Controllable Fuzzy Neutrosophic Soft Matrices

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1 Introduction

The concept of fuzzy sets was founded by Zadeh [19]. Intuitionistic Fuzzy Sets (InFuSes) introduced by Atanassov [2] are appropriate for such a situation. But the intuitionistic fuzzy sets can only handle the incomplete information considering both the truth-membership (or simply membership) and falsity-membership (or non-membership) values. It does not handle the indeterminate and inconsistent information, which exists in belief system. Smarandache [16] introduced the concept of Neutrosophic Set (NeSe), which is a mathematical tool for handling problems involving imprecise, indeterminacy, and inconsistent data.

In our regular everyday life, we face situations that require procedures allowing certain flexibility in information processing capacity. Molodtsov [12] addressed soft set theory problems successfully. In their early work, soft set was described purely as a mathematical method to model uncertainties. The researchers can pick any kind of parameters of any nature they wish in order to facilitate the decision-making procedure as there is a varied way of picturing the objects.

Maji [10] have done further research on soft set theory. Presence of vagueness demanded Fuzzy Soft Set (FuSoSe) to come into picture. But satisfactory evaluation of membership values is not always possible because of the insufficiency in the

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available information (besides the presence of vagueness) situation. Evaluation of non-membership values is also not always possible for the same reason, and as a result, there exists an indeterministic part upon which hesitation survives. Certainly fuzzy soft set theory is not suitable to solve such problems. In those situations, Intuitionistic Fuzzy Soft Set theory (InFuSoSs) [11] may be more applicable. Now in the parlance of soft set theory, there is hardly any limitation to select the nature of the criteria, and as most of the parameters or criteria (which are words or sentences) are neutrosophic in nature, Maji [9] has been motivated to combine the concept of soft set and neutrosophic set to make the new mathematical model neutrosophic soft set and has given an algorithm to solve a decision-making problem.

The theory of a fuzzy matrix is very useful in the discussion of fuzzy relations. We can represent basic propositions of the theory of fuzzy relations in terms of matrix operations. Furthermore, we can deal with the fuzzy relations in the matrix form. In the study of the theory of fuzzy matrix, a canonical form of some fuzzy matrices has received increasing attention. For example, Kim and Roush [8] studied the Idempotent fuzzy matrices. Xin [18] introduced the idea for Convergence of powers of controllable fuzzy matrices. Padder and Murugadas [15] are presented the max-min opetarion on InFuMa. Broumi et al. [3] redefined the notion of neutrosophic set in a new way and put forward the concept of neutrosophic soft matrix and different types of matrices in neutrosophic soft theory. They have introduced some new operations and properties on these matrices. The minimal solution was done by Kavitha et al. [5], based on the notion of FuNeSoMa given by Arokiarani and Sumathi [1]. As time goes, some works on FuNeSoMa were done by Kavitha et al. [4–7]. The Monotone interval fuzzy neutrosophic soft eigenproblem, and Monotone fuzzy neutrosophic soft eigenspace structures in max-min algebra and Solvability of system of neutrosophic soft linear equations were investigated by Murugadas et al. [13, 14]. Also, two kinds of fuzzy neutrosophic soft matrices presented by Uma et al. [17].

In this chapter, we study and prove some properties of controllable and Idompotent FuNeSoMas. However, we have developed an algorithm for controllable and nilpotent FuNeSoMas and reduce a controllable FuNeSoMa to canonical form. One of these results enables us to construct an idempotent and controllable FuNeSoMa from a given one, and this is the main result in the chapter.

2 Preliminaries

For basic result refer [5, 16–18].

2.1 Main Results

Let $R = \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle$ and $S = \langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle$ be square FuNeSoMa with elements in [0,1].

