot. However, the relationship between the number of squares, reciting the counting words and the number of pushes of the Forward button caused some difficulties.

C2 with the Blue-Bot nearby, counted, "One {touches the yellow square}, two {touches the blue square}, three, four, five {holds fist on the boat square for three counts}." T checked, "Will it go to the boat? {T touched the boat three times}." C2

Fig. 15 Blue-Bot going off the mat

Fig. 16 T clearing the previous program

replied "Yes, like this" while putting the Blue-Bot on a yellow square. T asked about where the Blue-Bot should be, as C2 pressed the Start button. The robot ran through the previous program and consequently moved off the mat. C2 tried to stop it with her hand (Fig. 15) and T had to assist C2 to stop the Blue-Bot running through the rest of its program (Fig. 16). C2 seemed surprised when it did not stop on the boat square, using her hand to impede its progress. This suggested she was uncertain about why this had occurred.

T finally stopped the robot and cleared its memory, "C2, where should we start?" C2 said "One, two, three, four {Touched the squares individually as she said the number word (see Fig. 17)}, five {touches boat square for second time}". T placed the Blue-Bot on the corner square, where C2 started to count. This suggested that C2 could identify a path for the robot by pointing at the squares.

Fig. 17 C2 touching yellow square on "Two"

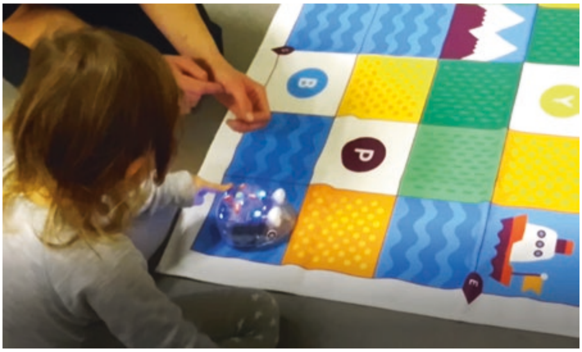

Fig. 18 C2 pressing the Clear button

T then told C2 how to program the robot to move three squares, "We have to press the Clear button first {T pointed and C2 pressed the Clear button (Fig. 18)}. Then we have to count how many times it is to there {T pointed to the squares}". C2 counted "One, two, three." As C2 could match the counting words to her pointing to individual squares, it seemed that she had understood the path the robot was to take and had some number understandings connected to one-to-one correspondence.

Although T asked C2 to press the Forward button three times, C2 kept pushing the button. T stated "You have to count! Wait. {C2 stopped}. Can you press the Clear button again? Then you can press the Forward button three times." C2 counted to three again, but looked like she might keep going. T said "Stop! Only three right?{C2 kept pushing the button while counting to seven} Oi! That was many. {T shrugged her shoulders}. Shall we see what happens?" C2 pressed the Start button and the Blue-Bot went past the boat square. T said, "He passes by! (Fig. 19)" C2 laughed and stopped the robot with her hands (Fig. 20).

After a lot of support from T, the robot was eventually programmed to go forward three steps. However, C2 caught the Blue-Bot with her hand as it approached the boat square as though she was unsure it would stop.

Fig. 19 Surprise at the robot going passed the boat

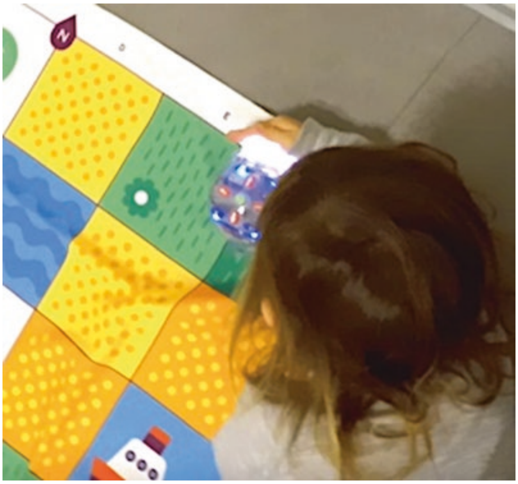

Fig. 20 Blocking the robot with her hand

C2 did not show uncertainty in the same way as C1, but indicated there was a problem when the robot did not go where she had expected, by physically picking it up while the program was running (Fig. 15), turning to the teacher when the robot did not stop where she wanted it to, putting her hand in front of the Blue-Bot to stop it from leaving the mat (Fig. 19), or holding it from behind (Fig. 20). C2 was aware that her programming of the robot did not result in it stopping on the boat square.

Although Lavigne et al. (2020) found that children were more able to debug a program if they could identify errors, it seems that this is only the case when they have the necessary skills or interest in learning how to fix the bugs. In C2's case, she seemed uninterested in matching the counting words to the squares the robot had to pass. The child's wish for the robot to stop on the boat square seemed to be secondary to her delight in reciting the number words. So, although C2 could identify the problem, fixing it did not seem to be incentive enough for her to focus on the one-to-one correspondence, even with the support of the teacher. Children of this age can be taught to recite the counting words, without an appreciation of how the counting words relate to amounts (Bruce & Threlfall, 2004). Programming the robot so it would stop where she wanted it to stop did not seem to be sufficient incentive to learn more about how counting could support her problem solving.

Problem 3

In the first problem, C1 showed uncertainty about incorporating the Right Turn command in between the two actions of going forward four steps. In this episode, C1 solved the issue by not including a turn into the robot's path. In so doing, she adopted a typical problem solving strategy of simplifying the problem so that it became more manageable, a common strategy promoted for older children to use at school (Barham, 2020).

