About Teaching Quantum Mechanics in High Schools

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Abstract Teaching quantum mechanics in classrooms other than those of (some) university departments, let alone speaking about it to the public, has been a taboo for almost one century. However, in the last few decades, it has become clear that it is time to release this taboo, both for an urgent need for professional figures that can understand the principles of quantum information and computation without being physicists, and for the intellectual honesty that obliges us to tell anyone, and young students in particular, the truth about how quantum mechanics shapes our world. In this chapter, we will first discuss the reasons that have relegated quantum mechanics into scientific academic education for such a long time and present our opinion about why these reasons are not valid anymore. We will then present a proposal to introduce the two main postulates of quantum mechanics, namely the state- and measurement-postulates, using a simple formalism that yet allows us to discuss some of the most revolutionary aspects of the theory. The proposal is then revisited to give some guidelines to design formally correct and yet educationally effective approaches to quantum mechanics. Finally, we will comment upon possible strategies to let the principles of quantum mechanics enter at least some high-school programs.

Keywords Classical physics · Quantum mechanics · Qubit · Vectors · Quantum physics for high schools

1 Quantum Versus Classical

Dealing with the introduction of quantum mechanics in high-school education is a process that extends well beyond its own declared objective, in so far as it requires understanding the way classical and quantum physics have talked to each other

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for almost one century. The dialog has been adversarial from the very beginning when messages about the inadequacy of classical physics (CP) in describing certain observed phenomena started to gather up in physics laboratories. These messages said that a fundamentally different theory was necessary, despite CP being a successful theory upon which no one dared cast doubts. The first reaction developed around the attempt of setting quantum mechanics (QM) into CP, with the former seen as a special case of the latter, whose formalism so nicely corresponds to our way of perceiving the world around us. CP is about what we can observe, see, and touch… and this is something quite powerful in the relationship between human beings and science in general, and physics. On the contrary, QM arose to explain very unusual phenomena, utterly out of our everyday life and yet so fundamental that one cannot explain why our universe exists as it is, without understanding them.

Confronted with the necessity of describing quantum phenomena with the formal tools of CP, scientists came up with a very complicated mathematical arsenal, from which paradoxes emerged, as well as true oddities such as the wave-particle dualism, or the infamous "spooky action at a distance", as Einstein dubbed what is now recognized as one of the key ingredients of our entire universe, namely the *entanglement* (that will be introduced later).

Differential equations, probability distributions, and path integrals, just to name a few, are not, and will never be, part of the mathematical background of a scientific high-school student, let alone of any student. Therefore, if these tools are truly necessary for dealing with QM, then QM is doomed to remain a mysterious and unintelligible corner of science for most of the population, despite its huge relevance in understanding the world around us.

The last century tries to exit this blind alley have been proposed along two main lines: bringing as much as the above-mentioned formalism at least in scientific high schools or simplifying the original structure of the theory to make it more like CP. None of the two strategies seemed to have worked, and while the former failed without doing much damage, the latter is responsible for misconceptions and a conceptual derailing [see Ref. Krijtenburg-Lewerissa (2020) for a recent analysis and a thorough bibliography] that can be summarized into the newly created verb "to quantize". The verb, still very much used by physicists, refers to a formal procedure that transforms a known problem of CP into some "quantum analog", as if a classical-to-quantum flow is legitimate. There are special cases in which such analog at least corresponds to a problem that makes sense in QM, but that is not necessarily the case and, even so, it may have no physical correspondence with the classical problem from which one has started. Ultimately, QM cannot be traced back to CP. Rather the opposite is true: CP emerges from QM, as a special case of it. As obvious as it is, this statement only recently entered the phrasebook of physicists (not all of them, in fact), the reason being that it claims some sort of quantum supremacy over CP, which is very difficult to accept, given the above-mentioned correspondence between the latter and our everyday experience. QM shapes our world, from the elementary units of quantum information, the tiny qubit, to the most massive objects of our universe, the extraordinary black holes.

