Chapter 15
Sequential Harmonic Component Tracking for Underdetermined Blind Source Separation in a Multitarget Tracking Framework

Romain Delabeye, Martin Ghienne, and Jean-Luc Dion

Abstract Smart factories are composed of heterogeneous cyber-physical systems. In light of their complexity and the lack of transparency in their design, monitoring the health of these machines in real time is made possible by the use of nonintrusive sensors. Such sensors produce mixed signals capturing component-specific signatures. Retrieving the activation statuses of the components (over the different operating modes of a machine) is essential for estimating their associated performance indicators. This is a special case of underdetermined blind source separation (UBSS), yet a sensor fusion perspective is adopted in this chapter. A harmonic component detector produces observations in the time-frequency (TF) domain, inherently entailing noise-induced false alarms. The main contribution of this chapter consists of a clutter-resilient multiharmonic component tracking algorithm, based on the sequential Monte Carlo probability hypothesis density (SMC-PHD) filter. Additionally, this chapter presents a track association algorithm adapting the results obtained in the multitarget tracking framework for unsupervised multilabel classification. The combination of the two algorithms mitigates typical difficulties encountered in traditional UBSS problems, such as nonstationary and partially coupled mode decomposition. The performance of the proposed technique is assessed on synthetic data.

Keywords Harmonic component tracking · Multitarget tracking · Sensor fusion · Underdetermined blind source separation · SMC-PHD filter

15.1 Introduction

Energy sustainability is one of the greatest challenges faced by the manufacturing industry. The manufacturing industry is energy-intensive by nature, making it worthwhile to put the emphasis on energy efficiency when aiming for substantial discounts in energy usage and associated carbon emissions. From a physical point of view, energy efficiency boils down to minimizing dissipated energy for a given production. General indicators such as the specific energy consumption (SEC), that is, the total energy consumption per unit of output, only allow for a shallow analysis of a system's energy efficiency. A key success factor in enhancing a production system's energy sustainability lies in the ability to allocate energy performance indicators (EnPI) to dedicated active components, actuators, and operating modes, designated as *components*, *actuators*, and *operations*, respectively. In this context, an actuator consists of a group of physical components always active simultaneously (e.g., a rotor and bearings), and an operation relates to the accomplishment of a task using a fixed group of actuators (e.g., drilling would use two motors to rotate and advance the drill). A machine thus obeys the same dynamics throughout an operation. This dynamics is more specifically made up of the actuators' dynamics and possible couplings between them. In practice, though it is common to monitor a manufacturing machine's total energy consumption (a mandatory requirement to compute EnPI), the activation sequences of (i) the actuators composing this machine and (ii) the different operations performed by the machine are seldom available. Hence, the times at which components, actuators, and operations are active need to be inferred from sensor data, without any prior regarding the studied system (no physical or process models). Furthermore, in order for such a process identification technique to scale, the use of nonintrusive sensors is preferred (e.g., accelerometers or current sensors). These sensors have the particularity of sensing much information from multiple remote sources, resulting in coupled dynamics from a sensor's point of view. Signal processing is thus required to uncouple these sources. This constitutes an underdetermined blind source separation (UBSS) problem, yet only activation statuses are sought

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rather than mixed signal. By putting the emphasis on the activation statuses of the components, actuators, and operations rather than source signal recovery, the latter aspect can be performed using an independent physics-informed regression algorithm instead of a statistical decomposition. Only the former aspect is considered here. Moreover, data are represented in the time-frequency (TF) domain.

UBSS problems are traditionally tackled using decomposition algorithms either identifying a mixing matrix and source signals or learning a sparse representation from a dictionary of representative vectors built iteratively. Such processes (subspace methods in particular) are very efficient when data are piecewise stationary [1], yet this assumption is too restrictive in this context since this would not cover controlled systems.

