Chapter 31
Footbridge Vibration Predictions and Interaction with Walking Load Model Decisions

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Abstract Vibrations in footbridges generated by pedestrians are a matter of concern, typically because there is the risk that vibration thresholds may exceed resulting in an unacceptable serviceability-limit-state.

There are challenges involved with predicting vibration levels at the design stage as the engineer in charge of predictions needs to make a number of choices for his calculations, for instance, regarding the load model for the pedestrians and the adjoining parameters (walking parameters). Through sensitivity studies employing artificial footbridges, the chapter will investigate the impact selected choices will have on the outcome of bridge vibration response predictions. In the chapter, a stochastic representation of the load will be considered, and hence the response calculations will end up in a stochastic representation of footbridge response.

The way to arrive at stochastic representations of bridge response will be by employing Newmark time integration and Monte Carlo simulations. The action in focus is the vertical load by pedestrians, and likewise, it will be the vertical footbridge response that is focused on.

Keywords Footbridge · Vibrations · Walking · Serviceability

Nomenclature

a	Bridge acceleration
F	Walking load
f_1	Bridge fundamental frequency
f_s	Step frequency
i	Integer
L	Bridge length
l_s	Step length
m_1	Bridge modal mass
Q	Modal load
t	Time
v	Pacing speed
W	Weight of pedestrian
Θ	Phase
Φ	Mode shape
α	Dynamic load factor
μ	Mean value
σ	Standard deviation
ζ_1	Bridge damping ratio

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31.1 Introduction

For footbridges, the serviceability-limit-state can be of concern. An unwanted scenario is excessive vibrations brought about by pedestrians. The Millennium Bridge vibrations [1] serve as an example, where resonant excitation caused by pedestrians resulted in excessive vibrations.

Different efforts have been devoted to describing the actions of humans whilst crossing a footbridge. Deterministic approaches were introduced in [2–4]. Later, it was recognised that it might be more appropriate to model the action of a pedestrian in a stochastic manner, modelling walking parameters as stochastic variables and to employ a stochastic framework for predicting footbridge vibrations [5–10].

Having settled on a stochastic approach for addressing the mechanisms of walking loads and recognised that a stochastic approach for studying predictions of footbridge vibrations is useful, there are yet other decisions to be made by the engineer in charge of computations.

In literature, there are different suggestions as to how to model the random nature of the different walking parameters, as will be shown in the chapter, and some of the different possible modelling approaches (in the form of the different potential settings of parameters for statistical distributions or fundamentally different relationships) will be applied in numerical simulations of pedestrian-induced vibrations in footbridges in this chapter. This is with a view to examine how different approaches affect estimates of bridge acceleration quantiles, the latter being considered a relevant parameter for assessing the serviceability-limit-state of a footbridge.

Occasionally, in a numerical simulation (modelling walking parameters as random variables and assuming mutual dependency between walking parameters), there will be scenarios where values for a walking parameter end up outside the range in which it was originally calibrated from experiments. These scenarios need to be appropriately handled in simulations. The authors of this chapter might not in all previous publications have addressed this issue thoroughly as they have been asked questions regarding this issue on different occasions. Hence, in this chapter, different assumptions as to how to possibly consider and address out-of-range properties of some walking parameters in numerical simulations are addressed and evaluated.

Section 31.2 describes the footbridges assumed for the studies of this chapter, and why a set of bridges are considered. Section 31.3 outlines the load modelling approach, different approaches for setting up a statistical framework for computing bridge acceleration response, and different approaches for handling out-of-range properties for walking parameters in numerical simulations. Section 31.4 outlines the overall methodology, and Sect. 31.5 presents the results. Conclusions are provided in Sect. 31.6.

31.2 Bridges Assumed for the Study

Assumed for this chapter are three single-span pin-supported single-degree-of-freedom (SDOF)-bridges. The general idea is to provide results for more than a single bridge so as to widen the basis for assessments and conclusions. The modal characteristics for the three bridges are provided in Table 31.1, along with adjoining assumptions regarding the bridge length, L.

The modal properties for the bridges are those valid for the bending axis in focus when considering the vertical action generated by pedestrians. The potential second, third, or higher bending modes are not considered for the studies.

Overall, the properties of the artificial footbridges, lined up in Table 31.1, are believed to be sensible for simple single-span pin-supported footbridges. For the investigations of this chapter, it is judged sensible to consider fairly simple bridges to allow the focus to be on possible implications of employing different load modelling assumptions in simulations.

