Chapter 13
Case Study on the Effect of Nonlinearity in Dynamic Environment Testing

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Abstract While recent research has greatly improved our ability to test and model nonlinear dynamic systems, it is rare that these studies quantify the effect that the nonlinearity would have on failure of the structure of interest. While several very notable exceptions certainly exist, such as the work of Hollkamp et al. on the failure of geometrically nonlinear skin panels for high speed vehicles (see, e.g., Gordon and Hollkamp, Reduced-order models for acoustic response prediction. Technical Report AFRL-RB-WP-TR-2011-3040, Air Force Research Laboratory, AFRL-RB-WP-TR-2011-3040, Dayton, 2011. Issue: AFRL-RB-WP-TR-2011-3040AFRL-RB-WP-TR-2011-3040), other studies have given little consideration to failure. This work studies the effect of common nonlinearities on the failure (and failure margins) of components that undergo durability testing in dynamic environments. This context differs from many engineering applications because one usually assumes that any nonlinearities have been fully exercised during the test.

Keywords Dynamic environment · Reliability · Quasi-linear

13.1 Introduction

Dynamic environment testing usually considers components that are fixed to a vehicle at their base. An environment is typically created that seeks to envelope the worst-case base accelerations that the component is expected to experience in operation. Then, the part is tested by replaying that environment on a large shaker and then checking whether the component is still functional. If the component survives, one typically repeats the tests with the environment increased to see if failure occurs. This continues until the part fails or until the shaker system is no longer able to exert sufficient force to increase the environment further. The difference between the environment at which the component fails (or the maximum tested environment) and the actual environment defines a margin; if the environment changes as more information becomes available this margin can be used to justify avoiding additional testing.

In this work, we consider cases in which the environmental testing is performed at various levels relative to that at which a nonlinearity becomes activated, to understand the consequences. To simplify the analysis, we consider quasi-linear behavior, in which one can define a linear model for a certain vibration level by choosing a constant value for the coefficients in

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an amplitude-dependent linear model. Measurements from a representative nonlinear structure are then used to predict the consequences on the environment testing.

13.2 Setting of Problem

We construct our model to match the behavior of the S4 benchmark beam structure [2]. For this study we assume a single degree-of-freedom modal model with the generalized coordinate being the displacement of Mode 2. This mode of vibration exhibits noticeable nonlinearity based on the data from [3], and is shown as the blue dots in Fig. 13.1. This model was then fit to a single-degree-of-freedom modal model with a mass, linear spring, linear damper, and an Iwan joint in order to be able to extrapolate the behavior to higher amplitudes (green dashed line in Fig. 13.1).

Approximating the system with a quasi-linear modal model, the equation of motion can be written in terms of the amplitude-dependent natural frequency $\omega_n(A)$ and damping ratio $\zeta(A)$ where A is the amplitude of the response q, i.e., $q = A\sin(\omega t - \phi)$. We assume a broad-band base excitation model for the force $F(t)$ so that the resonance frequency will be excited even if the natural frequency shifts at higher amplitude. Our modal equation of motion is

$$\ddot{q} + 2\zeta(A)\omega_n(A)\dot{q} + \omega_n^2(A)q = F(t) \tag{13.1}$$

The response of the system to a harmonic force is then given by the well-known equation, which gives the peak velocity in modal coordinates.

$$q(t) = Re\left[\frac{-F_p}{(\omega_n^2 - \omega^2) + i(2\zeta\omega_n\omega)}e^{i\omega t}\right] \quad \rightarrow \quad V_p = \frac{F_p}{2\zeta\omega_n}. \tag{13.2}$$

Now suppose that we know the response V_{p1} due to a force F_{p1} and we wish to find the response V_{p2} to a different force $F_{p2} = \alpha F_{p1}$. Using the prior equation, while allowing the damping and frequency to differ at each point because they are amplitude dependent, we obtain the following:

$$V_{p2} = \frac{\alpha F_{p1}}{2\zeta_2\omega_{n2}} = \alpha V_{p1}\left(\frac{\zeta_1}{\zeta_2}\right)\left(\frac{\omega_{n1}}{\omega_{n2}}\right) = V_{LinPred}\left(\frac{\zeta_1}{\zeta_2}\right)\left(\frac{\omega_{n1}}{\omega_{n2}}\right) \tag{13.3}$$

This equation must be solved iteratively because ζ_2 is a function of the amplitude V_{p2}. In doing so, one can discover how the response of the system changes as the force (i.e., the environment) increases or decreases. This can then be compared

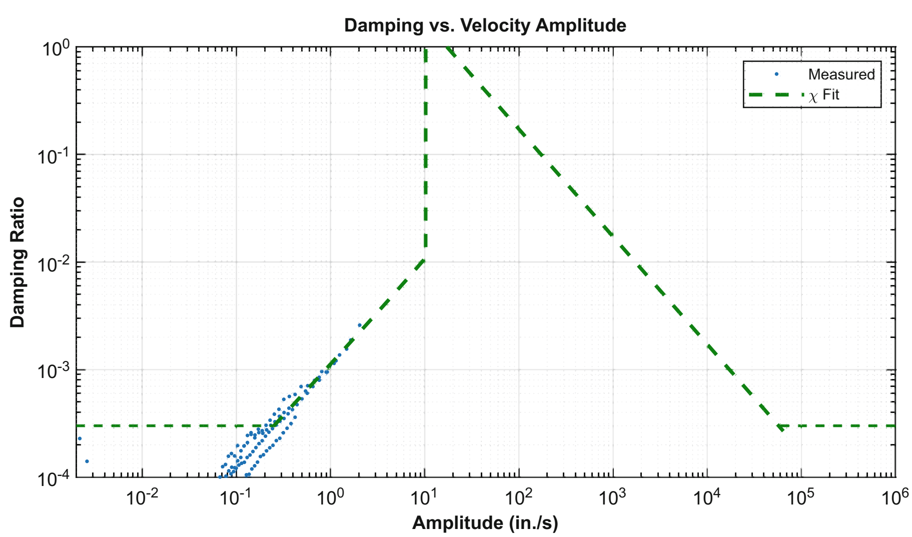

Fig. 13.1 Damping vs. velocity amplitude measurements from [3] and fit to an SDOF system with an Iwan element

