

# Reference Beam Method for Determining Thermal Fluctuation Constants



Aleksander Erofeev , Timofey Gorokhov , Sergey Emelyanov ,  
and Ekaterina Pakhomova 

**Abstract** A new method for determining the thermal fluctuation constants of the generalized Zhurkov equation is proposed. The method is based on the experimentally established dependences of the obtained thermal fluctuation constants on the coordinate of the pole point and the limiting temperature. The choice of the subject of the study is justified, methods of conducting experiments and processing experimental data are given. Solid polyvinyl chloride plates were selected as the sample material for the study, which were broken by transverse bending on a six-position bench. To increase the reliability of the obtained results, the static processing method was used according to GOST R 8.736-2011. According to the obtained experimental data, graphs were plotted in coordinates « $\lg \tau - \sigma$ ». A reference beam and the corresponding reference constants are proposed, as well as a system of coefficients necessary for finding the thermal fluctuation constants of a material on the basis of reference constants. To verify the adequacy of the developed technique, the thermofluctuation constants of the generalized Zhurkov equation for the decorative protective plate on the polyester resin binder were determined by the reference beam method, which were compared with values with constants for this material obtained by the graphical and graphoanalytic method.

**Keywords** Durability · Zhurkov equation · Forecasting · Performance · Coefficient system · Construction

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A. Erofeev (✉) · T. Gorokhov  
Tambov State Technical University, 106, Sovetskaya Ul, Tambov 392024, Russia  
e-mail: [av.erofeev@yandex.ru](mailto:av.erofeev@yandex.ru)

T. Gorokhov  
e-mail: [gorohowt@yandex.ru](mailto:gorohowt@yandex.ru)

S. Emelyanov · E. Pakhomova  
South West State University, 50 Let Oktyabrya Street, 94, Kursk 305040, Russia  
e-mail: [yz\\_swsu@mail.ru](mailto:yz_swsu@mail.ru)

## 1 Introduction

Thermofluctuation constants of the generalized Zhurkov equation are currently determined on the basis of experimentally obtained data of the dependence of durability from stress and temperature by a graphical analytical method [1–5]. This method is based on rearranging the experimentally obtained graph into the graph with coordinates of the logarithm of durability versus stress ( $\log\tau-\sigma$ ) and into a graph of the dependence of the logarithm of durability versus inverse temperature ( $\log\tau-1000/T$ ). Two constants  $\lg\tau_0$  and  $T_m$  are found from the position of the pole, and the other two,  $U_0$  and  $\gamma$ , from the graph of the dependence of the activation energy on stress [1, 6]. This method of determining the thermal fluctuation constants is rather complicated, and the need to perform graphical rearrangements reduces the accuracy of the results [7, 8]. This is due to the fact that according to the existing method for determining the constants, it is required to determine the durability (logarithm of durability) for each of three temperatures at five voltages. This is also due to the need to work in semi-logarithmic coordinates, when even minor deviations of the logarithm of durability in the zone of low durability (the range of experiments) leads to significant errors in further calculations of the durability of the material.