This motivates the use of multitarget tracking (MTT) for estimating both the state and the number of active components in a signal. The main contribution of this chapter hence consists of an MTT formulation allowing for tracking harmonic components over time. We propose a simple peak-based harmonic component detector (stemming from the signal's power spectral density). A particularity of the problem on hand is the time-varying number of false alarms per scan, which depends on the level of noise associated with each source signal. We hence propose a feature-aided tracking (FAT) formulation, based on the spectral kurtosis, increasing the clutter resilience of our tracking filter.

Related works focusing on MTT and frequency tracking are presented in the first section. Problem formulation is detailed in the second section, together with background knowledge regarding the techniques used in this chapter. The enhancements made to the tracker in order to increase its resilience to clutter are presented in the third section. The fourth section details how this MTT formulation is adapted to solve an unsupervised multilabel classification problem. The verification of the developed technique is performed on synthetic data; the results are presented and discussed in the fifth section.

15.2 Related Works

In blind source separation, independent component analysis (ICA) and its extension to the underdetermined case [2] have received great attention over the years, yet this method cannot separate sub-Gaussian distributions and is not well suited to discrete event data. Diverse techniques exist to estimate the number of source signals [3], often based on subspace methods. For the problem on hand, the number of actuators cannot be recovered using these methods [4], but the number of distinct operations can be retrieved. A traditional approach to tackle UBSS problems consists of clustering time-series data into chunks over which the number of sources is assumed to be constant; the signal is then factorized into a mixing matrix and unmixed source signals [5–7]. Another formulation consists of factorizing data as a dictionary of atoms (representing relevant modes) and a representation (linear combination of atoms) [8]. For these linear combinations to truly represent the sought labels, the representation must remain binary as in the semi-binary non-negative matrix factorization [9, 10]. Alternatively, dynamic time warping (DTW) can be coupled with hidden Markov models when clustering signals with different shapes [11, 12].

In this chapter, we adopt a multitarget tracking framework in which the number and states of target harmonic components are sought. Multifrequency tracking has been extensively studied in the literature as a data assimilation problem using Kalman filters [13, 14] and particle filters [15, 16] in particular. In these applications, tracks are initiated heuristically, and tracking is then treated solely as a state estimation problem.

Two additional challenges arise in the class of applications considered in this chapter. First, the number of harmonic components to track is not known a priori, evolves over time, and is not necessarily detected at each time step. A second important aspect is the presence of a time-varying number of false alarms (clutter). Suitable real-time compatible data association filters (responsible for mitigating the effect of clutter on state estimation performance) include the joint probabilistic data association (JPDA) filter, probabilistically associating measurements to tracks, and probability hypothesis density (PHD) filters [17, 18], implicitly fusing all states with all measurements at each time step. In between these two types of filter, a set JPDA has been proposed in [19].

With a view to increasing state estimation performance, feature-aided tracking has been investigated in the literature. The association probability of the JPDA was refined using signal-to-noise ratio (SNR) [20], radar high-resolution range (HRR) [21], and wavelet-based spectral features [22]. Target Doppler and down-range extent were implemented in feature-aided PHD filters [23, 24]. In these applications, the ingenious integration of feature information—essentially in the data association part of the filter—resulted in better tracking performance and exhibited clutter-resilient behaviors.

15.3 Problem Formulation

Among existing frameworks, the MTT formulation has the potential to track time-varying spectral components while detecting when a new component appears or disappears, thus alleviating major limitations in subspace decomposition methods.

As a machine operates through its manufacturing process, an actuator a with status $\delta_t^a \in \{0, 1\}$ at a time step t can switch on (1) or off (0) components with status $\delta_t^c \in \{0, 1\}$, producing sudden changes in sensor data, and bringing the machine into a new operating mode with status $\delta_t^o \in \{0, 1\}$. From a set of H_t harmonic components (targets) with states $\{\boldsymbol{x}_t^h\}_{h=1}^{H_t}$ at time step t in the TF domain, a set $\{z_t^m\}_{m=1}^{M_t}$ of M_t measurements is produced from a single sensor. Targets follow a Markov transition model $p(\boldsymbol{x}_t^h|\boldsymbol{x}_{t-1}^h)$.

