Chapter 12
Simulating Nonlinear Beating Phenomena Induced by Dry Friction in Dynamic Systems

Iyabo G. Lawal, Michael R. Haberman, and Keegan J. Moore

Abstract Self-excitation and beating phenomena are the result of nonlinear constitutive behavior of vibrating structures with nonlinear components. These behaviors require externally supplied excitation and can be induced by the combination of near equilibrium damping and nonlinear damping operating far from equilibrium. In this study, dry friction, a mechanism known for producing self-excitation in structures, is explored as a mechanism for producing beat phenomena in structures. The system consists of two masses connected via a linear spring with one of the masses damped via a grounded dashpot that is modeled using a five-parameter friction contact model. The system is modeled and solved using the RK-4 time-integration scheme. We perform system parameter identification of experimental data using the STFT (short-time Fourier transform) and wavelet-bounded empirical mode decomposition (WBEMD) to determine system model variables that may simulate the self-excitation and beat phenomena observed in the structural dynamics. Beat phenomena may also be a result of the existence of two or more closely separated damped natural frequencies. We also investigate the degree to which self-excitation in the structure is driving the nonlinear beat phenomenon as opposed to it being caused by choosing closely separated damped natural frequencies. We address this question using nonlinear normal modes (NNM) analysis which provides frequency-energy dependence of the modes as system parameters change. The approach developed here is useful for the design of energy harvesting and vibration isolation systems that are subjected to sliding friction.

Keywords Beating phenomena · Empirical mode decomposition · Friction · Mode lock-in · Self-excitation

12.1 Introduction

The emergence of "beating" has been observed in structures and depends on their stiffness and damping properties. In several studies, "beating" phenomena has been observed to be a result of closely separated modes in the structure, which in a linear system would produce a similar effect. It could also be caused by harmonic interactions driven by nonlinearities in the contact interface or by self-excitation within the structure. The root cause of this "beating" phenomena is not yet fully understood. The motivation in modeling this phenomena is to understand what set of parameters generate this effect in a 2-DOF mass system that consists of a driven linear oscillator and a nonlinear module with damping elements. This configuration of elements has also been known to produce targeted energy transfer (TET) from the linear oscillator to the strongly nonlinear module, known as a nonlinear energy sink (NES) [1, 2]. Kerschen and others [1] found that "beat phenomena" was a more efficient means of energy transfer than the reliance of either fundamental or sub-harmonic resonance capture. The efficiency of energy pumping or one-way energy transfer increases when "beating" phenomena is present. Another self-excitation effect is caused by friction-induced vibrations, where substructures separated by frictional contacts create self-excitation in the overall structure and in some cases may produce "brake squeal" [3]. This phenomena is identified as "mode lock-in" by the structural dynamics community. Prior studies explore how viscous elements can produce passive energy suppression in structures [4]. In this work, we explore another angle, by looking at friction as a dissipative element for TET. Friction has the

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additional challenge in that it can also be a cause of self-excitation in the structure, particularly with "stick-slip" contact [5]. At high velocities, it acts in a dissipative capacity; however, if the right system parameters are chosen, it can also produce self-excitation and the emergence of "beating" phenomena.

12.2 Background

The 2-DOF lumped parameter model with spring and viscous damping is shown in Fig. 12.1a. An alternative 2-DOF lumped parameter model is shown in Fig. 12.1b, where energy dissipation is from a frictional contact element represented with a Bouc-Wen contact model [6].

12.2.1 Lumped Parameter Model

Two systems are considered: one that is viscous damped and the other friction damped with the use of a BW element. The two systems are shown in Fig. 12.1. The driven mass behaves as a linear oscillator and is externally driven, while the dissipative element is either a viscous or friction element.

The characteristic equation for both cases is shown. The parameters can be tuned to produce the beating phenomena. The goal is to determine the set of parameters that generate "beating" phenomena. F_{ex} is the external driving force, and F_D is the friction force generated in the BW dissipation element. The viscous-damped model parameters are $\mathrm{m}_1 = m_2 = 1.2\,\mathrm{kg}$, $\mathrm{k}_1 = 1.4\,\mathrm{N}/\mu\mathrm{m}$, $\mathrm{k}_2 = 200\,\mathrm{N}/\mu\mathrm{m}$, $k_3 = 29\,\mathrm{N}/\mu\mathrm{m}$, and $c = 250\,\mathrm{kN/mm}\cdot\mathrm{s}$. In the Bouc-Wen model, the same parameters were used in addition to the five model parameters: $\mathrm{P_n} = 60\,\mathrm{N}$, $\mathrm{n} = 2$, $\mathrm{k_t} = 270\,\mathrm{N}/\mu\mathrm{m}$, $\sigma = 0.5$, $\mu = 0.5$, $\rho = \mathrm{k_t}/\mu|\mathrm{P_n}|$.