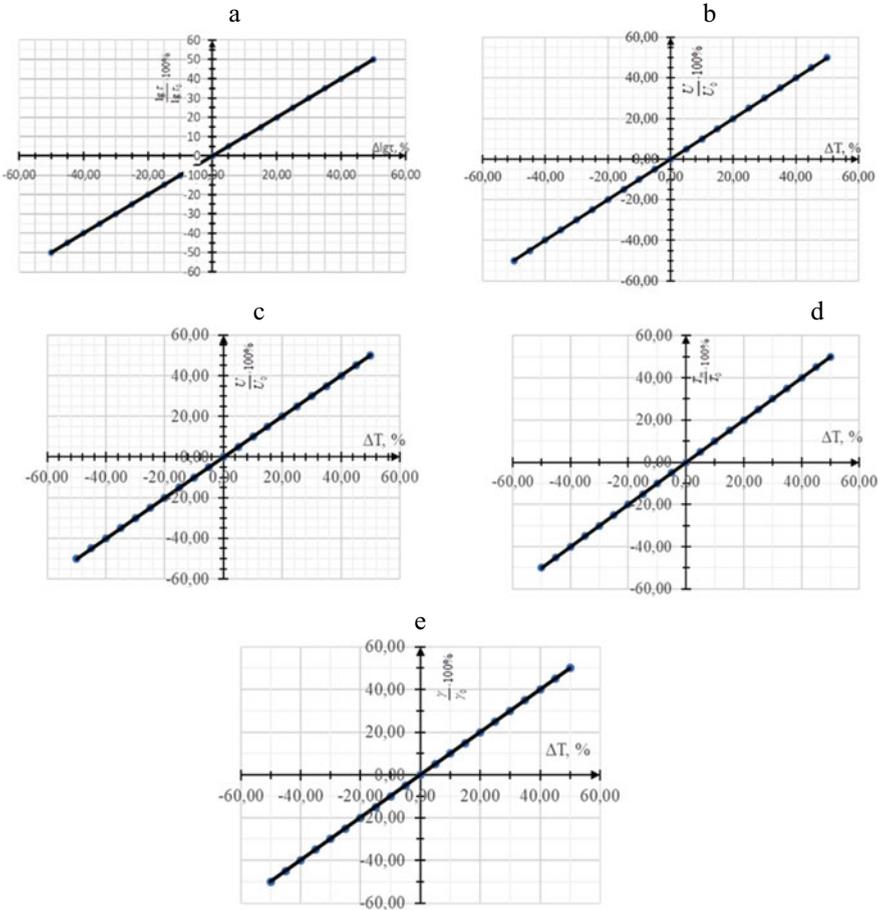
The established linear dependence of the change in the constant  $\log\tau_0$  on the magnitude of the pole displacement along the ordinate (Fig. 1a); the change in the thermal fluctuation constant  $U_0$  also has a linear dependence on the magnitude of the displacement of the beam pole only along the abscissa axis (Fig. 1b). The obtained dependences, along with other regularities due to the thermal fluctuation nature of fracture and deformation of solids (Fig. 1c–e), make it possible to propose a reference beam method for determining the thermal fluctuation constants of the generalized Zhurkov equation [9–11]. The proposed method does not imply rearranging the experimentally obtained graph. Its principle is based on manipulation with a reference beam.

## 2 Materials and Methods

For experimental verification of the hypothesis, polyvinyl chloride plates were chosen as the object of the study. This choice is due to the fact that it has a homogeneous structure and the presence of temperature–time force equivalence of the destruction process is clearly traced for it.

To identify the temperature–time–force equivalence, beams samples were made of polyvinyl chloride. The total number of samples was 150 pieces. The length of the samples was 6 cm. The cross section is rectangular ( $b \times h = 1.5 \text{ cm} \times 0.3 \text{ cm}$ ).

For transverse bending and fracture tests, a six-position stand was used (Fig. 2). This stand consists of a frame 1 made of corners. On the support platform of the frame 2, support rods 3 are installed at a distance from each other equal to the span of the beam. Sample 4 is placed on support pedestals and loaded with a load device

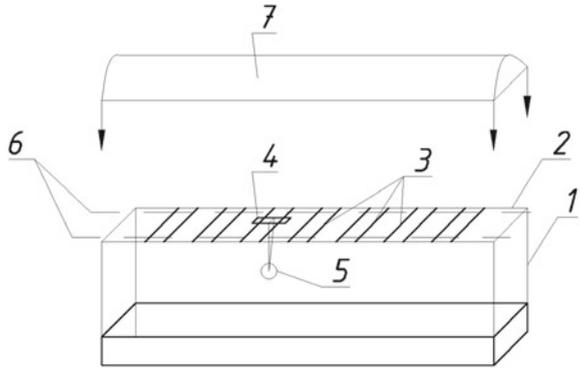


**Fig. 1** a dependence of the relative change in  $\lg \tau_0$  on  $\Delta \lg \tau$ , b dependence of the relative change in  $U_0$  on  $\Delta \sigma$ ; c dependence of the relative change in  $U_0$  on  $\Delta T$ ; d dependence of the relative change in  $T_m$  on  $\Delta T$ ; e dependence of the relative change in  $\gamma$  on  $\Delta T$

5. Elevated temperature is created by rod electric heaters 6. To reduce heat loss and create a directed heat flow, a casing 7 is installed and fixed to the frame on the support platform. The temperature is set by LATR 1 M 220 V-9A and is regulated by a potentiometer EPV2-11A gr. HC 0300 °C.