- $R \vee S = [\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \vee \langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle]$,
- $R \wedge S = [\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \wedge \langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle]$,
- $R \ominus S = [\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \ominus \langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle]$,

$$\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \ominus \langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle := \begin{Bmatrix} \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle & If & \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle > \langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle \\ \langle 0, 0, 1 \rangle & If & \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \leq \langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle \end{Bmatrix}$$

- $R \times S = [(\langle r_{i1}^T, r_{i1}^I, r_{i1}^F \rangle \wedge \langle s_{1j}^T, s_{1j}^I, s_{1j}^F \rangle) \vee (\langle r_{i2}^T, r_{i2}^I, r_{i2}^F \rangle \wedge \langle s_{2j}^T, s_{2j}^I, s_{2j}^F \rangle) \vee \ldots \vee (\langle r_{in}^T, r_{in}^I, r_{in}^F \rangle \wedge \langle s_{nj}^T, s_{nj}^I, s_{nj}^F \rangle)]$,
- $R^{k+1} = R^k \times R, \ \ k = \{0, 1, 2, \ldots\}$,
- $R^0 = I$,
- $R' = \langle r_{ji}^T, r_{ji}^I, r_{ji}^F \rangle$ the transpose of R,
- $\Delta R = R \ominus R' = \Delta \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle = \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \ominus \langle r_{ji}^T, r_{ji}^I, r_{ji}^F \rangle$,
- $\nabla R = R \wedge R' = \nabla \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle = \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \wedge \langle r_{ji}^T, r_{ji}^I, r_{ji}^F \rangle$,
- $R \leq S$ iff $(\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \leq \langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle \forall\, i, j \in \{1, 2, \ldots, n\})$,
- $R \Psi S$ iff $(\langle s_{ij}^T, s_{ij}^I, s_{ij}^F \rangle = \langle 0, 0, 1 \rangle \Rightarrow \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle = \langle 0, 0, 1 \rangle \quad \forall i, j \in \{1, 2, \ldots, n\})$,

FuNeSoMa R is said to be

- Transitive if $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle^2 \leq \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle$;
- Idempotent if $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle^2 = \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle$;
- Nilpotent if $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle^n = \langle 0, 0, 1 \rangle$;
- Symmetric if $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle = \langle r_{ji}^T, r_{ji}^I, r_{ji}^F \rangle$;
- ST iff for any index $i, j, k \in \{1, 2, \ldots, n\}$, with $i \neq j, i \neq k, j \neq k$, such that $\langle r_{ik}^T, r_{ik}^I, r_{ik}^F \rangle > \langle r_{ki}^T, r_{ki}^I, r_{ki}^F \rangle$ and $\langle r_{kj}^T, r_{kj}^I, r_{kj}^F \rangle > \langle r_{jk}^T, r_{jk}^I, r_{jk}^F \rangle$, we have $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle > \langle r_{ji}^T, r_{ji}^I, r_{ji}^F \rangle$;
- Strictly Lower (Upper) Triangular (SL(U)T) if $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle = \langle 0, 0, 1 \rangle \forall i \leq j (i \leq j)$.

Theorem 1 *Consider a NiFuNeSoMa N and Symmetric FuNeSoMa (SyFuNeSoMa) S. For a FuNeSoMa R given by* $R = N \vee S \exists$ *a Pemutation FuNeSoMa (PeFuNeSoMa)* $P \ni T = \langle t_{ij}^T, t_{ij}^I, t_{ij}^F \rangle = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F \rangle \times \langle r_{ij}^T, r_{ij}^I, r_{ij}^F \rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F \rangle$ *satisfies* $\langle t_{ij}^T, t_{ij}^I, t_{ij}^F \rangle \geq \langle t_{ji}^T, t_{ji}^I, t_{ji}^F \rangle$ *for* $i > j$.

Proof $\langle t_{ij}^T, t_{ij}^I, t_{ij}^F\rangle = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$
$= \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times (\langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle \vee \langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle) \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$
$= (\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle) \vee (\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle)$.
Since N is NiFuNeSoMa, $(\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle)$ becomes strictly lower triangler for some PeFuNeSoMa P.

Thus since $(\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle)$ is symmetric,

T satisfies $\langle t_{ij}^T, t_{ij}^I, t_{ij}^F\rangle \geq \langle t_{ji}^T, t_{ji}^I, t_{ji}^F\rangle$ for $i > j$ by choosing such a PeFuNeSoMa P.