Viscous Damping **BW Friction Damping**

$$\ddot{x_1} = \frac{1}{m_1}\left[F_{ex}(t) - k_2(x_1 - x_2) - k_1x_1 - c_1\dot{x}_1\right] \qquad \ddot{x_1} = \frac{1}{m_1}\left[F_{ex}(t) - k_2(x_1 - x_2) - k_1x_1 - c_1\dot{x}_1\right]$$

$$\ddot{x_2} = \frac{1}{m_2}\left[k_2(x_1 - x_2) - k_3x_2 - c_3\dot{x}_2\right] \qquad \ddot{x_2} = \frac{1}{m_2}\left[k_2(x_1 - x_2) - F_D(x_2, \dot{x}_2)\right]$$

$$\text{where} \quad F_D(x_2, \dot{x}_2) = \mu P_N \xi(t) \quad \text{and} \quad \dot{\xi} = \rho\left[1 - (\sigma\dot{x}_2 sgn(\xi) + 1 - \sigma)|\xi|^n\right]\dot{x}_2. \tag{12.1}$$

The matrix form of the equation of motion and the state-space representation of the viscous-damped system are shown with **F**, representing the external forcing vector.

$$\mathbf{M}\ddot{x}(t) + \mathbf{C}\dot{x}(t) + \mathbf{K}x(t) = \mathbf{F}(t) \tag{12.2}$$

$$\mathbf{z} = \begin{pmatrix}\mathbf{x}\\ \dot{\mathbf{x}}\end{pmatrix}, \quad \dot{\mathbf{z}} = \mathbf{g}(\mathbf{z}) \quad \text{and} \quad \mathbf{g}(\mathbf{z}) = \begin{pmatrix}\dot{\mathbf{x}}\\ \mathbf{M}^{-1}\left[\mathbf{F}(t) - \mathbf{K}\mathbf{x}(\mathbf{t}) - \mathbf{C}\dot{\mathbf{x}}(t)\right]\end{pmatrix} \tag{12.3}$$

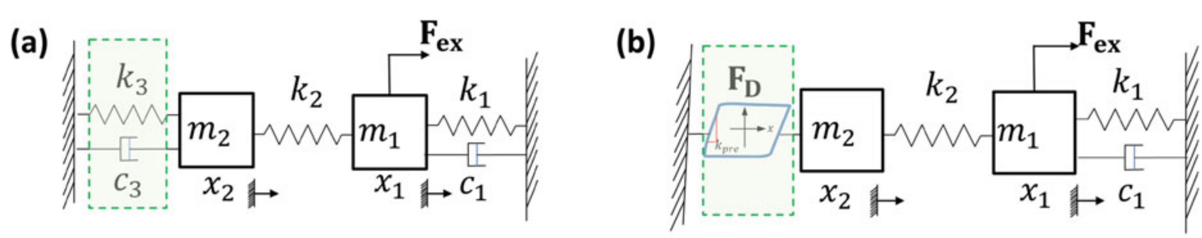

Fig. 12.1 Lumped parameter model with (**a**) representing viscous-damped system and (**b**) representing friction-damped system

12.3 Analysis

In the case where the system is being driven with an excitation force, F_{ex}, one technique to obtain the displacement and velocities is to use a numerical approach using implicit time-integration schemes such as Newark, Wilson, and Runge-Kutta (RK4) methods to get the displacements and velocities of a nonlinear system. A time-integration scheme based on Runge-Kutta is developed in MATLAB to determine the displacement and velocities in both systems. The simulation was completed over a 10-second interval and with a 0.01 time discretization for 10 seconds. The velocities of the masses are shown in Fig. 12.2 where the "beating" phenomena is observed in the viscous damped, but not the friction-damped system. For both models, the response of the displacement has a similar pattern to the velocity, with differences due to magnitudes.

A frequency domain analysis of the displacement in both systems is shown in Fig. 12.3. Figure 12.3a shows sub-harmonic interactions in the spectra. Analysis was done with a short-time Fourier transform (STFT) with six Gaussian windowing functions.

12.3.1 Numerical Integration Scheme

Beating phenomena was observed in the viscous-damped case. There also appears to be some energy transfer between the harmonics in the case of viscous damping. This can be seen clearly in Fig. 12.3a where the peak frequency at 0.03 Hz is transferring its energy into the other modes of the system. Beat phenomena occurred here when the difference between the damped frequencies, $\Delta\omega = \omega_1 - \omega_2$, is quite small. ω_1 represents the frequency with the highest amplitude for m_1 and ω_2 is the frequency with the highest amplitude for m_2. $\Delta\omega_{visc}$ was 0.001. This is different from the friction dissipated model, which at the given parameters did not demonstrate "beating" phenomena. $\Delta\omega_{BW}$ was 0.03. The other observation made from Fig. 12.3a is the presence of closely spaced frequencies near the peak frequency, ω_1.