This interaction began as the others had, by the teacher asking the child to chose a starting and finishing square. T asked, "Where will it go now?" C1 looked at the mat, stretched her index finger and slowly moved her hand towards the square with a tree (Fig. 21), then replied, "Tree. {C1 touched the middle of the tree, lifted her hand up then turned and smiled at T}." T checked with C1 that this was to be the end point, "To the tree? {C1 nodded and smiled}. Where should we start?" After a pause, C1 stated, "We start there! {She touched the yellow square above the tree square (Fig. 22)}."

T then suggested that C1 put the Blue-Bot on the start square. As C1 did this, T asked "And what [button] do we have to push first?". C1 looked at the mat, "This one {C1 pointed to the square with the tree}." Her mouth was open, suggesting she was a little uncertain. T then gave a direction in the form of a question, "But C1, first, we have to press the Clear button. Right?" C2 seemed to remain uncertain by

Fig. 21 Pointing to the end point

Fig. 22 Pointing to the starting point

Fig. 23 Showing uncertainty about pushing Clear

holding her mouth open as she pressed the Clear button (see Fig. 23). T reinforced her movement with, "Yes."

C1 moved her hand towards the Forward button, then took it away before holding it over the Turn Right button. She then moved her hand away from the robot (Fig. 24) and turned to T. C1 said, "No turn! {C1 smiled}." T replied, "No turn {T shook her head}. Okay. But what then?" C1 replied with, "It is one. {C1 pointed with index finger at the tree square while looking at T, suggesting that she was referring to the path being one square long}." T responded by asking, "Straight forward?" T and C1 nodded to each other. C1 followed with, "I have to push once. {C1 pressed the Forward button once}." After some reassurance from the teacher, C1 pressed the Start button and the Blue-Bot moved to the tree square and stopped.

According to Muñoz et al. (2020) 4-to-5-year-old children can provide an appropriate sequence of actions for moving a robot without help. However, C1 who was

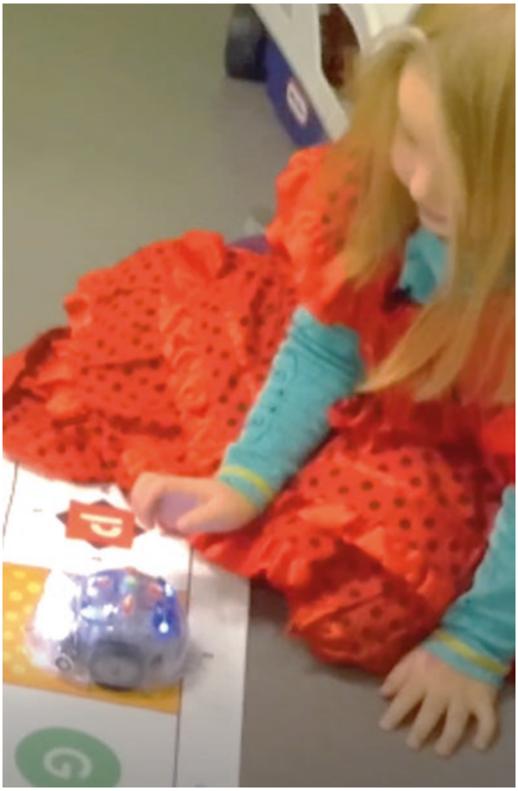

Fig. 24 Uncertainty about which button to press

4 years old solved the issue from Problem 1 by identifying a one-step path, which eliminated the need to incorporate a turn and split a path into two (or more) shorter ones. This can be seen in her exclamation "No turn!" She also simplified the number of steps the robot had to travel to the smallest amount possible, suggesting that she might have been aware that her understandings of how numbers worked was insufficient to solve more complex problems, such as Problem 1.

Conclusion

Earlier research on children's engagement with programming floor robots has mostly been through intervention studies (see for example, Muñoz et al., 2020). In our small study, we found similar overlaps between mathematics and computational thinking to those noted earlier, such as location with sequencing and decomposition (see Angeli & Valanides, 2020). However, by focusing on the children's uncertainty, we identified problems from their perspective. As a result, we have been able to show how different understandings about Counting contributed to their possibilities and willingness to solve those problems. Although Palmér (2017) noted the importance of number understandings, she highlighted one-to-one correspondence. However, C1 and C2 both showed some understanding of the need to match each number word to each push of the Forward button. However, C2 seemed uninterested in matching the number words to the squares in her path, often counting the final square more than once even if she pointed and counted simultaneously. C2 seemed to get more enjoyment from just reciting the counting words than programming the robot, so it would stop at the chosen square. C1 on the other hand showed that she was interested in having a program that resulted in the robot arriving at the end stop appropriately. Her problem seemed to be in inserting the turn because the 8 step path that she saw now consisted of two four-step paths (with the turn in the middle). This seemed to be connected to a lack of understanding about how eight steps could be made up of smaller amounts. C2 overcame this issue by identifying a path for the robot which did not require a turn.

Floor robots only have limited possibilities to move (forward or backward and turn left or turn right), so it was surprising to find that young children's understandings about Counting (Helenius et al., 2016) have not been documented as contributing to their understandings about sequencing, decomposition and debugging previously. Yet, as can be seen in our two examples, if the children do not have the appropriate Counting understandings, it becomes very difficult to determine by themselves or even with the teacher's help how to resolve the problem. Although the teacher in both episodes ensured that the problems were solved, it is unclear if either child understood how this had been achieved.

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