Chapter 17
Experimental Modal Analysis of Structures with High Nonlinear Damping by Using Response-Controlled Stepped-Sine Testing

Taylan Karaağaçlı and H. Nevzat Özgüven

Abstract In the last decade, various promising nonlinear modal identification techniques have been developed based on the nonlinear normal mode (NNM) concept. Most of these techniques rely on the phase resonance testing approach where the identification of nonlinear modal damping is still an unresolved issue. The response-controlled stepped-sine testing (RCT) framework provides a convenient way of accurately quantifying nonlinear modal damping by applying standard linear modal analysis techniques to frequency response functions (FRFs) measured at constant displacement amplitude levels with standard modal test equipment. Various studies by the authors have shown that these constant-response FRFs come out in quasi-linear form even in the case of a high degree of nonlinearities. The RCT approach has been validated so far on several systems including a real missile structure with moderate damping nonlinearity mostly due to bolted connections and a micro-electromechanical device with a stack-type piezo-actuator. This study makes a step further by validating the method on a real control fin actuation mechanism that exhibits very high and nonlinear modal damping; the maximum value of viscous modal damping ratio goes up to 15% and the percentage change of the damping with respect to vibration amplitude is about 70%.

Keywords High nonlinear damping · Nonlinear experimental modal analysis · Response-controlled stepped-sine testing · Control fin actuation mechanism · Unstable branch

17.1 Introduction

The ever-growing industrial competition always favors higher speed, lower energy consumption, and longer service life in aircraft and turbomachinery. In the achievement of these high-performance goals, lightweight design becomes an important objective. However, the lightweight design naturally results in more flexible structures that may exhibit large amplitude oscillations under dynamic loads, which may eventually lead to dynamic instabilities such as the aeroelastic flutter [1] or the limit cycle oscillation (LCO) [2].

Structural damping plays a key role in the suppression of aeroelastic instabilities [3, 4]. Therefore, accurate modeling of the damping mechanism is vital for the determination of realistic instability envelopes and for pushing the limits in the design process. However, under large amplitude oscillations as in the case of aeroelastic instabilities, the structural damping exhibits highly nonlinear behavior, which makes its identification a challenging task.

Almost all structural damping mechanisms in aircraft and turbomachinery are mainly due to friction. Friction and backlash in the actuation mechanisms of aircraft control surfaces, and special dissipative elements such as under-platform dampers in turbomachinery, are among important sources of structural damping. Another important source of damping common to both aircraft and turbomachinery is friction in mechanical joints. Identification of all these nonlinear damping mechanisms is already a difficult task as mentioned above. Another factor that complicates the identification process even more is the variability of nonlinear dynamics in mechanical joints in repeated testing [5].

The recent work of Al-Habibi et al. [6] gives a comprehensive review of the state-of-the-art techniques that can be used in the identification of nonlinear structural damping. Although it is possible to categorize these techniques in various ways, as is

T. Karaağaçlı (✉)
The Scientific and Technological Research Council of Turkey, Defense Industries Research and Development Institute, TÜBİTAK-SAGE, Mamak, Ankara, Turkey
e-mail: taylan.karaagacli@tubitak.gov.tr

H. Nevzat Özgüven
Department of Mechanical Engineering, Middle East Technical University, Ankara, Turkey

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done in the review paper, the methods can simply be divided into two main groups: nonmodal and modal techniques. Several important examples of nonmodal methods are the restoring force surface (RFS) [7] method, the nonlinear auto-regressive moving average with exogenous input (NARMAX) method [8], the wavelet transform [9], and the describing surface method (DSM) [10]. Although these techniques give satisfactory results in the case of local nonlinearities, their extension to general nonlinear multi-degree-of-freedom (MDOF) systems is not practical in general. The main advantage of modal methods over nonmodal approaches is that they are capable of quantifying the resultant effect of multiple nonlinearities spread over a nonlinear MDOF structure. The last decade witnessed the development of various promising nonlinear modal identification techniques. In 2011, Peeters et al. [11] extended the phase resonance testing approach to nonlinear systems and successfully isolated a single NNM during the experiment. In 2015, Londono et al. [12] adapted the nonlinear resonant decay method (NLRDM) [13] to extract the backbone curves of nonlinear systems. A year later, Renson et al. [14] implemented the control-based-continuation (CBC) [15] to trace out the NNM backbone curves. In 2017, Peter and Leine [16] proposed another experimental continuation technique based on the phase-locked-loop (PLL) control strategy commonly used in electrical engineering. A very recent NNM backbone curve identification technique is the one proposed by Kwarta and Allen [17], where near-resonant steady-state harmonic responses measured by open-loop sine testing are processed by using the single nonlinear mode formula [18]. Another very recent nonlinear modal identification technique is the velocity feedback approach proposed by Scheel [19].