This evidence makes the discussion on the possibility of bringing QM into high-school classrooms essential for designing strategies for teaching modern physics. The discussion is very well represented by the Quantum Technology Education (QTEdu) Project of the European Community, whose purpose is *"to assist the European Quantum Flagship with the creation of the learning ecosystem necessary to inform and educate society about quantum technologies"* (from the homepage of the QTEdu official site). The project is based on the activity of five working groups, *School education, and Public outreach, Educational initiatives in higher education, Lifelong learning and workforce training, Educational research in Quantum Technology, Equity, and Inclusion for QT educational initiatives,* which are making valuable resources available online (there included the *European Competence Framework for Quantum Technologies)* such as programs, courses, training, and evaluation tools, for primary and secondary schools (follow the link "Resources for everyone" from the main menu of the official site).

It is not by chance that the qubit is mentioned above, as quantum information has a fundamental role in the story we are telling. At the end of the last century, QM started being viewed not only as a useful, though disturbing, physical theory, but also as the carrier of an original logical system, based on a small number of postulates, fully consistent, and extraordinarily powerful (Nielsen and Chuang 2011). In this context, the connection with CP was not necessary, and QM could speak its native language: its laws were finally expressed in the simplest possible way, and the cumbersome translation into classical-like expressions became needless. The ensuing process is quite impressive, in so far as it simplifies the mathematical formalism without depriving QM of any of its elements of novelty, as we try to explain in the next section, briefly introducing the two postulates of QM that are today recognized as the cornerstone of the theory, namely the state- and the measurement-postulates.

Before moving on, though, let us consider the following. The formulation of most scientific theories begins with some statement about the "state" of the system under scrutiny. The state is a generic concept: we say that a garden is in a poor state or a friend is in a very good state. However, the concept becomes scientifically useful when embodied in a formal tool, capable of conveying information about the properties of the system to which it refers. In CP such a tool is the "point", whose coordinates represent position and momentum, as in Fig. 1a–c, or pressure and volume, electric and magnetic field, or other possible sets[1] of physical quantities. Dealing with points implies working in the geometrical space where the points are defined, for instance, a two-dimensional plane or a sphere. This is neither the three-dimensional space where we move nor the $3+1$-dimensional space time used in relativity. Rather, it is a space, technically dubbed *phase-space*, whose shape is determined by some essential physical constraints and whose dimension is the minimum number of coordinates that are necessary to identify a point on it. Once the formal tool that describes the state is identified, the main goal of CP is that of providing equations, as simple and general as possible, via which one can predict how the point describing the initial state of the system will move, in the above-mentioned space, as time goes by; in

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[1] By *possible* we here refer to sets of canonically conjugated variables, in the Hamiltonian formalism.

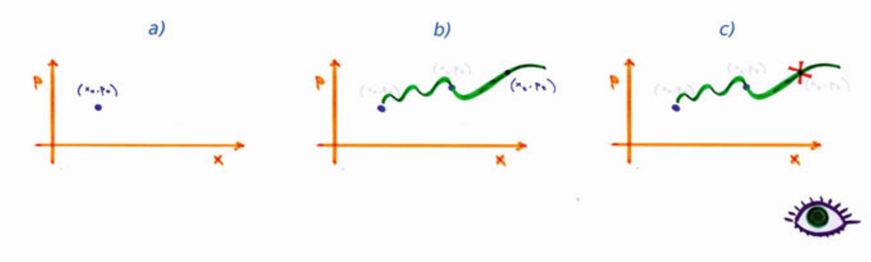

Fig. 1 The state of a system in classical physics is represented by a point: here is the case when the coordinates are the position x and momentum p of a particle moving along a one-dimensional path. **a** the state at some initial time t_0; **b** the trajectory that describes the dynamics of the state at any later time t; **c** the measurement process as a harmless snapshot

other terms, CP aims at drawing the so-called *trajectory* of the initial point, in so far doing allowing us to predict the dynamics of the system at whatever later time, shown as a line in Fig. 1b. Notice that once the trajectory is drawn, the theory has accomplished its task, as the possible observation of the system is just a practical implementation of some experimental procedure that does not alter the state of the system, as represented in Fig. 1c.