A simple harmonic component detector is proposed. The discrete Fourier transform (DFT) is evaluated on successive overlapping windows with time index t. Then a detection $z_t^m = \left[a_t^m, b_t^m, A_t^m, \omega_t^m\right]^T + \boldsymbol{w}_t$ is triggered every time the power spectral density reaches a peak above a threshold τ, where $\boldsymbol{w}_t \sim \mathcal{N}\Big(.; 0; diag(\left[\sigma_a, \sigma_b, \sigma_A, \sigma_\omega\right])\Big)$; A_t^m, ω_t^m, a_t^m, b_t^m are a harmonic component's amplitude, pulse, and complex coefficients, respectively, and σ_A, σ_ω, σ_a, σ_b denote their corresponding standard deviations. This measurement m corresponds to a complex yet undamped modulated sine wave supplemented with Gaussian noise $w_t^s \sim \mathcal{N}(.; 0; \sigma_s)$, i.e., $s_t^m = A_t^m \exp\left(i\, \omega_t^m t\right) + w_t^s = a_t^m + i\, b_t^m$. An immediate consequence of this detection technique is the time-varying nature of the number of false alarms per scan λ_{FA}. In this formulation, λ_{FA} is implicitly defined by the noise level σ_s. The underlying detection probability p_D is unknown, yet close to unity.

The trajectory $\boldsymbol{T}_{t_i:t_f}^h$ of a target h is made of associated states $\boldsymbol{x}_t^h$ (defined by the same variables as the measurement vector) between times t_i and t_f. For simplicity, the same target index h is kept over time.

For interpretation, a trajectory represents the behavior of a physical component, unless its frequency can be expressed as a positive integer multiple of another trajectory's (i.e., as a harmonic of the fundamental frequency).

Formally, an occurrence of an operation o thus corresponds to a set of trajectories $\boldsymbol{\Omega}_{t_i^o:t_f^o}^o = \{\boldsymbol{T}_{t_i^o:t_f^o}^h\}_{h=1}^{H^o}$ made of H^o components between times t_i^o and t_f^o. Similarly, each time an actuator a is activated, it induces a set of trajectories $\boldsymbol{\Omega}_{t_i^a:t_f^a}^a = \{\boldsymbol{T}_{t_i^a:t_f^a}^h\}_{h=1}^{H^a}$ composed of H^a components between times t_i^a and t_f^a.

15.4 Feature-Aided SMC-PHD for Harmonic Component Tracking

The SMC-PHD filter is considered in this chapter due to its inherent clutter-resilience and computational efficiency (from a data association point of view) [17]. Indeed, in this MTT framework, target dynamics is nonlinear and much clutter (false alarms) is expected, yet the detection probability is high. The SMC-PHD filter is particularly well suited to such problems. This filter consists of five steps: (i) particle sampling, (ii) prediction, (iii) update, (iv) resampling, (v) clustering, and (vi) assignment. Particle sampling is responsible for exploring the state space and spotting new targets. Prediction, update, resampling, and clustering constitute the multitarget state estimation activities. Assignment binds tracks (states and associated covariances) to one another across time steps, resulting in trajectories. The SMC-PHD filter relies on two assumptions [24]:

Assumption 1 *The targets are independent of one another and generate at most one measurement per scan.*

Assumption 2 *Clutter and target birth distributions are Poisson and target-independent.*

We apply uniform particle sampling over the field of view (FOV), that is, the whole spectrum. This degrades the state estimation performance but allows locating any target appearing within the FOV.