All the modal approaches mentioned above successfully identify NNM backbone curves. However, the identification of nonlinear modal damping is still an unresolved issue for many of them [11, 14, 16]. Although in recent papers [17, 19, 20], a special emphasis is placed on nonlinear modal damping, its identification is still a difficult problem especially when the degree of nonlinearity is very high. However, in the RCT method [21], the nonlinear modal damping is very conveniently identified by applying standard linear modal analysis techniques to constant-response FRFs that are measured in the quasi-linear form. The quasi-linearity of these FRFs is achieved by keeping the displacement amplitude of the driving point constant throughout the frequency sweep. The RCT approach has been validated so far on a benchmark beam with local cubic stiffness nonlinearity and a real missile structure with moderate damping nonlinearity mostly due to bolted connections [21], a double-clamped beam that exhibits strong geometrical nonlinearity [22, 23], a micro-electromechanical device with a stack-type piezo-actuator [24], and a benchmark beam with a bolted lap joint [25]. The main contribution of this chapter is to show that the RCT method is capable of identifying a very high and nonlinear modal damping; the maximum value of viscous modal damping ratio goes up to 15% and the percentage change of the damping with respect to vibration amplitude is about 70%. The mechanism exhibits not only a high degree of damping nonlinearity but also a high degree of stiffness nonlinear that results in a 50% shift of the natural frequency, which is also successfully identified by the RCT technique.

17.2 Nonlinear Experimental Modal Analysis with the RCT-HFS Framework

Nonlinear experimental modal analysis by using response-controlled stepped-sine testing (RCT) [21] consists of the following steps:

1. Measure constant-response FRFs at several different displacement amplitude levels by keeping the displacement amplitude of the driving point constant throughout the stepped-sine testing at each level.
2. Extract modal parameters (modal constant, natural frequency, and modal damping ratio) at each displacement amplitude level by applying standard linear modal analysis techniques to constant-response FRFs that come out in the quasi-linear form. Construct the nonlinear modal model by expressing the identified modal parameters as functions of the modal amplitude.
3. Synthesize required constant-force FRFs by using the identified nonlinear modal parameters in a Newton-Raphson solution scheme with the arc-length continuation algorithm.

The quasi-linearity of constant-response FRFs mentioned in step 2 is theoretically based on the following receptance formula derived from the Nonlinearity Matrix concept [26] and the single nonlinear mode theory [18] as explained in [21]:

$$\alpha_{jk}(\omega, q_r) = \frac{\overline{A}_{jkr}(q_r)}{\overline{\omega}_r^2(q_r) - \omega^2 + i2\overline{\xi}_r(q_r)\,\omega\overline{\omega}_r(q_r)} \tag{17.1}$$

where α_{jk} is the near-resonant receptance corresponding to the displacement at point j for a given excitation at point k, and ω denotes the excitation frequency. $\overline{A}_{jkr}(q_r)$, $\overline{\omega}_r(q_r)$, and $\overline{\xi}_r(q_r)$ are the modal constant, natural frequency, and viscous modal damping ratio corresponding to the r th nonlinear mode, respectively.

All the modal parameters shown in Eq. (17.1) are functions of a single parameter: the modal amplitude q_r. Therefore, if the modal amplitude is kept constant throughout the stepped-sine testing, the measured FRFs come out in the quasi-linear form. In [21], it is shown that in the case of single-point excitation, the constant modal amplitude condition can be achieved by keeping the displacement amplitude of the driving (excitation) point constant in closed-loop control. It is important to note that the constant displacement amplitude condition does not necessarily require a displacement transducer as the control sensor. In most of the applications of the RCT method conducted so far, the accelerometer has been used as the control sensor due to its popularity and availability. The constant displacement amplitude condition can be achieved indirectly by feeding the closed-loop controller with an appropriate acceleration profile.