Remark 1 If $N = \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle$ is NiFuNeSoMa, then there exists a PeFuNeSoMa P such that $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$ is SL(U)T.

Remark 2 The NiFuNeSoMa, R has not less than a null row and atleast one null column.

Remark 3 If R is NiFuNeSoMa iff $\langle r_{ii}^T, r_{ii}^I, r_{ii}^F\rangle^{(k)} = \langle 0, 0, 1\rangle$, to a little $i \in I_n$(Index) and little $k \in I_n$, for $R^k := [\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle^{(k)}]$.

Theorem 2 *For any FuNeSoMa R,* $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle = \Delta\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \vee \nabla\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle$.

3 Controllable Fuzzy Neutrosophic Soft Matrices

Here we establish some basic properties of FuNeSoMas. In the ensuing discussion, we pact only with SqFuNeSoMas.

Proposition 1 *For a FuNeSoMa N. If* $\exists$ *a PeFuNeSoMa P* $\ni$ $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$ *is SL(U)T, then N is a NiFuNeSoMa.*

Proof Let

$$S = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle = \begin{bmatrix} \langle 0,0,1\rangle & \langle 0,0,1\rangle \\ & \ddots \\ * & \langle 0,0,1\rangle \end{bmatrix}$$

we can prove the direct multiplication that S^2 is also SL(U)T, and consequently $S^3, S^4, \ldots$ all powers of S. All diagonals are zero in $S, S^2, S^3, \ldots,$ so by Remark 3, S is nilpotent. As $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle = \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle = \langle 1, 1, 0\rangle$, then multiplying S on the left by $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle$, we get $\langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle \times \langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle = \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle$, so we find the N^n,

that is $\langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle^n = \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle \times \langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle \times \langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \ldots \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle \times \langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle$
$= \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle \times \langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle^n \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle$
$= \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle \times \langle 0, 0, 1\rangle \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle = \langle 0, 0, 1\rangle$.

Theorem 3 *A FuNeSoMa N is nilpotent iff* $\exists$ *a PeFuNeSoMa* $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \ni \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle n_{ij}^T, n_{ij}^I, n_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$ *is SL(U)T.*

Note 1 Let $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle$ be a FuNeSoMa, $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle$ is PeFuNeSoMa. Let $T = \langle t_{ij}^T, t_{ij}^I, t_{ij}^F\rangle = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$. The element which lies in the $(i, j)^{th}$ entry of R lies next in the $(h, k)^{th}$ of T iff $\langle p_{hi}^T, p_{hi}^I, p_{hi}^F\rangle = \langle p_{kj}^T, p_{kj}^I, p_{kj}^F\rangle = \langle 1, 1, 0\rangle$.

Theorem 4 *Let* $R = \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle$ *be a FuNeSoMa,* $P = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle$ *is PeFuNeSoMa. Then*

$$\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times (\Delta\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle) \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle = \Delta(\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle) \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle, \quad (1)$$

$$\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times (\nabla\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle) \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle = \nabla(\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle). \quad (2)$$

Definition 1 We say a FuNeSoMa R is controllabel from belove (above), if $\exists$ a PeFuNeSoMa $P \ni \langle t_{ij}^T, t_{ij}^I, t_{ij}^F\rangle = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$ satisfies $\langle t_{ij}^T, t_{ij}^I, t_{ij}^F\rangle \geq \langle t_{ji}^T, t_{ji}^I, t_{ji}^F\rangle$ $(\langle t_{ij}^T, t_{ij}^I, t_{ij}^F\rangle \leq \langle t_{ji}^T, t_{ji}^I, t_{ji}^F\rangle)$ a long as $i > j$.
A FuNeSoMa $R = \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle$ is said to be controlled from below (above), if $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \geq \langle r_{ji}^T, r_{ji}^I, r_{ji}^F\rangle(\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \leq \langle r_{ji}^T, r_{ji}^I, r_{ji}^F\rangle)$as long $i > j$.