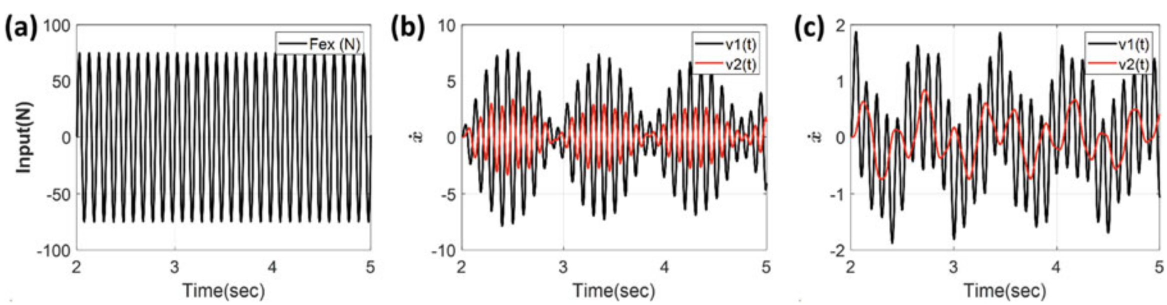

Fig. 12.2 Input excitation force shown in (**a**) with amplitude of 75 and $T = 0.1$. The viscous-damped system velocities ($\dot{\mathbf{x}}$) are shown in (**b**) and the friction-damped velocities ($\dot{\mathbf{x}}$) are shown in (**c**)

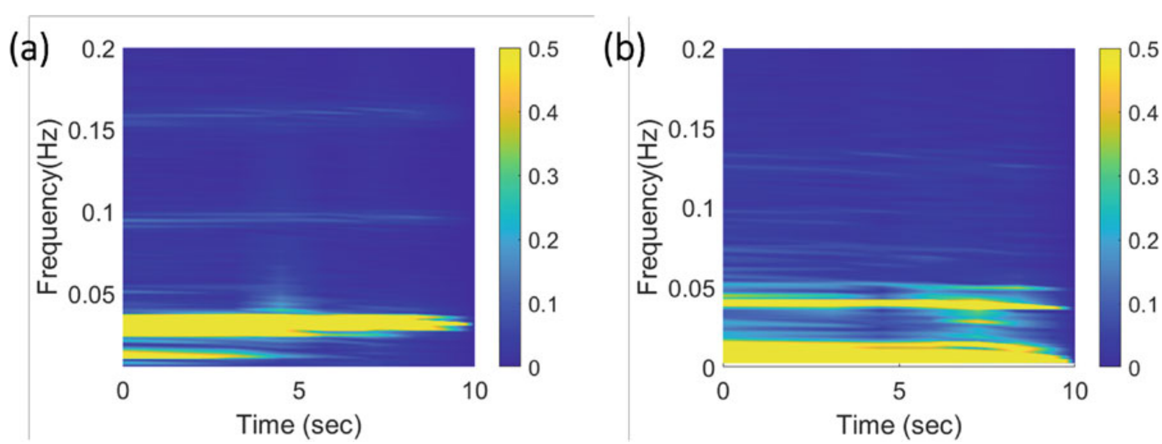

Fig. 12.3 STFT of the displacement, x_2, for (**a**) viscous-damped system and (**b**) friction-damped system with Bouc-Wen friction element with maximum frequency of 0.2 Hz

12.4 Conclusion

If the "beating" phenomena is caused by self-excitation in the structure, it can be suppressed by changing damping and stiffness parameters. Lu and others [4] achieve this by including stiffness and damping elements in directions orthogonal or at an angle to the direction of motion. In a friction element, both dissipative properties and self-excitation in the structure may emerge. Self-excitation from friction elements has been observed experimentally in structures; however, it isn't clear where the switch from self-excitation to dissipation or the reverse occurs in the structure. Chatterjee found that for the LuGre friction model, there exists a critical normal load modulation factor that reduces "stick-slip" behavior in the assembly [5]. In that study, cyclical normal force, P_N, present was used to reduce "stick-slip"-based oscillations. Future studies will explore how cyclical normal force, P_N, in the BW module can be used to reduce "stick-slip"-based oscillations and even possibly understand where the shift from self-excitation to dissipation occurs in the structure.

The use of normal modes (NNM) using the arc-length continuation method [7, 8] may help to evaluate energy transfer within the system. As observed in the viscous-damped case, there seems to be some level of sub-harmonic transfer when "beating" phenomena is present. However, there might also be an energy transfer mechanism, known as targeted energy transfer (TET), present as well. In TET, energy transfer between the linear oscillator and the dissipative element occurs passively. To ascertain if this mechanism is present, the use of NNM is needed to generate energy-frequency curves and to see the interactions between the two masses in the system [9, 10].

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