2 Simply Quantum

The above familiar picture is revolutionized in QM. The state-postulate of QM tells us that the state of a system Ψ is formally represented by a vector, as shown in Fig. 2a, whose length is set equal to 1, aka *unit vector*, which is consistently dubbed *state-vector*. The symbol $|\cdot\rangle$ was first introduced by Dirac to indicate state-vectors, and by him dubbed *ket*, from the noun *bra-ket*; the graphical sign $|\Psi\rangle$ is hence read "*ket* Ψ". Replacing points with vectors may seem a harmless process: vectors, like points, are simple mathematical objects, oriented segments introduced well before QM, largely used in CP for describing forces, velocities, angular momenta, or other properties, and consequently familiar to many of us. What makes the step revolutionary, and to some extent baffling, is the fact that a vector is now used to describe the state of a system and not one of its properties. Why baffling? Because vectors are algebraic objects, meaning that they can be added together. Some people may remember the parallelogram law of vector addition, shown in Fig. 2b; others may anyway recognize the horizontal and vertical components of an oblique vector, instinctively understanding that these components sum together to set an inclined direction; similarly, a blue paint can be added to a yellow one, and a green color is obtained. With these examples in

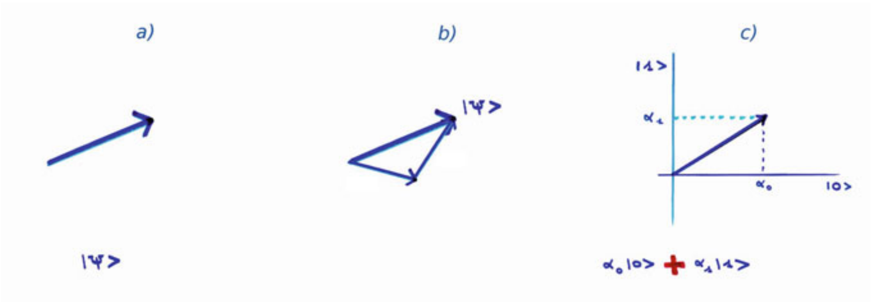

Fig. 2 The state of a system in quantum mechanics is represented by a unit vector: here the case of a qubit (see text). **a** The state; **b** the state as a sum of any two other vectors; **c** the state as a weighted sum of two other states

mind, particularly the last one, we see why the state-postulate of QM, here reasonably simplified into “the state of a system is described by a unit vector”[2] delivers the revolutionary *superposition principle*, according to which different states of a system can coexist, which is the same as saying that their respective state-vectors can be summed up. It is thus possible for a system to be at the same time horizontal AND vertical, yellow AND blue, 0 AND 1, or even alive AND dead, as the infamous Schrödinger cat. In fact, one of the most fascinating aspects of QM is the way, in which the mathematical language of the theory, which is called linear algebra, corresponds to a description of reality: the sign “ + ” that enters the algebraic sum between vectors shown in Fig. 2b precisely corresponds to the logical conjunction AND in the description of a world where systems can be simultaneously in different states. It is not a matter of being either horizontal OR vertical, perhaps yellow OR maybe blue, 0 OR more likely 1, as Einstein kept on thinking, never recognizing state-superposition as a real phenomenon (he rather thought it was an improper way to treat a lack of information of the same sort of that dealt with by any statistical analysis (Einstein et al. 1935, 1971)): QM informs us that a system can be in two or more different states at the same time, possibly weighted differently. This is nicely expressed by the most general form of the simplest possible state-vector

$$\Psi\rangle = \alpha_0|0\rangle + \alpha_1|1\rangle, \tag{1}$$

which is that of the simplest possible quantum system, today called *qubit*. A qubit is a system that can be in any superposition of two different states, as in Fig. 2c, here indicated by $|0\rangle$ and $|1\rangle$ with reference to the usual notation adopted in quantum information to describe qubits (usually called Alice, Bob, Charlie…). In Eq. (1), α_0 and α_1 are numbers, such that $|\alpha_0|^2 + |\alpha_1|^2 = 1$, to ensure $|\Psi\rangle$ is a unit vector and hence a proper state-vector. Once physical states are given their formal representation

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[2] For those who know some basic linear algebra, the postulate says “to every system is associated a Hilbert space: Any normalized vector of this space describes a possible state of the system, and vice versa”.