An important merit of the RCT method is that it is based on frequency sweep at constant amplitude and therefore it can be implemented by using standard modal test equipment available in the market (e.g., LMS SCADAS & LMS Test Lab). This is an important advantage of the RCT approach over state-of-the-art techniques such as CBC [15] and PLL [16]. CBC technique, which is based on amplitude sweep at constant frequency, and the PLL technique, which is based on phase control, cannot be implemented with standard equipment.

The validation of the nonlinear modal parameters identified by the RCT method can be achieved by using the harmonic force surface (HFS) concept proposed by the authors in [21]. The prominent feature of the HFS is the accurate identification of the turning points and unstable branches of constant-force frequency response curves, directly from the experiment. This makes the HFS a valuable tool to accurately determine the backbone curves of strongly nonlinear structures [27]. The HFS technique consists of the following steps:

1. Measure harmonic excitation force spectra at several different constant displacement amplitude levels by keeping the displacement amplitude of the driving point constant throughout the stepped-sine testing at each level.
2. Construct the HFS by merging the measured harmonic excitation force spectra and using linear interpolation.
3. Extract constant-force frequency response curves by cutting the HFS with constant-force planes.

The third step of the HFS technique provides a model-less identification of stable and unstable branches of constant-force frequency response curves. Theoretically, these curves should be the same as the curves synthesized in the third step of the RCT method and therefore can be used to validate the nonlinear modal model identified by RCT. It is important to note that the harmonic excitation force spectra used to construct the HFS are the ones measured during RCT and used to construct the constant-response FRFs. Therefore, since no additional test is required, the construction of the HFS can be regarded as an integral part of the RCT method together with the identification of the nonlinear modal model. In this context, the RCT-HFS can be regarded as a self-validating framework.

17.3 Experiment

Control fins play an important role in the aeroelastic behavior of guided missiles [28, 29]. For realistic aeroelastic analyses, the accurate modeling of the structural dynamics of control fins is of vital importance. Unfortunately, backlash and friction between various moving parts in the actuation mechanism of a control fin may result in a very complex nonlinear behavior. The identification of such a high and complex nonlinearity is a very challenging problem for the current state of the art.

Figure 17.1 shows the sketch of the experimental setup used to identify the nonlinear behavior of a real control fin actuation mechanism. The casing of the actuation mechanism is rigidly fixed to the ground. The control fin is instrumented with 10 miniature accelerometers (Dytran 3225M23). The structure is excited at point 1 in the z-direction with a stinger attached to an electrodynamic shaker (B&K). The excitation force is measured with a force transducer (Dytran 1022V) attached between the stinger and the control fin.

In this study, the vibration mode of interest is the first mode of the control fin actuation mechanism assembly which is a torsional mode. Preliminary broadband random test results indicate that the fin surface acts as a rigid body in that mode. Therefore, it can be concluded that this mode simply results from the nonlinear dynamics of the actuation mechanism. Accordingly, the system is treated as an SDOF rotational system and the nonlinear system identification is accomplished simply by using the driving point FRFs of the system.

Similar to the previous applications of the RCT method, all the data acquisition and closed-loop control tasks are accomplished by using the LMS SCADAS Mobile data acquisition system and the LMS Test Lab. software package. The upper and lower frequency limits of the stepped-sine tests covering the mode of interest are determined based on preliminary broadband random tests. The frequency step used in stepped-sine testing is 0.125 Hz.

In a recent study by the authors [27], the NNM backbone curve corresponding to the first mode of the control fin actuation mechanism has been successfully identified by using the HFS technique as shown in Fig. 17.2. The evolution of the backbone

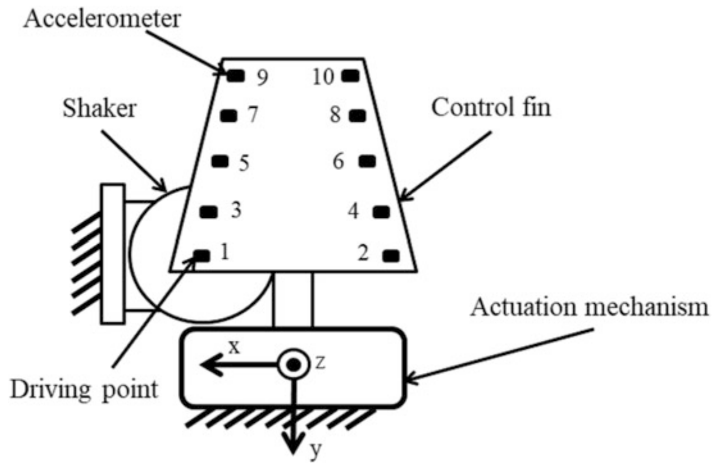

Fig. 17.1 Sketch of the experimental setup for the nonlinear system identification of the control fin and its actuation mechanism