Table 31.1 Modal properties of bridges (f_1, ζ_1, m_1) and bridge lengths (L)

Property	f_1	ζ_1	m_1	L
Bridge I	1.6	0.5	61.7	53.8
Bridge II	1.9	0.5	43.8	45.3
Bridge III	2.2	0.5	32.6	39.1
Unit	Hz	%	10^3 kg	m

31.3 Modelling of Walking Loads

31.3.1 Basic Load Model Assumptions

The vertical action of a pedestrian, $F(t)$, is modelled as a summation of the two contributions outlined in Eqs. (31.1) and (31.2).

$$F_i(t) = W\alpha_i \sum_{\overline{f}_j=i-0.25}^{i+0.25} \overline{\alpha}_i\left(\overline{f}_j\right)\cos\left(2\pi\overline{f}_j f_s t + \theta\left(\overline{f}_j\right)\right) \tag{31.1}$$

$$F_i^{S}(t) = W\alpha_i^{S} \sum_{\overline{f}_j^{S}=i-0.75}^{i-0.25} \overline{\alpha}_i^{S}\left(\overline{f}_j^{S}\right)\cos\left(2\pi\overline{f}_j^{S} f_s t + \theta\left(\overline{f}_j^{S}\right)\right) \tag{31.2}$$

In this chapter not all details about the modelling approach are explained, neither is the summation of load contributions which is somewhat more complex than explained right above. Interested readers can study [9]. The general idea is that the modelling approach accounts for the main harmonics and subharmonics of excitation to be present and that it accounts for the leakage of energy around the excitation frequencies, which some load models do not. Basically, it is considered the most refined time-domain load model of walking loads available. Hence, this model is chosen for the investigations of this chapter.

Having determined $F(t)$, the modal load $Q(t)$ can be derived using Eq. (31.3).

$$Q(t) = \Phi(t)F(t) \tag{31.3}$$

Here, $\Phi(t)$ is the mode shape function assumed for the first mode of bending action and Eqs. (31.4) and (31.5) outline the assumptions in that regard.

$$\Phi(t) = \sin(\pi v t/L) \tag{31.4}$$

$$v = f_s l_s \tag{31.5}$$

For employing Eq. (31.4), the pacing speed, v, is required and it is calculated using Eq. (31.5). In this equation, f_s is the step frequency and l_s is the step length of the pedestrian.

For the studies of this chapter, in most cases, these two parameters will be modelled as random variables, assuming Gaussian distributions. Mean values and standard deviations are shown in Table 31.2.

In Table 31.2 two different statistical distributions for f_s (both suggested in the literature) are introduced to allow examining the sensitivity of choosing one or the other distribution as the basis for computing bridge accelerations.

The static weight of the pedestrian, W, is assumed to take on a value of 750 N, hence it is considered being a deterministic property for the studies of this chapter.

Adding to the complexity of possible choices to be made by the engineer in charge of computations is that there would be another way to arrive at the modal load $Q(t)$ in that it would be possible to employ Eq. (31.6) for determining the step length of the pedestrian.

$$l_s = 0.2011 f_s^3 - 0.6021 f_s^2 + 0.6462 f_s + 0.2547 \tag{31.6}$$

This is a relationship introduced for the first time in [8] based on a graph shown in [12]. Equation (31.6) is a polynomial approximation to the graph presented in [12], which had no adjoining mathematical formula. The polynomial fit was made for f_s in the range [1.0 Hz; 2.7 Hz]. In Eq. (31.6), the value of f_s is to be inserted in the unit Hz for arriving at l_s –values in the unit meters.

Table 31.2 Mean values (μ) and standard deviations (σ)

–	μ	σ	Reference
l_s	0.71 m	0.071 m	[9]
f_s	1.87 Hz	0.186 Hz	[9]
f_s	2.20 Hz	0.300 Hz	[11]

Table 31.3 Mean values (μ) and standard deviations (σ) [7, 9]

–	α_2	α_3	α_4	α_5
μ	0.07	0.05	0.05	0.03
σ	0.030	0.020	0.020	0.015

Table 31.4 Handling of out-of-range properties for Eq. (31.7)

Approach	Description
A	Setting values of μ equal to zero when f_s is outside the range [1.0 Hz; 2.7 Hz]
B	Setting values of μ equal to the value attained at $f_s = 1.0$ Hz also for values of $f_s < 1$ Hz and setting the values of μ equal to values attained at $f_s = 2.7$ Hz also for values of $f_s > 2.7$ Hz
C	Using Eq. (31.7) also when f_s is outside the range [1.0 Hz; 2.7 Hz]

As for the first main harmonic, α_1, for use in Eq. (31.1), its mean value, μ, and standard variation, σ, may be calculated using Eq. (31.7).

$$\mu = -0.2649 f_s^3 + 1.3206 f_s^2 - 1.7597 f_s + 0.7613; \quad \sigma = 0.16\mu \tag{31.7}$$

This is a suggestion introduced in [7] as regards μ, calibrated to measured data for values of f_s in the range [1.0 Hz; 2.7 Hz].