as unit vectors, QM deals with the dynamical evolution of systems along the same path of CP, i.e., providing equations to describe how the state-vector changes with time. Notice that, as its length is set equal to 1 by the postulate, the only variation possible for the state-vector resides in its *direction*, where the italic fonts serve to remind us that we are dealing with vectors in an abstract space, technically named *Hilbert space*, whose dimension can go from 2 to infinity, a space where the notion of direction still exists, but must be treated with a bit of care. It is not fair to say that QM deals with time evolution without introducing elements of novelty, in fact, the so-called *problem of time* in QM is at the heart of any attempt to reconcile QM with General Relativity (Anderson 2017). However, QM describes the dynamics of systems in quite a standard way, without tearing with respect to CP, and we will not dwell on this aspect, here, but rather move toward the observation stage, where things change drastically again.

The superposition principle cannot be satisfactorily processed without considering what happens when making a measurement of a system. Suppose we ask the oblique state-vector in Fig. 3a "are you horizontal or vertical"? Following strict logic, QM teaches us that all the different answers provided by the definition of our question are possible, and the predictive capacity of the theory consists in furnishing the probability with which each possible answer will be given. Moreover, when the specific answer is given, i.e., some result is produced by the experimental apparatus, a true change in the state-vector occurs: answering "I am horizontal (vertical)", the oblique state-vector will lay down (stand up), as seen in Fig. 3b, c. In fact, the quantum measurement process is the subject of the measurement postulate, which provides us with a formula, known as Born's rule, for getting the probability that each possible result (the answer) is obtained, given the specific measurement done (the question). The formula is simple: referring to Fig. 3a, the probability of getting the answer "I am horizontal" equals $|\alpha_0|^2$, whenever "I am vertical" consistently occurs with probability $|\alpha_1|^2$. Despite having been the subject of heated discussions and disparate interpretations, the formal content of this postulate is quite simple and embodies the predictive power of QM. Without entering the realm of QM interpretations, we notice that the idea that a measurement may alter the state of the experimentally tested system is quite reasonable: the information transfer implied by the question–answer scheme needs some sort of interaction, that may well cause a change in the state of the parties involved.

Getting back to the above-mentioned predictive power of QM, one may wonder whether it is too poor for giving the theory the status of a proper scientific tool. To this respect, besides considering the exceptionally precise predictions that QM provides for the most diverse experiments, it is to be noticed that there always exists a question, sometimes called "the right question", for which the answer is perfectly determined, in so far as the probability to get it equals 1. Referring to Fig. 4, for instance, if one asks the state-vector: "do you point toward northeast or northwest", the answer will be with absolute certainty "toward northeast", and the system will remain in its state after the measurement has occurred. Obviously, it is not always possible to guess the right question, unless one accurately designs and controls the dynamical evolution of the system, but the fact that such a specific question exists

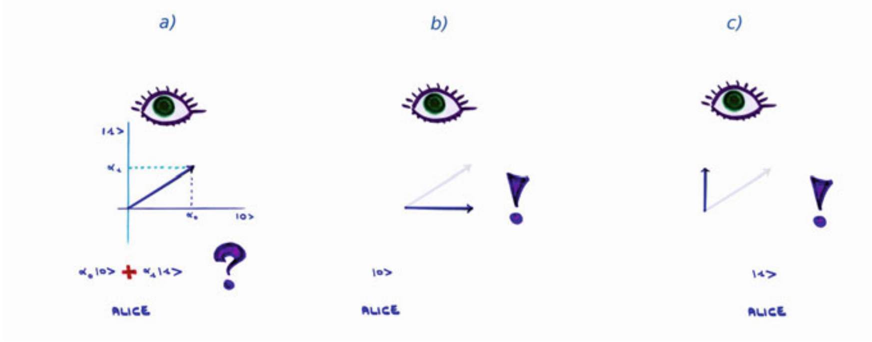

Fig. 3 The measurement process in QM: here is the case of a qubit (see text). **a** The measurement seen as a question; **b** the effect associated with the answer 0; **c** the effect associated with the answer 1

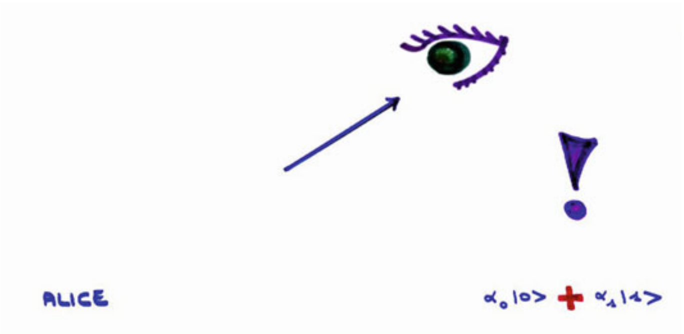

Fig. 4 The "right question" in quantum mechanics (see text)

allows, for instance, to obtain exact results from quantum devices such as quantum computers or simulators.

