Chapter 12
Optimal Sensor Placement for Developing Reliable Digital Twins of Structures

Tulay Ercan and Costas Papadimitriou

Abstract Sensor networks are mounted on structures to collect information for addressing a number of important but competing tasks involved in building a reliable digital twin from the collected data. These monitoring tasks include (1) modal identification under low vibration measurements assuming that the system can behave linearly, (2) physics-based model selection and model parameter estimation under various vibration levels activating nonlinear mechanisms at subsystem levels, (3) virtual sensing and response reconstruction over the whole body of the structure using the information from the limited number of sensors, and finally (4) structural health monitoring and damage identification (location and severity). Optimal sensor configuration (OSC) designs (type, number and location of sensors) have been developed in the past to address individual tasks, making assumptions about the loads, models and environmental conditions. However, the sensor network should be designed to collect data that are informative for all tasks simultaneously. In addition, the OSC design should be made robust to modelling, loading and environmental uncertainties. Cost issues related to budget availability for implementing and maintaining a sensor configuration should also be considered in the sensor network design. In this work, a multi-objective OSC framework based on utility functions that are built from information theoretic measures and cost considerations is presented for accounting simultaneously for the aforementioned tasks and thus using cost-effective information extracted from the physical sensing system for developing reliable digital twins. The Kullback-Liebler divergence is used to quantify the information gain from a sensor network, and heuristic algorithms to solve the multi-objective optimization problem are proposed.

Keywords Bayesian inference · Optimal experimental design · Information gain · Virtual sensing · Parameter estimation · Nonlinear structural dynamics · Multi-objective optimization

12.1 Introduction

The objective of an optimal sensor configuration (OSC) design is to maximize the quality of the data collected from a monitoring system. The instrumentation should be designed to collect data that are most informative for different and competing monitoring tasks, including model selection; model updating and parameter estimation; identification of location and magnitude of damage; as well as response reconstruction or virtual sensing of important quantities of interest that are deemed useful to evaluate the condition of structures, detect damages, and make decisions regarding structural health, safety, and performance. Realizing that an OSC for one monitoring task can be suboptimal for another task, a trade-off between the information gained from multiple tasks is needed when designing a monitoring system to be cost-effective and optimal for all the monitoring tasks.

Let $\underline{\delta}$ be a sensor configuration involving the type, location and number of sensors in a structure. Information theoretic measures [1] such as mutual information, Kullback-Liebler divergence, information entropy, joint information and value of information can be used to measure the information content in the data obtained from a sensor network installed on a structure. For a monitoring task i the information gained by the sensor configuration $\underline{\delta}$ is denoted by $\overline{U}_i\left(\underline{\delta}, \underline{\varphi}\right)$. The information contained in the data depends on the type, location and number of sensors in $\underline{\delta}$. Uncertainties included in the parameter set $\underline{\varphi}$ arise from modelling and measurement errors, as well as environmental and operational variabilities. Cost

T. Ercan · C. Papadimitriou (✉)
Department of Mechanical Engineering, University of Thessaly, Volos, Greece
e-mail: costasp@uth.gr

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of the monitoring system is also very important in the design of a sensor configuration in order to limit the overall lifetime cost associated with installing and maintaining the sensor network.

The present study uses previous developments on the information gain indices $\overline{U}_i\left(\underline{\delta}, \underline{\varphi}\right)$ for measuring the information contained in a sensor configuration for each monitoring task and presents a multi-objective methodology for designing an OSC for building a reliable digital twin of an engineering system for the purpose of monitoring its state, performance, reliability and safety. In particular, the information gain for various monitoring tasks has been studied in the literature. Information gain measures have been developed for modal identification [2], for model updating [3] and parameter estimation [1, 4] of nonlinear models of structures, for damage identification [5], and for virtual sensing using modal expansion techniques [6, 7] and sequential Bayesian techniques [1] for linear systems. The sensor network is designed in this work to be optimal for several monitoring tasks simultaneously, cost-effectively trading off the information gained for each of the aforementioned monitoring tasks.

12.2 Information Gain Accounting for Uncertainties

The parameter set $\underline{\varphi}$ introduced in $\overline{U}_i\left(\underline{\delta}, \underline{\varphi}\right)$ accounts for uncertainties in the model parameters such as stiffness and mass properties of the finite element model of the structure, model error uncertainties, as well as operational and environmental (e.g. input) uncertainties [1]. Probability distributions are used to quantify the uncertainty in the values of these model parameters. For this, the uncertain parameter vector $\underline{\varphi}$ is modelled by a prior probability distribution $\pi\left(\underline{\varphi}\right)$. Then the information gain $\overline{U}_i\left(\underline{\delta}, \underline{\varphi}\right)$ is extended to account for the uncertainty in the parameter vector $\underline{\varphi}$ so that the optimal design is robust to uncertainties involved in $\underline{\varphi}$. For this, the information contained in a sensor configuration for the i-th monitoring task is defined to be the expected information gain given by

$$U_i\left(\underline{\delta}\right) = \int \overline{U}_i\left(\underline{\delta}, \underline{\varphi}\right) \pi\left(\underline{\varphi}\right) d\underline{\varphi} \tag{12.1}$$

over all possible values of the parameter set $\underline{\varphi}$. The sources of uncertainties in the parameter set $\underline{\varphi}$ vary from excitation uncertainties to structural model and prediction error model uncertainties. The integral in (12.1) can be computed using Monte Carlo techniques.