The load model, Eq. (31.1), also requires settling on mean values and standard deviations for other main harmonics beyond the first harmonic. Table 31.3 defines the assumptions made for the remaining main dynamic load factors assumed in the load model.

There is also a set of subharmonic dynamic load factors to be entered into Eq. (31.2). These are computed following lines of procedures outlined in [9]. Here it is just mentioned that they depend on the main harmonic load factor, α_1.

31.3.2 Out-of-Range Considerations

Having outlined the general load assumptions, the focus is on Eqs. (31.6) and (31.7). These are assumed valid for f_s in the range [1.0 Hz; 2.7 Hz]. In numerical simulations, handling f_s as a random property, there will be outcomes of f_s outside this range.

This can be handled in different ways. For the studies of this chapter, three different approaches (A, B, and C) are considered. The approaches are outlined in Table 31.4.

Table 31.4 has focused on μ for the first main harmonic load factor as function of f_s. Exact similar approaches to handling l_s as function of f_s (in Eq. (31.6)) are also considered for the studies of this chapter. The general idea is to examine how sensitive the predicted bridge acceleration response is to the choice between the three different approaches.

31.4 Methodology

Using Newmark time integration, it is possible to generate a load-time history and to compute vertical bridge acceleration response whilst a pedestrian is assumed to cross the bridge. Repeating this exercise using Monte Carlo simulations allows for obtaining a statistical representation of bridge acceleration response. From each simulated bridge crossing, the maximum acceleration at the bridge midspan, denoted a, is extracted.

This exercise is carried out for the three different bridges, for instance, for the different assumptions regarding how to handle out-of-range properties of f_s as well as for other studied scenarios. For each bridge and study condition, 100,000 bridge crossings by a single pedestrian (at each crossing assuming different values for f_s and l_s) are simulated.

From the simulation results, it is possible to extract different quantiles of a. It is believed that the higher quantiles are those most relevant for consideration as it is believed to be these that might be problematic in the matter of bridge serviceability, but the result section of this chapter (in some cases) will also present acceleration quantiles lower than a_{95}, for the sake of completeness.

31.5 Results

This section presents and evaluates results obtained for acceleration quantiles for the three different bridges. Three investigations are made (I, II, and III).

31.5.1 Investigation I

Table 31.5 shows the results obtained for a_{95} for bridge I, II, and III computed using the relationship $v = f_s l_s$ where both f_s and l_s are handled as random variables. The assumption as for the distribution of l_s is the values set out in Table 31.2 for mean value and standard deviation of this parameter. However, computations were made on the two different assumptions for the distribution of f_s also presented in Table 31.2, and Table 31.5 compares the results.

In Table 31.5, model FS1 refers to the model for f_s in which $(\mu, \sigma) = (1.87$ Hz, 0.186 Hz$)$ is assumed and model FS2 to the model in which $(\mu, \sigma) = (2.20$ Hz, 0.30 Hz$)$ is assumed. For all calculations, approach B for handling out-of-range conditions is employed.

It is apparent that there is a significant difference between the a_{95}-values computed for the three different bridges (for instance, compare results for FS1). This is not surprising, as the likelihood of resonant excitation is different for the three bridges. For bridge II (having a fundamental frequency of 1.9 Hz), the a_{95}-value is quite high for model FS1, which is because the mean value of f_s is relatively close to the fundamental frequency of the bridge. For bridges I and III, again looking at results for FS1, the a_{95}-values are lower, as would be expected.

A similar tendency is seen in results obtained assuming FS2, where it is apparent that the a_{95}-value peaks when the mean value of f_s equals the bridge frequency (being the case for bridge III).

Overall, the results suggest that assumptions made about the statistical representation of f_s have a relatively high impact on a_{95}-values. This is because the values of this acceleration quantile differ quite a lot assuming model FS1 or model FS2 for simulations.