So far, we have only seen two postulates, and yet even textbooks that introduce QM via postulates[3] often refer to 4 or even more postulates [see for instance Ref. Kaye et al. (2006)]. In fact, most of them can be derived from either the state-postulate or the measurement one, which means that the others are not ultimately necessary to define QM,[4] which is why we will not consider them here.

Let us rather concentrate upon one of the most surprising consequences of the above-considered postulates, namely what happens when two systems, say the famous Alice and Bob that inhabit any quantum information textbook, are put

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[3] QM can be alternatively presented by means of some principles, a choice that has been often adopted in the last century, but has been mainly dismissed in the last decades.

[4] Recent works on the problem of time (Foti et al. 2021) and on the way systems are jointly described (Carcassi et al. 2021) indicate that indeed QM can be formulated using only the state- and the measurement-postulates.

together. Suppose the two friends A and B are qubits, meaning that they can separately be in any superposition of the two basis states introduced in Eq. (1), namely $|0\rangle$ and $|1\rangle$; quite obviously, the system "Alice with Bob" will possibly be in any superposition of the four basis states $\{|00\rangle, |1\rangle, |10\rangle, |11\rangle\}$, where we may think that the digit on the left refers to Alice, and that on the right is for Bob, so that the state $|01\rangle$ sees Alice in $|0\rangle$ and Bob in $|1\rangle$. Among the infinite superpositions possible, some are characterized by the fact that despite representing a well-defined state of the AB-pair, they do not, and cannot, convey information about the individual state of each qubit separately. Consider for instance the following superposition.

$$|AB\rangle_{\uparrow\uparrow} = \frac{1}{\sqrt{2}}\left(|00\rangle + |11\rangle\right)_{,} \tag{2}$$

or also

$$|AB\rangle_{\uparrow\downarrow} = \frac{1}{\sqrt{2}}\left(|01\rangle + |10\rangle\right)\ : \tag{3}$$

We cannot see what the state of Alice is, nor that of Bob, and yet we have a perfectly defined state for the two of them: the two friends now form a pair, where the state of Alice establishes the state of Bob and vice-versa.

The state of the two friends, now become a couple, is said to be *entangled* or, quite equivalently, we say that Alice and Bob are entangled, or quantumly correlated, where the adverb stands to recall that the bond between them holds up against any spatial separation and the passing of time unless some external disturbance occurs. In fact, in the above reasoning, there is no reference to the fact that Alice and Bob be close or distant, that they communicate or not, that they recently met or do not see each other for ages. And yet, by asking a question, Alice not only will change her state according to the answer received but will also affect the state of Bob.

In the case of the state reported in Eq. (2), for instance, when the result of a measurement done upon Alice tells us that she is in the state $|0\rangle$, we also know, without any further measurement or communication procedure, that Bob is also in the state $|0\rangle$. This fact made Einstein crazy, as he thought it meant that Bob was instantaneously informed about the result of the measurement on Alice, no matter how distant they were, via some sort of "spooky action at a distance" (Kaye et al. 2006) already mentioned. In fact, he never came to terms with the fact that Alice and Bob had already shared the whole information content of their pair-state, when prepared in such an entangled state. Whereupon they can be separated at will: if no event occurs that changes their pair-state in Eq. (2) they will stay quantumly correlated forever, and the above reasoning on the measurement will stay valid.

The state- and measurement-postulates introduced in this section can be, and have been, the subject of many, extremely relevant and interesting discussions and considerations, and further research is still needed to fully understand their physical meaning and their logical and philosophical content. Besides these possible developments, there is already quite a lot to digest in the simple narrative presented here, as briefly discussed in the next section.