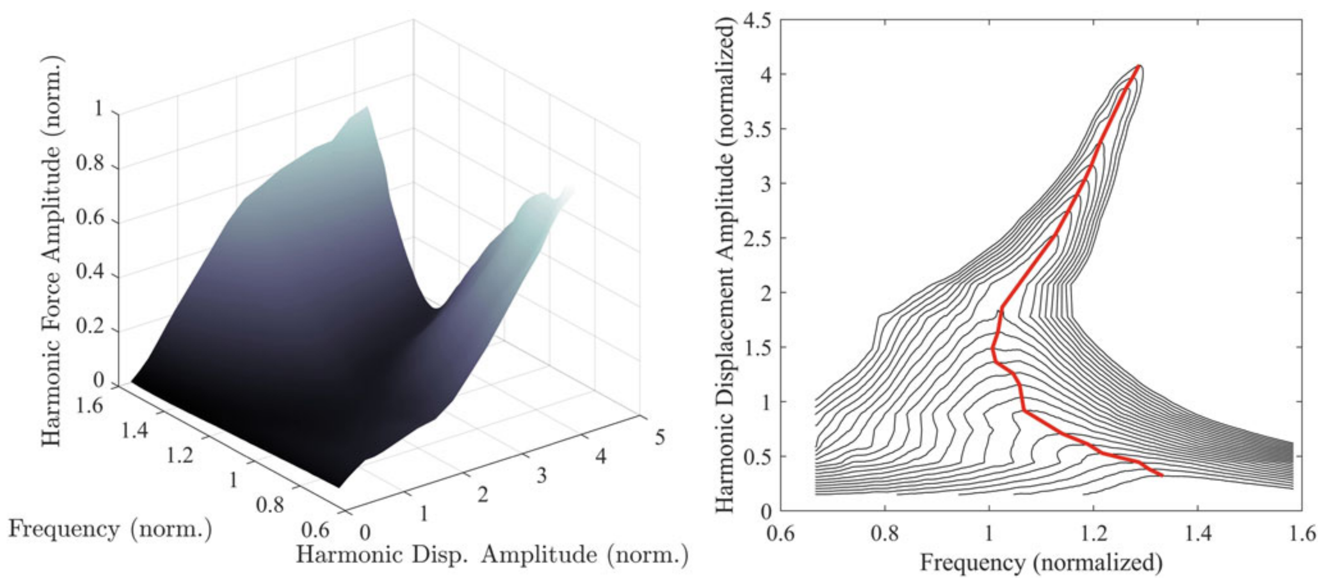

Fig. 17.2 Identification of the NNM backbone curve of the control fin actuation mechanism by using the HFS technique [27]: (**a**) harmonic force surface (HFS) (**b**) driving point constant-force frequency response curves (black) extracted from the HFS and the NNM backbone curve (red)

curve demonstrates the complexity of the nonlinear behavior: the resonance frequency first decreases until it reaches a minimum value and then it increases monotonically. The initial softening effect could be related to the stick-to-slip transition and succeeding hardening behavior is probably related to backlash.

Unfortunately, the identification of the NNM backbone curve only reveals the evolution of the stiffness nonlinearity of the control fin actuation mechanism, but it does not say anything about the damping nonlinearity. The main contribution of this chapter is the accurate identification of nonlinear modal damping.

In order to identify the nonlinear modal model of the control fin actuation mechanism, first of all, the constant-response receptances of the system are measured at 15 different displacement amplitude levels (labeled D1 to D15) by using the RCT strategy. For the sake of clarity, Fig. 17.3 shows the receptance curves corresponding to only 10 different constant displacement amplitude levels.