31.5.2 Investigation II

Next up is addressing the influence of the choice made regarding setting up a model for arriving at values for l_s. For the investigation, there are two possible ways referred to as model LS1 and LS2. For both models, the basic assumption $v = f_s l_s$ will be used, and it will be assumed that f_s follows a Gaussian distribution which $(\mu, \sigma) = (1.87$ Hz, 0.186 Hz$)$. In model LS1, it is assumed that the value of l_s is derived also assuming a Gaussian distribution with values $(\mu, \sigma) = (0.71$ m, 0.071 m$)$, in accordance with Table 31.2. In model LS2, the values of l_s are computed using Eq. (31.6). Table 31.6 presents the results obtained using the two different approaches. For all calculations, approach B for handling out-of-range conditions is employed.

Table 31.5 Computed values of acceleration quantile a_{95} for two different assumptions for modelling step frequency

Bridge	I		II		III	
Bridge frequency f_1	1.60 Hz		1.90 Hz		2.20 Hz	
Model for f_s	FS1	FS2	FS1	FS2	FS1	FS2
Mean value for f_s	1.87 Hz	2.20 Hz	1.87 Hz	2.20 Hz	1.87 Hz	2.20 Hz
a_{95}	0.0932 m/s^2	0.0338 m/s^2	0.3456 m/s^2	0.2267 m/s^2	0.2728 m/s^2	0.4973 m/s^2

Table 31.6 Computed values of acceleration quantiles a_{95} for two different assumptions for modelling step length

Bridge	I		II		III	
Bridge frequency f_1	1.60 Hz		1.90 Hz		2.20 Hz	
Model for l_s	LS1	LS2	LS1	LS2	LS1	LS2
a_{95}	0.0932 m/s^2	0.0932 m/s^2	0.3456 m/s^2	0.3456 m/s^2	0.2728 m/s^2	0.2728 m/s^2

Table 31.7 Computed values of acceleration quantiles (on the basis of the FS1 assumption)

Bridge	I			II			III		
Approach	A	B	C	A	B	C	A	B	C
a_{95}	0.0909	0.0932	0.0932	0.3442	0.3456	0.3456	0.2674	0.2728	0.2728
a_{75}	0.0306	0.0308	0.0308	0.1316	0.1308	0.1308	0.0630	0.0634	0.0634
a_{50}	0.0227	0.0227	0.0227	0.0629	0.0629	0.0629	0.0364	0.0367	0.0367
Unit	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2

Table 31.8 Computed values of acceleration quantiles (on the basis of the FS2 assumption)

Bridge	I			II			III		
Approach	A	B	C	A	B	C	A	B	C
a_{95}	0.1818	0.0338	0.0338	0.4646	0.2267	0.2267	0.8082	0.4973	0.4973
a_{75}	0.0216	0.0204	0.0204	0.0705	0.0604	0.0604	0.1946	0.1562	0.1562
a_{50}	0.0171	0.0165	0.0165	0.0406	0.0379	0.0379	0.0833	0.0746	0.0746
Unit	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2	m/s^2

Studying the a_{95}-values, it is apparent that they differ for the different bridges, as would be expected. Another thing to look for is the difference between values obtained employing model LS1 and model LS2 for simulations. In fact, no difference is seen in results even when employing four digits for displaying a_{95}-values, suggesting that the choice between employing LS1 and LS2 is not important for obtaining values of a_{95}.

31.5.3 Investigation III

In this section, the focus is on the results of computed acceleration quantiles for the three different bridges (I, II, and III) assuming the three different approaches (A, B, and C) for handling out-of-range conditions for the equations for the walking parameters f_s and l_s.

First up is a presentation of results of acceleration quantiles obtained using $v = f_s l_s$ where both f_s and l_s are handled as random variables. The walking parameter f_s is modelled with a distribution in which $(\mu, \sigma) = (1.87$ Hz, 0.186 Hz) is assumed (FS1) and l_s is modelled with a distribution in which $(\mu, \sigma) = (0.71$ m, 0.071 m) is assumed, in accordance with Table 31.2. Table 31.7 presents the results.

As expected, the values of acceleration quantiles turn out different for the three bridges, and highest for Bridge II, explained by the fact that the fundamental frequency of this bridge is very close to the mean value assumed for f_s. The results also show that for any of the presented acceleration quantiles for a specific bridge, there appears to be a very limited difference having calculated them on the basis of approach A, B or C. It suggests that the choice of approach for handling out-of-range conditions for f_s is not of significant importance. At least in the studied case, with the mean value of f_s at 1.87 Hz and a relatively small standard deviation (0.186 Hz), the likelihood of scenarios in simulations with a pedestrian crossing using a step frequency $f_s < 1.0$ Hz or $f_s > 2.7$ Hz (the out-of-range conditions for Eq. (31.7)) is very small, at the same time explaining why almost similar values of acceleration quantiles are obtained in simulation for the three approaches (A, B, and C).