3 Layered Strategies

In this last section, we want to consider possible strategies to speak about QM in high schools or, in general, to the public. No doubt the subject is difficult and may be tricky to deal with it, but this is no excuse for renouncing or, which is worse, for speaking about it in a simple but unfaithful way, as ever too often done (see the above-mentioned site of the QTEdu project for reports, studies, and articles about these points). Let us better explain our viewpoint in this respect: it is the duty of those who know the details and understand the meaning of a difficult topic to find a way to explain the latter without the former, which is not the same as creating a new, simpler, story, that resembles the original one, but with a different meaning. Whoever is the recipient of our teachings, we must consider that she or he will always have the possibility to get a more detailed education on the taught subject, equipped with more refined formal tools and a richer background. It is hence important that there is no misunderstanding, no confusion, not even the slightest derailment from the conceptual backbone of the subject, in the first encounter with it.

Consider the case of QM: as already mentioned in the first section, attempts of making it more similar to CP thinking that this could help understand the theory were unsuccessful and detrimental. One should rather think about some *layered strategy*, where each layer speaks about QM, but with a different level of detail. Take the above section: there is nothing truly unfaithful to QM and, whenever something is not exactly as we would like it to be, were the recipients equipped with the necessary formal tools, yet there is no lie, in such a way that when the above tools will possibly become available, it will simply be a matter of adding up. Consider the *superposition principle*, or state-postulate: if introduced without mentioning the fundamental fact that states in QM are represented by vectors, there is no way to make the sentence "systems can be in different states at the same time" more precise. We have not mentioned the fact the two coefficients α_0 and α_1 in Eq. (1) are complex numbers, but this is a piece of information that can be easily added to the overall picture if complex numbers will ever become part of the student mathematical background. On the other hand, it is not by chance that we have not mentioned decoherence or the double-slit experiment: there is no way to understand them without referring to the fact that QM uses complex numbers. It is possible to introduce the double-slit experiment to schools in a didactic way, but this implies either evoking a qualitative knowledge about how waves behave or introducing the (complex) mathematical formalism of wave-dynamics.

And yet, the above section alone, without complex numbers, allows one to precisely describe quantum teleportation, quantum cryptography, the Stern-Gerlach experiment, and many other quantum phenomena. Decoherence and the double-slit experiment may come later, when a second layer of knowledge is built upon the solid, despite the simple, basement created. The same is true for the measurement process: we have not mentioned the fact that the process corresponds to a non-unitary dynamic, which is essential and troublesome at the same time. And yet, the idea that observation may alter the state of what is observed is conveyed, Born's rule is provided, and the concept of the right question is introduced. When, and if, the student will meet Hilbert spaces and their orthonormal basis, will see that asking a question means choosing an orthonormal basis and dealing with eigenvectors and eigenvalues of Hermitian operators; the whole formalism will overlap with the previously acquired knowledge and the new information will adhere to the background. Similar considerations can be done for entanglement, where we have only considered two qubits, without speaking about entanglement in more complex systems: this prevents us to comment upon the fact that we do not see a quantum world in front of us, and yet it is enough to define the CNOT quantum gate, an essential element of any universal quantum computer, or explain the Bell inequality (1966) and the reason why experiments demonstrating its violation earned Aspect, Clauder, and Zeilinger the 2022 Nobel prize in physics (2022). Among the existent proposals for introducing QM in high schools, working environments, or to the public in general, which are based on a layered strategy, we like to mention the one designed by *qplayLearn* (https://qplaylearn.com/), the outreach division of the Finnish society Algorithmiq (https://algorithmiq.fi/): on their site one can find multimedia content, such as the *Quantum Pills*, that have been carefully designed in such a way that adding more details become a seamless process.

Let us end this Chapter by underlying that besides the strategy here briefly presented, there are many other proposals for bringing QM into high-school classrooms, and different viewpoints about the goal itself. However, the absolute relevance of creating *"a quantum-ready society, with knowledge about and positive attitudes toward quantum technologies"*, as stated on the homepage of the QTEdu project, is finally emerging, and a dialog between experts in physics, philosophy, sociology, education, and other related disciplines, is the essential tool for moving forward.

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