State estimation relies on a transition model, here the amplitude and the frequency follow a generic random walk, whereas the complex coefficients rotate at the harmonic component's angular frequency. The transition equation is given by Stephan et al. [25]

$$\begin{bmatrix} a_{t+1}^h \\ b_{t+1}^h \\ A_{t+1}^h \\ \omega_{t+1}^h \end{bmatrix} = \begin{bmatrix} \cos(\omega_t^h \Delta t) & -\sin(\omega_t^h \Delta t) & 0 & 0 \\ \sin(\omega_t^h \Delta t) & \cos(\omega_t^h \Delta t) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a_t^h \\ b_t^h \\ A_t^h \\ \omega_t^h \end{bmatrix} + \boldsymbol{v}_t \tag{15.1}$$

where $\boldsymbol{v}_t$ is a zero-mean Gaussian noise and Δt is the duration of a time step.

Similarly to [24], we augment the likelihood (Eq. (15.3a)) as well as the PHD of the posterior (Eq. (15.3b)) using a feature likelihood, that is, the probability for a measurement to be target-originated, based on the spectral kurtosis. The *spectral kurtosis* κ_t^m is obtained by evaluating the kurtosis on a window t filtered using a band-pass filter (BPF) with a $\Delta\omega_{bp}$ bandwidth centered around ω_t^m; a more detailed definition is presented in [26, 27]. In order to interpret the spectral kurtosis and elicit a feature likelihood, the following assumption is made [27]:

Assumption 3 *Noise and excitations highlighting normal modes are assumed to be mesokurtic or leptokurtic.*

That is, the deterministic part of the signals of interest (in their bandwidth) must have a kurtosis strictly below 3 (kurtosis of a Gaussian distribution, i.e., noise in this context). This assumption also leads to a restriction on transient responses. Fast and spiky transients will be leptokurtic and difficult to distinguish from noise and exceptional events. Such transients will thus be treated as clutter. For this reason, we use the spectral kurtosis to correct the estimation in a probabilistic framework rather than triggering measurements solely based on this information.

With a view to lower the weight of clutter-influenced particles, a feature likelihood (assessing the extent to which a measurement was target- or clutter-originated) is elicited from the spectral kurtosis. To this end, we fit a gamma distribution such that the cumulative probability function (cdf) reaches 95% at $\kappa = 3$ (shape parameter $\alpha = 2.615$ and scale parameter $\theta = 0.525$), resulting in the spectral kurtosis likelihood:

$$p_f(\kappa_t^m) = \frac{1}{\Gamma(\alpha)\theta^\alpha}\left(\kappa_t^m\right)^{\alpha-1}\exp\left(-\frac{\kappa_t^m}{\theta}\right) \tag{15.2}$$

This feature likelihood is assumed to be independent of the kinematic likelihood $g_t(\boldsymbol{z}_t^m|\boldsymbol{x}_t^h)$. It refines the weights $\left\{w_{t|t}^{(p)}\right\}_{p=1}^{L_t}$ of the L_t particles approximating the PHD of the posterior at time step t, expressed with respect to prior weights $\left\{w_{t|t-1}^{(p)}\right\}_{p=1}^{L_t}$:

$$g_t(\boldsymbol{z}_t^m|\boldsymbol{x}_t^h) \leftarrow g_t(\boldsymbol{z}_t^m|\boldsymbol{x}_t^h)\,p_f(\kappa_t^m) \tag{15.3a}$$

$$w_{t|t}^{(p)} = \left[1 - p_D + \sum_{m\in[\![1,M_t]\!]} \frac{p_D\ \ g_k(\boldsymbol{z}_t^m|\boldsymbol{x}_t^{(p)})}{K + \sum_{p'=1}^{L_t} p_D\ \ g_t(\boldsymbol{z}_t^m|\boldsymbol{x}_t^{(p')})w_{t|t-1}^{(p')}}\right] w_{t|t-1}^{(p)} \tag{15.3b}$$

where the detection probability p_D and the clutter spatial density K are assumed to be constant and uniformly distributed over the FOV. These weights are also further scaled up or down (and carefully re-normalized to their original mass) according to their probability $p_f(\kappa_t^{(p)})$ to represent a target, that is, $w_{t|t}^{(p)} \leftarrow w_{t|t}^{(p)} p_f(\kappa_t^{(p)})$. However, evaluating the spectral kurtosis of each particle at its estimated frequency would be computationally intractable. Instead, the spectral kurtosis of a particle is computed as a linear interpolation of the one calculated during preprocessing (for each frequency bin of the DFT). Alternatively, this step can be skipped for real-time applications.