Theorem 5 *The next statements are analogous:*

- *(1)* $\langle r_{ji}^T, r_{ji}^I, r_{ji}^F\rangle$ *is CFB(A).*
- *(2) There exists a PeFuNeSoMa* $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle$ *such that* $\Delta(\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle)$ *is SL(U)T.*
- *(3) There exists a PeFuNeSoMa* $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle$ *such that* $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times (\Delta\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle) \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle)$ *is SL(U)T.*
- *(4)*$\Delta\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle$

Corollary 1 $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle$ *is CFB iff* $\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle$ *is CFA.*

Note 2 Let R and S be FuNeSoMas, and P is a PeFuNeSoM. Then $R\Psi S$ iff $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle \Psi \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times \langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$.

Proof: The proof is obvious.

Theorem 6 *Let R, S be a FuNeSoMas, and $\Delta R\Psi\Delta S$. If S is controllable, then R is controllable.*

4 Reduction of Controllable Matrix to Canonical Form

Lemma 1 *Let $R = (\langle r_{ij}^T, r_{ij}^I, r_{ij}^F\rangle)$ and $S = (\langle s_{ij}^T, s_{ij}^I, s_{ij}^F\rangle)$ be $n \times n$ FuNeSoMa of the form*

$$R = \begin{bmatrix} \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \cdots \langle 0,0,1\rangle \\ & \cdots\cdots\cdots\cdots\cdots & \\ \alpha & \vdots & R_1 \end{bmatrix},$$

$$S = \begin{bmatrix} \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \cdots \langle 0,0,1\rangle \\ & \cdots\cdots\cdots\cdots\cdots & \\ \beta & \vdots & S_1 \end{bmatrix},$$

where α and β are $(n-1)\times 1$ FuNeSoMa and R_1 and S_1 are FuNeSoMa of order (n-1). Then

(i)

$$R \times S = \begin{bmatrix} \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \cdots \langle 0,0,1\rangle \\ & \cdots\cdots\cdots\cdots\cdots & \\ R_1\times\beta & \vdots & R_1 \times s_1 \end{bmatrix}$$

(ii)

$$R^n = \begin{bmatrix} \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \cdots \langle 0,0,1\rangle \\ & \cdots\cdots\cdots\cdots\cdots & \\ R_1^{n-1}\times\alpha & \vdots & R_1^n \end{bmatrix}$$

(iii) R is NiFuNeSoMa iff R_1 is nilpotent.

Remark 4 Let $R = (\langle r^T_{ij}, r^I_{ij}, r^F_{ij}\rangle) \in FuNeSoMa_n$, and R have no less than one $\langle 0, 0, 1\rangle$ row (say, the i^{th} row). Let i^{th} row $\rightarrow$ I-row and vice versa and do the same for column; then we have

$$R^* = \begin{bmatrix} \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \cdots \langle 0,0,1\rangle \\ & \cdots\cdots\cdots\cdots\cdots & \\ * & \vdots & \\ \vdots & \vdots & \\ * & \vdots & R_1 \end{bmatrix}$$

Lemma 2 *R is NiFuNeSoMa iff R_1 is NiFuNeSoMa. By Lemma 1, we have the following for NiFuNeSoMa R.*

Algorithm 1 **Step 1.** Check R, for a null row and null column; if anyone is missing, then R fails to be nilpotent. End.
Check R for both zero row and zero column. If not, then R is not nilpotent. If R has both conditions, then do interchange as mentioned; then, we have

$$\langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle_1 \times R \times \langle p^T_{ji}, p^I_{ji}, p^F_{ji}\rangle_1 = \begin{bmatrix} \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \cdots \langle 0,0,1\rangle \\ & \cdots\cdots\cdots\cdots\cdots & \\ * & \vdots & R_1 \end{bmatrix}$$

where $\langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle_1 = \langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle(1, i_1)$. Next step
Step 2. Check R for both zero row and zero column. If not then R_1 is not nilpotent, Stop.
If R_1 satisfies desired conditions, i.e., in R_1, the i_2^th row is a null row. The new form $\langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle_2 \times \langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle_1 \times R \times \langle p^T_{ji}, p^I_{ji}, p^F_{ji}\rangle_1 \times \langle p^T_{ji}, p^I_{ji}, p^F_{ji}\rangle_2$ from FuNeSoMa $\langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle_1 \times R \times \langle p^T_{ji}, p^I_{ji}, p^F_{ji}\rangle_1$ by interchanging the $(i_2 + 1)$-th row with II row and $(i_2 + 1)$-th column with the II column such that