For each temperature, five voltages are selected in the range from 0.75 to 0.95 of breaking. The temperatures at which the measurements were carried out were 15, 30, 45 °C. To obtain each point, at least 8 samples were tested under similar conditions. The results obtained were subjected to statistical processing according to the method according to GOST R 8.736–2011 to eliminate gross errors and establish the boundaries of the confidence interval [12, 13].

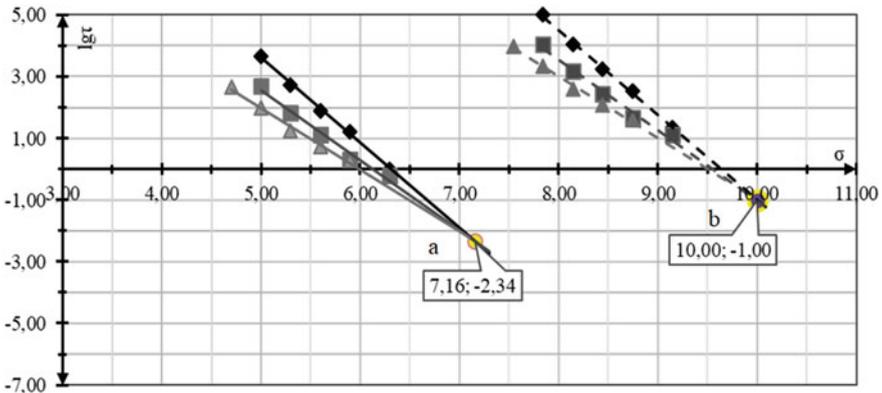
**Fig. 2** Experiment equipment



Thermofluctuation constants for the reference beam (Fig. 3a) were obtained on the basis of experimentally determined constants of PVC plates at transverse bending ( $\log\tau_0 = -2.34$ ;  $\gamma = 43.85$  kJ/(mol MPa);  $T_m = 437.95$  K,  $U_0 = 313.85$  kJ/mol) [14–16]. The beam with the coordinates of the pole point (10; 11) was taken as the reference beam, i.e.  $\log\tau = -1$ ,  $\sigma = 10$  MPa and the maximum temperature possible  $T_m = 500$  K (Fig. 3b).

The obtained thermal fluctuation constants of the reference beam are summarized in Table 1.

To determine the thermal fluctuation constants by the reference beam method, the reference constants of Table 1 must be multiplied by the conversion factors (system of coefficients):  $k_\sigma$ ,  $k_k$ . However, initially it is required to determine the



**Fig. 3** a base beam; b reference beam

**Table 1** Thermofluctuation constants of the reference beam

$\gamma$ , kJ/(mol $\times$ MPa)	$U_0$ , kJ/mol	$T_m$ , K	$\lg\tau_0$
50	500	500	-1

limiting temperature of material possible [6], which is determined based on the linear dependence of the change in the slope of the equation:

$$\lg \tau(\sigma) = a \cdot \sigma + b \quad (1)$$

where  $a$ —is the slope or the tangent of the angle of inclination of a straight line, the physical meaning of which is the rate of the process;

$b$ —the free term of the equation, which determines the logarithm of the life in the absence of stresses.

In fact, Eq. (1) is the equation of the direct temperature of the graph, experimentally constructed in the coordinate system of the dependence of the logarithm of durability on stresses.

The linearity of the dependence of the slope on temperature (inverse temperature) is verified in three ways: theoretical, proof against the contrary, and practical [17]. Let the dependence of the change in the coefficients of “Eq. 1” be linear and described by the equation:

$$a = k \cdot T + d, \quad (2)$$

Then, substituting them into “Eq. 1”, we obtain:

$$\lg \tau = (k \cdot \sigma + d) \cdot T + c \cdot T + \mu \quad (3)$$

solving which relative to the variable (temperature) or its reciprocal, taking into account that the stress is in this case a constant ( $\sigma = const$ ), it turns out:

$$\lg \tau = (k \cdot \sigma + c) \cdot T + (d \cdot \sigma + \mu) \quad (4)$$

thus we have a linear dependence.

In this case, it is assumed that the dependence of the change in the coefficient  $b$  on temperature or its inverse value is also linear:

$$b = c \cdot T + \mu \quad (5)$$

Thus, the resulting “Eq. 4” is the equation of forward stresses constructed in the coordinate system of the logarithm of durability versus temperature (reciprocal temperature), which are obtained by remaking the graph in the coordinate system.