This feature-aided SMC-PHD filter finally yields tracks (states and associated covariances) $\left\{\boldsymbol{T}^h\right\}_{h=1}^H$, where H is the total number of harmonic components detected in a dataset.

15.5 Multitarget Tracking to Unsupervised Multilabel Classification

The tracks obtained in the previous section at most represent the behavior of physical components. In this section, a technique is proposed to process and interpret these tracking results in order to recover the activation sequences of the actuators, and the operations they perform.

A first step consists of grouping harmonic components according to their trajectories' states. For simplicity, a descriptive vector is computed for each component, namely $\boldsymbol{\mu}^h = \left[A^h, \omega^h, \sigma_A^h, \sigma_\omega^h\right]^T$, corresponding to the average amplitude, angular frequency, and associated covariances over a trajectory $\boldsymbol{T}^h$. Pairwise distances are computed, allowing for harmonic components to be grouped with one another according to these descriptors. Euclidean distance upon standardized features was considered in this chapter. In more complex cases than those considered, other metrics can be used to associate the tracks between them. For instance, Fréchet distance [28] takes the shape of the trajectory into account, and distances based on Gaussian processes (GP) [29] can take advantage of uncertainty information (i.e., state covariance along each trajectory) provided by the MTT framework. Furthermore, actual harmonics are grouped together. That is, given two components h and h', h is paired with h' as one of its harmonics if there exist $k \in \mathbb{N}$ such that $\omega_t^h \approx k\ \omega_t^{h'}$ throughout the tracks' lifespans. This step results in sets of trajectories $\boldsymbol{\Omega}^c$ for each component c. The activation statuses $\boldsymbol{\delta}^c \in \{0, 1\}^{1\times T}$ over T time steps are immediately deducted from these groups.

In a second step, components are grouped together according to their activation sequences. Pairwise similarities are computed to identify components that are always simultaneously active. The *Jaccard index* [30] is considered here, yet other clustering metrics can be used to compare label sequences with each other. This step results in the activation statuses $\boldsymbol{\delta}^a$ of the actuators. At last, each operation can be characterized by a set of actuators simultaneously active. Operation activation statuses $\boldsymbol{\delta}^o$ hence immediately stem from the actuators' statuses.

Interestingly, the transition from multitarget tracking to unsupervised multilabel classification follows a bottom-up approach (gradually building the operation activation statuses from atomic components), whereas traditional underdetermined blind source separation methods use top-down approaches (from operation clustering to their decomposition into atoms) [4, 5].

15.6 Numerical Simulations and Discussion

A study has been conducted to assess the ability of the proposed approach to identify a machine's production process. A representative synthetic scenario has been designed. A univariate signal was composed as the superimposition of actuator-originated signals, according to the pattern "$AC - AB - BC - ABC$", where A, B, C denote both actuators and atomic operations (i.e., originated by a single actuator).

These actuators produce a 50 Hz-triangle wave, a 700–800 Hz second order (with rise time $\tau_1^a = 0.3$ s and damping $\zeta = 0.3$), and a 400–500 Hz first order (with rise time $\tau_2^a = 4s$) modulated sine waves as source signals respectively, with amplitudes 1, 1 and 3 units; noise standard deviations $\sigma_{s,0}^a = 0.2$, $\sigma_{s,1}^a = 0.8$, $\sigma_{s,2}^a = 1.2$. The signal is sampled at $f_s = 6250$ Hz, and windowed at $w = 0.3$ s with a 50% overlap for short-time Fourier transform (STFT) and spectral kurtosis computation. Measurements are generated according to a detection threshold $\tau = -5.5$ on the log power spectral density.