12.3 Cost-Effective OSP for Multiple Monitoring Tasks

The OSC design $\underline{\delta}_{\text{opt}}$ has to trade-off information provided for different monitoring tasks such as modal identification, structural identification, structural health monitoring, and response reconstruction (virtual sensing). It can be obtained by maximizing the normalized information gain values for each monitoring task. Herein, the OSC design is formulating as a multi-objective optimization problem of finding the optimal type and location of sensors that simultaneously maximizes the objectives

$$\underline{u}\left(\underline{\delta}\right) = \left\{u_1\left(\underline{\delta}\right), u_2\left(\underline{\delta}\right), \ldots, u_n\left(\underline{\delta}\right)\right\} \tag{12.2}$$

over all possible sensor configurations $\underline{\delta}$, where $u_i\left(\underline{\delta}\right) = U_i\left(\underline{\delta}\right) / U_{i,\max}$ is the normalized robust information gain and $U_{i,\max}$ is the maximum information gain that could be achieved for the monitoring task i by placing sensors at all possible sensor locations. The normalized information gain for each monitoring task guarantees that each term $u_i\left(\underline{\delta}\right)$ in Eq. (12.2) varies from 0 (no information gain) to 1 (maximum information gain). The different information gains can also be combined into a single measure of the total information gain for all tasks, as follows:

$$u\left(\underline{\delta}\right) = \sum_{i=1}^{n} w_i u_i\left(\underline{\delta}\right) \tag{12.3}$$

where the weights w_i, $i = 1, \ldots, n$, sum to 1 and they measure the contribution of each monitoring task on the OSC. Heuristic algorithms such as forward and backward sequential sensor placement (FSSP/BSSP) algorithms [8] can be employed to solve the optimization problem.

Analytical derivations [3] have shown that the information gain for each individual task increases as the number of sensors increases. As a result, the optimal number of sensors cannot be found by information gain considerations only, although after a number of sensors is optimally placed in the structure, the additional information gain is insignificant as one keeps adding sensors in the structure and this could be used as a criterion to select the optimal number of sensors. In practical applications, cost issues should be considered to select the optimal number of sensors. Specifically, the optimal number of sensors should be a trade-off between the information gain from the data and the lifetime cost of instrumentation and maintenance of the sensor system. Let $c\left(\underline{\delta}\right)$ be the total cost of a sensor configuration $\underline{\delta}$, including the cost of sensors, the installation cost and the maintenance cost over the lifetime of the sensor system. The optimal number and location of sensors is obtained as the one that maximizes the information gain vector $\underline{u}\left(\underline{\delta}\right)$ and minimizes the cost $c\left(\underline{\delta}\right)$. Using the total information gain in Eq. (12.3), the selection of the OSC (type, location and number of sensors) can be setup as a two-objective optimization problem of minimizing the weighted sum $u\left(\underline{\delta}\right)$ and maximizing $c\left(\underline{\delta}\right)$. The problem can be readily solved (e.g. [7]) to find the Pareto optimal solutions. Alternatively, given cost constrains (a fixed budget c_t available for designing a monitoring system), the optimal sensor configurations can be reformulated as a constrained optimization problem of maximizing the objectives $\underline{u}\left(\underline{\delta}\right)$ or the single objective $u\left(\underline{\delta}\right)$ subject to the cost constrain $c\left(\underline{\delta}\right) \leq c_t$. A special case of cost consideration in optimal sensor placement (OSP) can be found in [7] for a single monitoring task.

12.4 Conclusions

The conceptual design of a cost-effective OSC is formulated as a multi-objective optimization problem that trades off the information gained for each monitoring task and the installation and maintenance cost of instrumentation. The formulation presented can account for a variety of monitoring tasks provided that an information gain index is built for each monitoring task. The tasks may include modal identification, model selection, model updating, parameter estimation, damage identification and virtual sensing. The proposed methodology can accommodate the environmental and operational uncertainties, including input as well as modelling uncertainties manifested in building information gain indices. Monte Carlo techniques can estimate the resulting probability integrals, and a number of optimization strategies are available to use for solving the resulting multi-objective optimization problem and estimate the Pareto optimal sensor configurations.

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