« $\lg \tau - \sigma$ » into the coordinate system « $\lg \tau - T$ » (« $\lg \tau - \frac{1}{T}$ ») in practice. Consequently, the hypothesis put forward is confirmed.

In the case of proof from the opposite, let the dependences of the change in the coefficients of “Eq. 1” are not linear, but obey, say, parabolic dependences. Then, solving “Eq. 1” with respect to a variable  $T$  at a constant voltage, the dependence of the logarithm of durability on temperature (reciprocal temperature) will not have a linear form, which contradicts the experimental data.

Having established the values of the coefficients  $c$  and  $d$  of “Eq. 2” and equating  $a$  to zero, the limiting temperature of the existence of a solid is found.

The constant  $\lg\tau_0$  is determined by the ordinate of the pole point of the resulting beam [1].

To determine the thermal fluctuation constants by the reference beam method, a system of coefficients is used:  $k_\sigma, k_k$ . It seems possible to determine the structural-mechanical constant  $\gamma$  and the value of the activation energy of destruction  $U_0$  on the basis of the ratio of the angular coefficients of “Eq. 2” of the reference and the desired beam, expressed by the coefficient  $k_k$ .

The coefficient  $k_\sigma$  is determined as follows:

$$k_\sigma = \frac{\sigma}{\sigma_\sigma}, \quad (6)$$

where  $\sigma$ —the abscissa point of the pole of the obtained direct temperature graph;  
 $\sigma_\sigma$ —abscissa point of the pole of the reference direct temperature graph;

$$k_k = \frac{k}{k_\sigma}, \quad (7)$$

where  $k$ —slope of “Eq. 2” for experimental data;

$k_\sigma = -2,619$ —slope of “Eq. 2” of the reference graph.

The constant  $U_0$  is determined by multiplying the reference constant  $U_{0,\sigma}$  by the system of coefficients  $k_\sigma$  and  $k_k$ :

$$U_0 = k_\sigma \cdot k_k \cdot U_{0,\sigma}, \quad (8)$$

The constant  $\gamma$  is determined by multiplying the reference constant  $\gamma_\sigma$  by the coefficient  $k_\sigma$ :

$$\gamma = k_\sigma \cdot \gamma_\sigma. \quad (9)$$

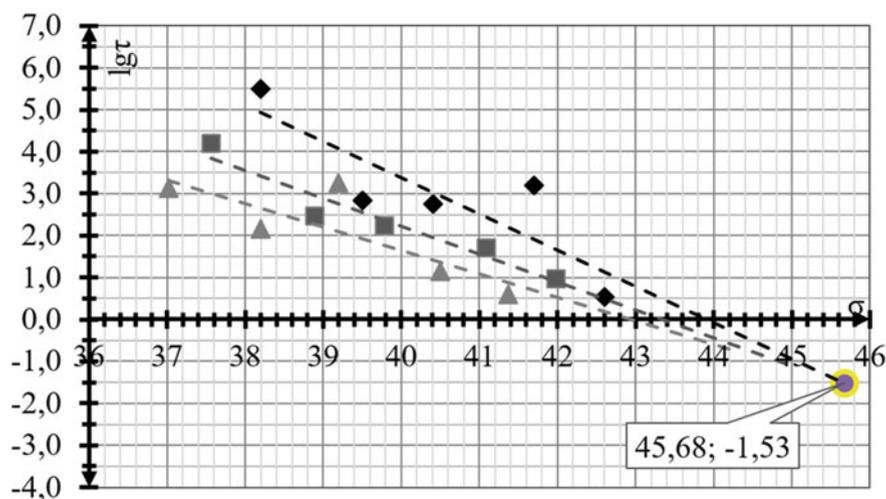
### 3 Results and Discussion

To check the developed technique, let us determine the thermal fluctuation constants of the generalized Zhurkov equation for a decorative protective slab based on a polyester resin binder using the reference beam method and compare the obtained values with the constants for this material obtained by the graphic and graphic-analytical method [19, 20]. Experimental data are presented in Table 2 [21].

On the basis of the experimentally obtained data (Table 2), a graph of direct temperatures in the coordinates  $\lg\tau - \sigma$  for a decorative protective board on a binder made of polyester resin was built (Fig. 4).