To compute the spectral kurtosis, a second-order Butterworth band-pass filter is used with a bandwidth $\Delta\omega_{bp} = 3\Delta f$, depending on the spectral resolution $\Delta f = 1/w$.

This scenario has several specificities. An actuator is never active alone to begin with. This case would typically be misinterpreted by traditional decomposition algorithms, in that the pairs AB, BC, and AC would form atomic unseparated modes. Furthermore, the second-order source signal is characterized by its fast rise time and important overshoot. This signal highlights the expected difficulties encountered by the filter when presented with such transients, as mentioned in the third section.

The SMC-PHD is parametered by a clutter rate $\hat{\lambda}_{FA} = 20$ false alarms per scan, a probability of detection $p_D = 99\%$, noise standard deviations $\sigma_A = 0.3$, $\sigma_\omega = 2 \cdot 2\pi\ rad \cdot s^{-1}$, and 1500 particles per expected target. As a birth model, particles are sampled uniformly over the FOV in order to spot targets as they appear. Due to the high-frequency resolution and the fast convergence of target states, we apply the roughening strategy proposed in [31] in order to limit the risk of sample impoverishment.

Despite obvious difficulties with fast-rising transients, we observe that smooth transients are correctly tracked. In comparison to other UBSS frameworks in which data is represented as successive vectors, the orthogonality between the dimensions would make the associated techniques fail (e.g., singular value decomposition [SVD] or sparse regression).

The results of the proposed approach are presented in Fig. 15.1.

Numerical experiments highlighted the little sensitivity the SMC-PHD filter has with respect to its estimated clutter rate. Furthermore, slightly overestimating $\hat{\lambda}_{FA}$ results in better estimation performance, yet at this stage, artifacts remained. This motivated the use of the spectral kurtosis feature to make the filter less dependent on the true (noise-induced) clutter rate across the different operations. Although this enhancement had a very positive effect on clutter resilience, the formulation proposed to elicit a feature likelihood out of the spectral kurtosis experimentally suffered the (theoretically expected) drawback of preventing the tracker to pick up on fast transient responses. Additionally, the birth model is corrected immediately after particle sampling using the proposed feature likelihood. This prevents erroneous tracks from being generated.

Another major advantage of the proposed method for UBSS problems is the ability to decompose a signal despite nonlinearly mixed signals. In practice, actuators emit component-specific signatures, that is, harmonic components that