$$\langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle_2 \times \langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle_1 \times R \times \langle p^T_{ji}, p^I_{ji}, p^F_{ji}\rangle_1 \times \langle p^T_{ji}, p^I_{ji}, p^F_{ji}\rangle_2$$

$$= \begin{bmatrix} \langle 0,0,1\rangle & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \ldots \langle 0,0,1\rangle \\ * & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle \ldots \langle 0,0,1\rangle \\ & \cdots\cdots\cdots & \cdots\cdots\cdots & \\ * & * & \vdots & \\ \vdots & \vdots & \vdots & R_2 \\ * & * & \vdots & \end{bmatrix}$$

where $\langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle_2 = \langle p^T_{ij}, p^I_{ij}, p^F_{ij}\rangle(2, i_2 + 1)$. Next step.

Step 3. Check R_2, for a null row and null column. If not, R_2 is not nilpotent; thus, Lemma 2 implies R_1 and R are not nilpotent, stop.
Else if in R_1, the i_3^{th} row of R_2 is null, the new FuNeSoMa of the form $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_3 \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_2 \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_1 \times R \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_1 \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_2 \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_3$ from matrix $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_2 \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_1 \times R \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_1 \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_2$ then changes the (i_3+2)-th row with II row and (i_3+2)-th column with the II column $\ni$,

$$\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_3 \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_2 \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_1 \times R \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_1 \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_2 \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_3$$

$$= \begin{bmatrix} \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle & \ldots & \langle 0,0,1\rangle \\ * & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle & \ldots & \langle 0,0,1\rangle \\ * & * & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle & \ldots & \langle 0,0,1\rangle \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ * & * & * & \vdots & & & \\ \vdots & \vdots & \vdots & \vdots & R_3 & & \\ * & * & * & \vdots & & & \end{bmatrix}$$

where $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_3 = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle(3, i_3+2)$. Next step.

Continuing like this, finally we get

$$\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_n \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_{n-1} \times \ldots \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_2 \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_1 \times R \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_1 \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_2 \times \ldots \times P_{n-1} \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle_n =$$

$$\begin{bmatrix} \langle 0,0,1\rangle & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle & \ldots & \langle 0,0,1\rangle \\ * & \langle 0,0,1\rangle & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle & \ldots & \langle 0,0,1\rangle \\ * & * & \langle 0,0,1\rangle & \vdots & \langle 0,0,1\rangle & \ldots & \langle 0,0,1\rangle \\ & \cdots & \cdots & \cdots & \cdots & \cdots & \\ * & * & * & \vdots & & & \\ \vdots & \vdots & \vdots & \vdots & R_n & & \\ * & * & * & \vdots & & & \end{bmatrix}$$

where $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_m = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle(m, i_m+m-1), m \in I_n$.

In the event if R_n fails to satisfy both the condition, then R is not nilpotent.
Else,

$$R_n = \begin{bmatrix} \langle 0, 0, 1\rangle & \langle 0, 0, 1\rangle \\ \ddots & \\ * & \langle 0, 0, 1\rangle \end{bmatrix}$$

then by Lemma 2, R is nilpotent by the sequence of actions. Then, $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle = \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_t \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_{t-1} \times \ldots \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_2 \times \langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle_1$.
We obtain,

$$\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times R \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle = \begin{bmatrix} \langle 0, 0, 1\rangle & \langle 0, 0, 1\rangle \\ \ddots & \\ * & \langle 0, 0, 1\rangle \end{bmatrix}$$

which indeed SLT. Algorithm to curtail CoFuNeSoMa to canonical form.

Algorithm 2 **Step 1.** By Algorithm 1, we can check if ΔR is nilpotent or not. Thus R is CoFuNeSoMa or not by Theorem 5.
Step 2. If R is CoFuNeSoMa, then by Step I, we get a permutation matrix P, i.e., $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times (\Delta R) \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$, which is SLT. So $\langle p_{ij}^T, p_{ij}^I, p_{ij}^F\rangle \times R \times \langle p_{ji}^T, p_{ji}^I, p_{ji}^F\rangle$ is canonical form of R. Stop.