However, there are other assumptions for the statistical distribution for f_s available in the literature. There is the proposal suggested in [11], see Table 31.2. In this $(\mu, \sigma) = (2.20$ Hz, 0.30 Hz) is assumed (FS2). Here the mean value of f_s is higher than assumed for the previous study and so is the standard deviation. Hence, the likelihood of reaching simulation conditions where $f_s > 2.7$ Hz is higher than in the previous study.

Table 31.8 presents the results for acceleration quantiles for the three different bridges, again using $v = f_s l_s$ and the model for l_s is yet again based on the assumption $(\mu, \sigma) = (0.71$ m, 0.071 m).

It comes as no surprise that on these assumptions for computation of acceleration quantiles, it is bridge III that experiences the highest acceleration levels as bridge III has a fundamental frequency in fact equal to the mean value assumed for f_s (2.20 Hz). The interesting part is to examine the difference between outcomes of acceleration quantiles computed for the three different approaches (A, B, and C) employed for handling outcomes of f_s outside the range [1.0 Hz; 2.70 Hz]. It is computed that the probability of obtaining values of $f_s > 2.70$ Hz in simulations is at 4.78% when assuming $(\mu, \sigma) = (2.20$ Hz, 0.30 Hz) for f_s as done for these simulations whereas it was only at 0.0004% when assuming $(\mu, \sigma) = (1.87$ Hz, 0.186 Hz).

For all three bridges, it is seen that there is no difference between the acceleration quantiles computed employing approaches B and C. Furthermore, it is seen that the acceleration quantiles computed employing approach A differ rather significantly from those computed on the assumption of approaches B and C, at least for the a_{95}-estimates. Thinking it through, this is not all that surprising as in approach A for every outcome of $f_s > 2.70$ Hz in simulations, a figure of 0 enters for the first main harmonic load factor, α_1. This basically has the effect that the pedestrian force is set to zero for this pedestrian crossing, which cannot be meaningful, and consequently, the computed distribution function for bridge acceleration is messed up, rendering the acceleration quantiles not meaningful. However, it is comforting to find that approaches B and C result in close to identical estimates of the acceleration quantiles and that approach B has been used by the authors of this chapter in previous studies published in the literature.

Other simulation runs were made studying the implications of choosing approach A, B, or C for handling out-of-range properties for l_s when using the relationship outlined in Eq. (31.6) for l_s. Results are not shown here but they generally confirm that there are no differences in estimates of acceleration quantiles between choosing approach B or C for these simulations. A fun fact is that it turned out that using approach A resulted in infinitely long simulation runs. This was and is because setting values of l_s to zero (and this will happen during simulations using approach A), the value of v using $v = f_s l_s$ will be set to zero, resulting in an infinitely long time for the pedestrian to cross the bridge. Hence, these simulation runs are not meaningful.

31.6 Conclusion and Discussion

The chapter has addressed the implications of decision-making for the engineer in charge of predicting bridge accelerations using a probability-based approach for modelling the action. This is by doing simulation studies.

The results suggest that acceleration quantiles of bridge response are sensitive to choices made for the statistical distribution representing the step frequency of walking, f_s. Different ways to modelling the walking parameter l_s representing the step length of a pedestrian were also studied. It turned out so that the manner in which this parameter (l_s) was modelled did not have much bearing on the outcome of simulation results for bridge acceleration quantiles.

Another matter of interest for the studies of this chapter was to examine how sensitive simulation results in terms of bridge acceleration quantiles would be to different ways of handling out-of-range conditions for walking parameters in numerical simulations. As mentioned in the introduction, questions have been asked and the chapter has provided results.

The results suggest that basically either approach B or approach C studied in this chapter can be employed without inflicting differences in estimates of bridge acceleration quantiles, at least for the SDOF bridges assumed for the studies of this chapter. Approach A would be the approach not to choose, for the reasons described in Sect. 31.5.

For the study of possible out-of-range study conditions for the dynamic load factor, it should be mentioned that it encompassed an assumption where the probability of arriving at out-of-range conditions is relatively high (by employing model FS2). It is worth mentioning that this is the only model for the step frequency that the authors of this chapter have encountered that translates into out-of-range probabilities for the main dynamic load factor that are that high. The data basis and sample size for calibrating this model are not very-well described in [11]. However, as it is a model available in the literature, it was considered relevant to address it for the context of the studies of this chapter.

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