In the second step, the constant-response receptance curves measured at each displacement amplitude level are subjected to linear modal analysis with LMS PolyMAX. The outcome of each modal analysis is a specific set of modal parameters (modal constant, natural frequency, and modal damping ratio). The good overlap between the linear receptance curves synthesized from each set of modal parameters and the corresponding constant-response receptance curves validates the quasi-linearity of receptances shown in Fig. 17.3. The variations of the modal parameters (identified at 15 different displacement amplitude levels) with respect to the modal amplitude are shown in Figs. 17.4 and 17.5.

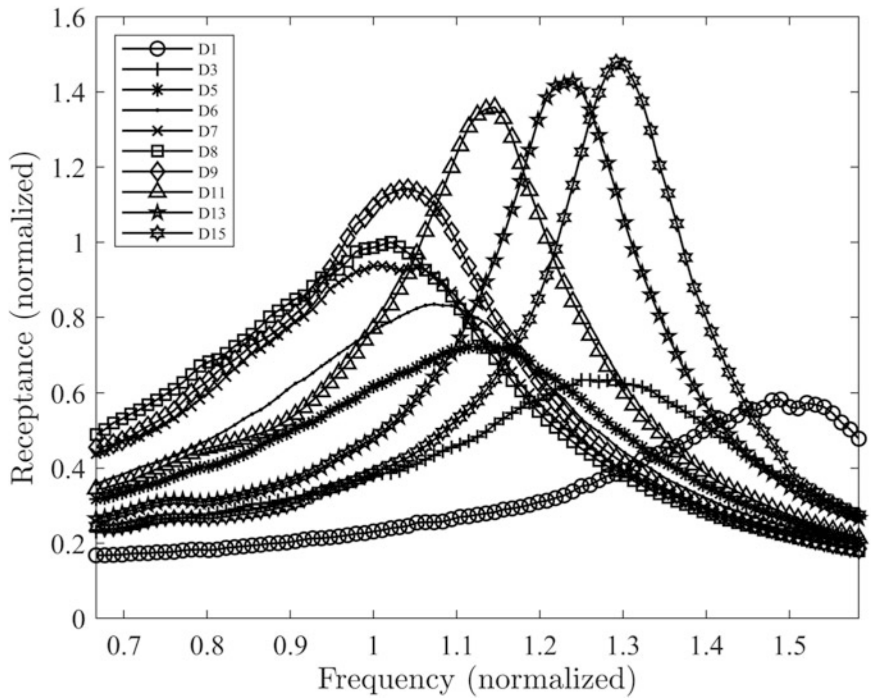

Fig. 17.3 Constant-response receptances of the control fin actuation mechanism measured at the driving point by RCT