5 Conclusion

In this article the controllable fuzzy neutrosophic soft matrix is defined. Further, various properties of nilpotent and controllable fuzzy neutrosophic soft matrices are showed. We have developed an algorithm for controllable and nilpotent fuzzy neutrosophic soft matrices and reduced a controllable fuzzy neutrosophic soft matrix to canonical form.

References

1. Arockiarani, I., Sumathi, I.R.: A fuzzy neutrosophic soft matrix approach in decision making. JGRMA **2**(2), 14–23 (2014)
2. Atanassov, K.: Intuitionistic fuzzy fets. Fuzzy Sets and System, **20**, 87–96 (1986)
3. Broumi, S., Sahin, R., Smarandache, F.: Generalized interval neutrosophic soft set and its decision making problem. J. New Results Sci. **7**, 29–47 (2014). https://doi.org/10.5281/zenodo.49000
4. Kavitha, M., Murugadas, P.: Eigenspace of a circulant fuzzy neutrosophic soft matrix. Neutrosophic Sets Syst. **50**, 591–601 (2022)

5. Kavitha, M., Murugadas, P., Sriram. S.: Minimal solution of fuzzy neutrosophic soft matrix. J. Linear Topol. Algebra **6**, 171–189 (2017)
6. Kavitha, M., Murugadas, P., Sriram, S.: On the power of fuzzy neutrosophic soft matrix. J. Linear Topol. Algebra **7**, 133–147 (2018)
7. Kavitha, M., Murugadas, P., Sriram, S.: Priodicity of interval fuzzy neutrosophic soft matrices. Adv. Math. Sci. J. **9**, 1661–1670 (2020). https://doi.org/10.37418/amsj.9.4.22
8. Kim, K.H., Roush,F.W.: Idempotent fuzzy matrices. Seminar Report, Alabama State Univ., Montgomery, AL (1981)
9. Maji, P.K.: A neutrosophic soft set approach to a decision making problem. Ann. Fuzzy Math. Inf. **3**(2), 313–319 (2012)
10. Maji, P.K., Biswas. R., Ray. R.: Fuzzy soft set. J. Fuzzy Math. **9**, 589–602 (2001)
11. Maji, P.K., Roy, A.R., Biswas. R.: On intuitionistic fuzzy soft set. Int. J. Fuzzy Math. **12**(3), 669–684 (2004). https://doi.org/10.3233/IFS-2012-0601
12. Molodtsov, D.: Soft set theory first results. Comput. Math. Appl. **37**(4–5), 19–31 (1999). https://doi.org/10.1016/S0898-1221(99)00056-5
13. Murugadas, P., Kavitha, M., Sriram, S.: Monotone interval fuzzy neutrosophic soft eigenproblem. Malaya J. Matematik **1**, 342–350 (2019). https://doi.org/10.26637/MJM0S01/0062
14. Murugadas, P., Kavitha, M.: Solvability of system of neutrosophic soft linear equations. Neutrosophic Sets Syst. **40**, 254–269 (2021)
15. Padder, R.A., Murugadas, P.: Max-max operation on intuitionistic fuzzy matrix. Ann. Fuzzy Math. Inf. Article in Press
16. Smarandache, F.: Neutrosophic set, a generalization of the intuitionistic fuzzy set. Int. J. Pure Appl. Math. **24**(3), 287–297 (2005)
17. Uma, R., Sriram, S., Murugadas, P.: Fuzzy neutrosophic soft matrices of type-I and type-II. Fuzzy Inf. Eng. **13**(2) (2021). https://doi.org/10.1080/16168658.2021.1923621
18. Xin, L.J.: Convergence of powers of controllable fuzzy matrices. Fuzzy Sets Syst. **45**, 83–88 (1994). https://doi.org/10.21917/ijsc.2016.0184
19. Zadeh, L.A.: Fuzzy sets. J. Inf. Control **8**(3), 338–353 (1965)