to what one would obtain if the system were linear, i.e., $V_{LinPred} = \alpha V_{p1}$, as this is the behavior that is typically assumed when the nonlinearity in a system is not characterized.

13.3 Results

Several possible cases are presented in Table 13.1. In our analogy, the force or the factor α represents how much the force is amplified compared to that for the reference point. The force is analogous to the strength of the vibration environment, which is typically quantified as the base acceleration in environmental testing; the base acceleration on a component translates into an inertial force that can excite its modes of vibration. In the case considered here the system has only one mode of vibration.

The first lesson learned from the results in Table 13.1 is that, for this type of nonlinearity, the response and hence the stress can increase very slowly relative to the environment. For example, in Case C we began with a response level $V_1 = 0.1$ in/s and then the environment was increased by a factor of 1000 ($\alpha = 1000$), and yet the response V_{2NL} (and hence the stress in the part) increased by only a factor of $5.3/0.1 \approx 53$. A typical test might actually only involve an environment that was 20 dB or an order of magnitude above the expected environment, in which case the stresses in the part may not increase much beyond those in the expected environment. As a result, one should be wary in using linear thinking (i.e., that a 10x environment equates to 10x stress) when assessing environments. On the other hand, this case study also shows that a system with friction nonlinearity can have very high robustness, being capable of experiencing very large environments without large increases in stress. Conversely, if analysts were to take this nonlinearity into account ,then they might be able to exploit it to reduce the mass and cost of the component quite significantly.

The situation is worse in Cases D–F, where the test was performed with the system on the verge of macroslip. In Case D, increasing the environment by a factor of 10 increases the response by a factor of $9.54/2 \approx 3.2$, but then increasing the environment by a further factor of 100 (Case F) results in a further increase of only $10.25/9.54 \approx 7\%$. Once again the system would have great ability to increase in the level of the environment so that the tests would do little to increase the stresses in the part. While this is desirable, one is on the verge of catastrophe. From Case F, if the amplitude of the environment is increased by a factor of 2 (i.e., to a total of 2000), then Case G shows that the response would suddenly increase by more than a factor of 6000! In such a case, we would expect the part to break very quickly and fail the environmental test. One might then increase the strength of the part by an order of magnitude and repeat the test only to see it fail again.

Cases with $\alpha < 1$ in Table 13.1 are also particularly relevant to environment testing because one usually presumes that the test was performed at a higher level than the field environment. For example, Case K states that if testing was performed at a very high level (i.e., beyond the macroslip regime), that decreasing the response by a factor of 10 ($\alpha = 0.1$) reduces the response by a factor of 1000 (i.e., to 10.23 instead of 10,000). Hence, the environment test would be dramatically over-conservative. While 100,000 in/s may sound like a tremendously large environment for the S4 Beam, one should bear in mind that the curve above can shift a lot if the design of the bolts is changed or if they are just tightened less. Hence, small levels of base excitation could be needed to achieve this phenomenon in a different structure.

Table 13.1 Data comparing the response at higher input amplitude predicted by the linear model versus the nonlinear model. All of these results assume that $\omega_{n1} = \omega_{n2}$ in Eq. 13.3

Case	V_1	ζ_1	α	ζ_2	V_{2Lin}	V_{2NL}
A	0.1	0.0003	10	0.0006	1	0.5
B	0.1	0.0003	100	0.0018	10	1.65
C	0.1	0.0003	1000	0.00566	100	5.3
D	3	0.00324	10	0.01	30	9.54
E	3	0.00324	100	0.095	300	10.25
F	3	0.00324	1000	0.95	3000	10.25
G	3	0.00324	2000	0.0003	6000	65000
H	10.25	0.95	2	0.0003	20.5	65000
I	3	0.00324	0.1	0.001	0.3	0.928
J	3	0.00324	0.01	0.00034	0.03	0.286
K	100000	0.0003	0.1	0.29	10000	10.23

13.4 Conclusions

This paper has highlighted the danger in applying linear thinking to a nonlinear system and has also demonstrated a procedure by which the damping versus amplitude behavior of a system can be used to obtain more reliable estimates of the response of a system as its environment changes. Even if the test drives the structure well into the nonlinear regime, nonlinearity affects how the stresses in the part increase or decrease in response to that environment, and one should be aware of that when determining what environment to test to and when interpreting the results.

While it was not discussed here, the results could be even more serious if testing is performed at low levels and then that model is used to predict the response at higher levels. That extrapolation can be highly over-conservative or highly under-conservative depending on whether the joints are exercised above or below macroslip.

For the conference presentation, these case studies will be investigated further, including quantifying the effect on fatigue life, and additional case studies will be presented for a bilinear contact nonlinearity.

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