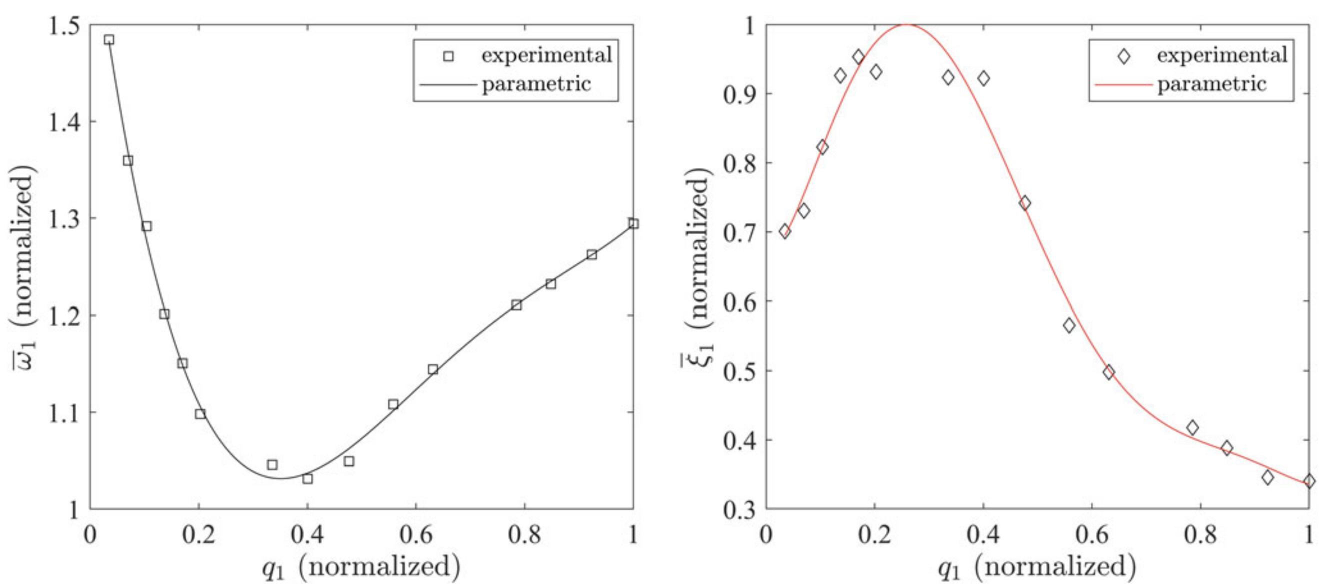

Fig. 17.4 Variation of the modal parameters with modal amplitude: (**a**) natural frequency (**b**) modal damping ratio

In Fig. 17.4, about a 50% change in the natural frequency and a 70% change in the modal damping ratio indicate a very high degree of stiffness and damping nonlinearities that cannot be reached in many studies. In Fig. 17.4b, the modal damping ratio is given in the normalized form; the actual value of the viscous damping ratio goes up to 15%. This means that not only the damping nonlinearity but also the value of damping itself is very high. All these results clearly demonstrate the power of the RCT method.

In order to achieve the validation, first of all, constant-force receptances are synthesized from the identified modal parameters in a Newton-Raphson solution scheme with the arc-length continuation algorithm at four different force levels: F1, F2, F3, and F4 in ascending order. Then, these synthesized receptances are compared with the ones extracted from the HFS as shown in Fig. 17.6. As can be seen from the figure, the match between the synthesized and extracted receptances is almost perfect, which shows that the identified nonlinear modal parameters are very accurate.