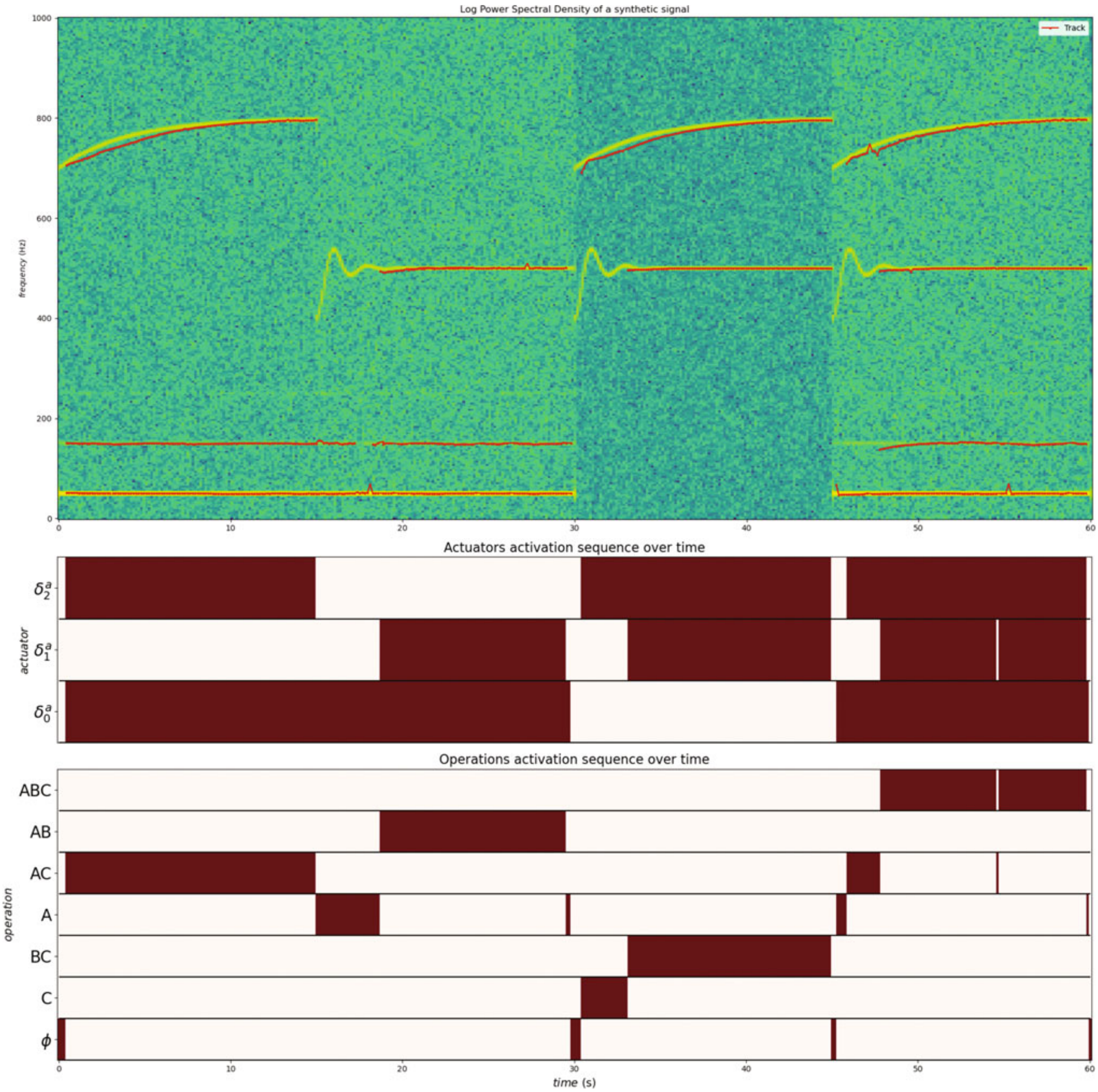

Fig. 15.1 Tracking and decomposition results on a synthetic use case ("$AB - BC - AC - ABC$" sequence of operations with actuators $A = \delta_0^a$, $B = \delta_1^a$, $C = \delta_2^a$; active operations and actuators are shown in red)

uniquely define them in a machine. Hence, by removing coupled harmonics (same frequencies, but different amplitudes), actuators can be well separated regardless of the way they were aggregated in the first place by the remote nonintrusive sensor.

15.7 Conclusions

In this chapter, a feature-aided SMC-PHD was proposed to track harmonic components using the spectral kurtosis to distinguish targets from noise-originated clutter. An algorithm was developed to convert MTT results (trajectories, states, and covariances) into component, actuator, and operation activation sequences. This end-to-end unsupervised process identification approach was verified on synthetic data in a nontrivial scenario in which typical UBSS methods would underperform.

Future work will focus on increasing clutter resilience, handling stronger nonlinearities and transient responses, and validating the generalizability of the approach. Indeed, real-world signals often exhibit a variety of mixed behaviors, from stationary to nonstationary and nonlinear harmonic components. The use of heterogeneous models will be investigated. Moreover, uncertainty information provided by the MTT framework will be leveraged using statistical distances between tracked harmonic components for a better multilabel clustering performance.

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