**Table 2** Experimental data on the dependence of durability on voltage and temperature for a decorative protective board on a binder made of polyester resin

T = 20,00 °C		T = 40,00 °C		T = 60,00 °C	
$\sigma$	$\lg\tau$	$\sigma$	$\lg\tau$	$\sigma$	$\lg\tau$
42,60	0,540	41,99	0,950	41,37	0,600
40,40	2,740	41,10	1,700	40,50	1,150
38,20	5,500	39,78	2,220	39,20	3,260
41,70	3,200	38,90	2,460	38,20	2,170
39,50	2,830	37,57	4,200	37,02	3,120

**Fig. 4** Graph of the dependence of the decimal logarithm of durability on voltage at a given temperature

According to the obtained graph (Fig. 4), the pole point has coordinates along the abscissa axis  $\sigma = 45.68$  MPa and along the ordinate axis  $\lg\tau = -1.53$ , and the resulting straight lines are expressed by the equations:

- at T = 20 °C  $y = -0,863x + 37,896$ ;
- at T = 40 °C  $y = -0,6629x + 28,734$ ;
- at T = 60 °C  $y = -0,5581x + 23,969$ .

The limiting temperature of the existence of the material is determined from the linear dependence of the change in the slope of the slope of direct temperatures, shown in Fig. 5.

The equation of direct change in the slope of the formula (1) for a decorative-protective plate on a binder made of polyester resin (Fig. 5) with  $R_2 = 0.98$  is expressed by the equation:

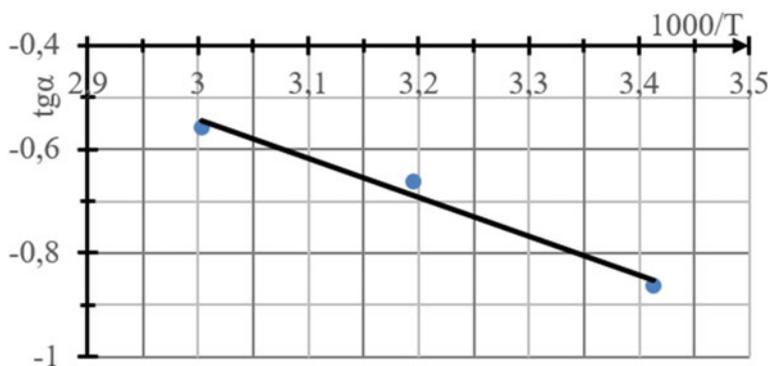


Fig. 5 Graph of the change in the slope of the «Eq. 1»

$$a = -0.7477(1000/T_m) + 1.7008,$$

hence at  $a = 0$   $T_m = 439.64$  [22, 23].

The values of the coefficients  $k_\sigma$  and  $k_k$  are determined by “Eqs. 5, 6”, respectively:

$$k_\sigma = \frac{45,68}{10} = 4,57,$$

$$k_k = \frac{-0,748}{-2,619} = 0,285$$

Let us determine the thermal fluctuation constants  $U_0$  and  $\gamma$  for a decorative protective board based on a polyester resin binder according to “Eqs. 5, 6”, respectively:

$$U_0 = 4,57 \cdot 0,285 \cdot 500 = 652,79 \text{ kJ/mol.}$$

$$\gamma = 0,29 \cdot 50 = 14,50 \text{ (mol MPa).}$$

The obtained values coincide with the results obtained by graphic and graphic-analytical methods and presented in Table 3.

Analysis of Table 3 shows a high convergence of the results of calculations performed by the graphical-analytical method and by the method of the reference

**Table 3** Thermofluctuation constants for a decorative protective board based on a polyester resin binder

Method of obtaining constants	Thermofluctuation constants			
	$\gamma$ , kJ/(mol MPa)	$U_0$ , kJ/mol	$T_m$ , K	$\lg \tau_m$
Graphic	15,85	701,00	454,50	-1,00
Graphoanalytical	14,29	652,79	439,64	-1,53
Reference beam method	15,50	652,79	439,64	-1,53

beam, which indicates the adequacy of the proposed method for determining the thermal fluctuation constants [24].

## 4 Conclusion

Thus, the technique of the reference beam makes it possible to determine the durability of the material according to the generalized Zhurkov formula without additional plotting in the coordinates  $\text{tga} - (1000/T)$  and  $U - \sigma$ , which reduces labor intensity, and also, more importantly, avoids errors and errors that arise in the process of graphic constructions.

The next stage of work is the need to test the proposed methodology on a wide range of building materials.

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