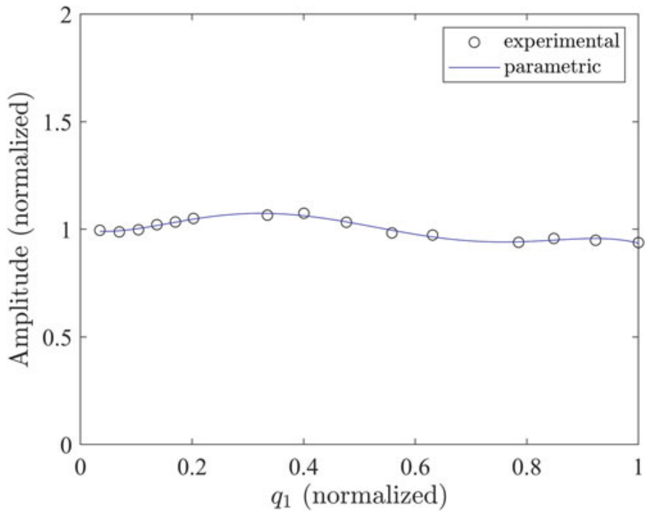

Fig. 17.5 Variation of the modal constant with modal amplitude

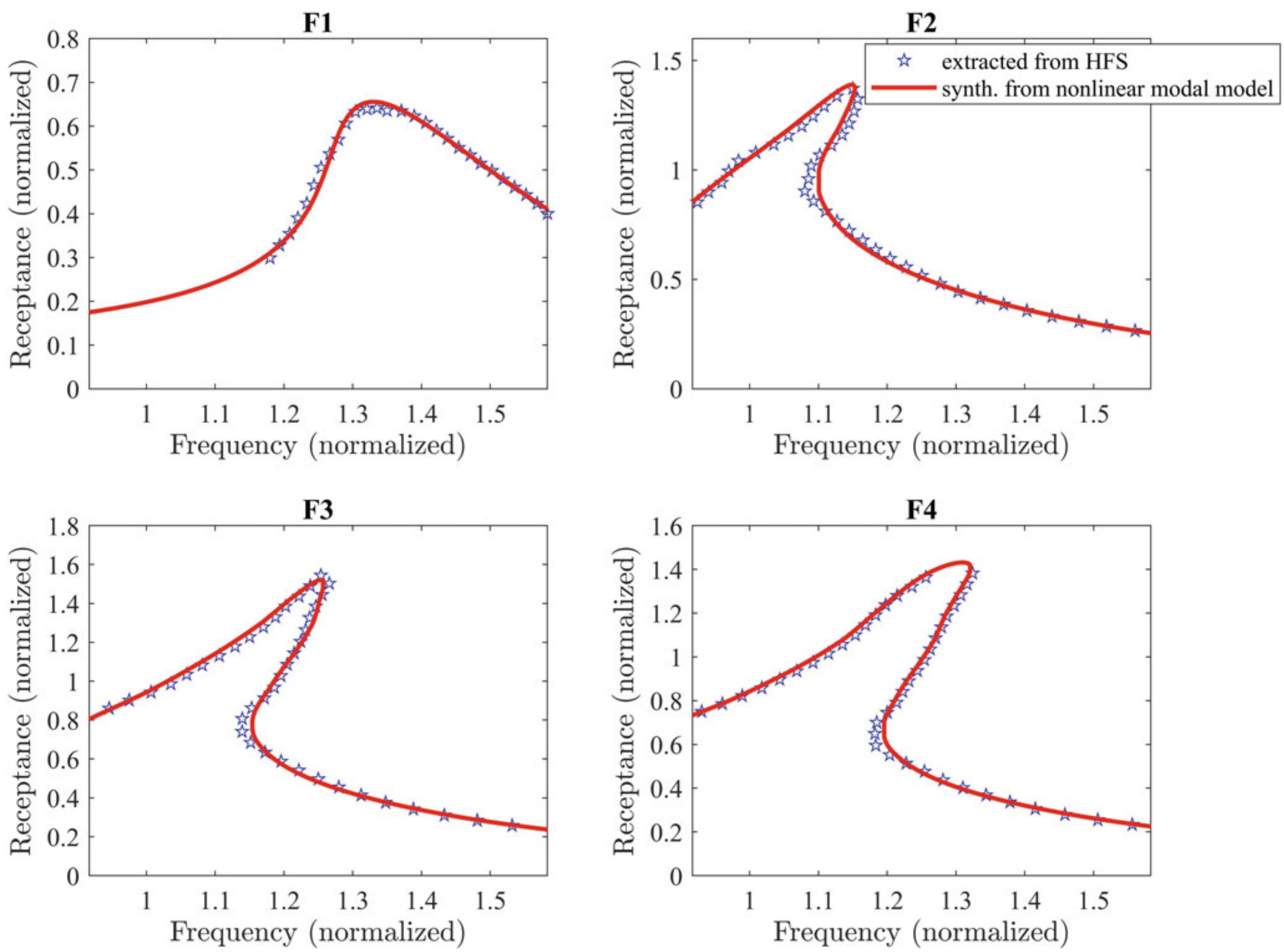

Fig. 17.6 Validation of the identified nonlinear modal model by comparing the driving point receptances synthesized from the model with the ones extracted from the HFS

17.4 Discussions and Conclusions

In the last decade, very promising experimental continuation techniques have been developed based on the nonlinear normal mode (NNM) concept. Most of these techniques proved to be successful in the accurate identification of NNM backbone curves. However, the rigorous mathematical treatment and the accurate identification of nonlinear modal damping is still a very challenging task for most of the state-of-the-art techniques, especially in the case of very high and nonlinear damping. However, the response-controlled stepped-sine testing (RCT) method provides a very convenient and accurate way of identifying nonlinear modal damping by applying standard linear modal analysis techniques to constant-response FRFs that are measured in the quasi-linear form. The quasi-linearity of these FRFs is achieved by keeping the displacement amplitude of the driving point constant throughout the stepped-sine testing. The RCT method has been successfully applied so far on various systems including a real missile structure with moderate damping nonlinearity mostly due to bolted connections and a micro-electromechanical device with a stack-type piezo-actuator. This study makes a step further by validating the method on a real control fin actuation mechanism that exhibits a very high and nonlinear modal damping; the maximum value of viscous damping ratio goes up to 15% and the percentage change of the damping with respect to vibration amplitude is about 70%. The mechanism exhibits not only a high degree of damping nonlinearity but also a high degree of stiffness nonlinear that results in a 50% shift of the natural frequency, which is also successfully identified by the RCT technique. The perfect match between the constant-force FRFs synthesized from the identified modal parameters and the ones directly extracted from the experiment by using the harmonic force surface (HFS) technique shows that the nonlinear modal parameters are very accurately identified.

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