Chapter 10
A Reconsideration of Appropriation from a Sociocultural Perspective

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10.1 Introduction

Why does interaction in the learning community deepen mathematics learning? How does individual learning contribute to the learning community through dialogue and deepen mathematics learning? These questions can be answered from not the viewpoint of dualism, which considers the learning of society and the individual separately, but from a sociocultural perspective, which considers these aspects jointly. Sfard (2008), who pioneered the unique concept of "commognition" based on psychology and philosophy, emphasized that communication represents thinking itself, focusing on the connection between individual thinking and community learning from a sociocultural approach.

Ernest (1998, 2010) demonstrated the structural deepening of mathematics learning within an individual from the perspective of social constructivism based on sociocultural perspective. Ernest (2010) showed that the key to facilitating mathematics learning was the publication of individual sign use and appropriation. The concept of appropriation took shape from Bakhtin's linguistic philosophy and Vygotsky's psychology, and it plays an important role in the study of learning from a sociocultural perspective. Appropriation is defined as a "process that has as its end result the individual's reproduction of historically formed human properties, capacities, and modes of behavior" (Leontyev, 1981, p. 422). Following the above studies, Matsushima (2020, 2021) connected individual thinking with learning communities from the perspective of appropriation to present the structure of deepening mathematical learning through dialogue as shown in Fig. 10.1. The two main features of appropriation are given below.

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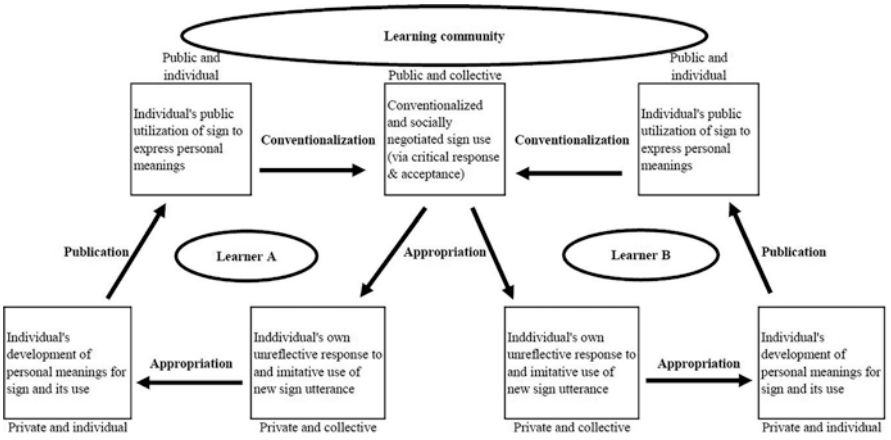

Fig. 10.1 Extended model of sign appropriation and use (Matsushima, 2020, p. 113)

Feature 1 (dynamic composition):

Gradually forming one's concept by speaking, while borrowing the concept of others.

Feature 2 (mutual composition):

The concept of learning community is formed in the process of forming the concept of self. (Matsushima, 2021)

As the above two appropriation features indicate, appropriation is dynamically composed of the concept of learner and learning community and mutually composed of both by bidirectionally influencing the learner's concept and learning community's concept. In other words, appropriation facilitates the concepts of individual learners and learning community to develop interactively (Brown et al., 1993). Conceptual development here refers to the ability to use language appropriately. In mathematics learning, being able to use mathematical words and signs appropriately indicates a deep understanding of mathematics.

> However, only a few studies have so far examined the concept of appropriation, with the concept itself appearing confusing. For example, few bidirectional discussions on appropriation can be found in the literature, with many studies examining the concept within each individual (e.g., Moschkovich, 2004; Solomon et al., 2021). Furthermore, it is difficult to distinguish between appropriation and similar concepts (e.g., Brown et al., 1993). One study points out the problem how to think about the effects of appropriation when the learner does not speak to others (Carlsen, 2010).

To solve these research problems, we need to first clarify the features of appropriation. Therefore, the purpose of this chapter is to reconsider the concept of appropriation and clarify its features.

10.2 Structure of This Chapter

In this chapter, we first review previous research on appropriation and point out three problems with the concept of internalization, which is closely related to the concept of appropriation. To show that the appropriation as a concept that overcomes the three problems of internalization, we first discuss how the learner's concept may be transformed by appropriation based on Figure 10.1. In the discussion, we proceed with the discussion separately for the case where the learner is the speaker of the dialogue and the case where the learner is the listener, and solve the first two problems of internalization. Then we will cite the discussion of intersubjectivity regarding the connections between individual learners and communities from knowledge of developmental psychology to overcome the third problem. Through these discussions, the concept of appropriation is clarified as a concept to overcome the three problems of internalization, and its six characteristics are pointed out. Finally, from the standpoint of a sociocultural approach, we will answer the following questions: "Why does dialogue deepen mathematics learning?" "Will mathematics learning deepen without dialogue?"

10.3 Problems Related to the Concept of Appropriation

As stated in the previous section, the concept of appropriation took shape from Bakhtin's philosophy of language and Vygotsky's psychology. Bakhtin (1981) explains the polyphonic nature of spoken language as follows:

> The word in language is half someone else's. It becomes "one's own" only when the speaker populates it with his own intention, his own accent, when he appropriates the word adapting it to his own semantic and expressive intention. (Bakhtin, 1981, p. 293)

This quotation outlines the concept of appropriation. People discuss a concept at the beginning using the concept of others and gradually form their concept based on it. Note the start point of concept formation here. The first point is borrowing the concept from others. Conventions such as concepts, ideas, and values of others are shared with the community even before we join the community. Individuals in the learning community assimilate conventions individually through appropriation, the start point of concept formation. Discussing and acting on these conventions also affect the concept formation of others and learning community. This bidirectional concept formation chain between individuals and the learning community transforms the conventions of individuals and the learning community. In other words, individuals and the learning community continue to form new conventions (Cazden, 2001; Rogoff, 2003). These features involve both dynamic and mutual composition (Matsushima, 2021). However, a new question arises here. How does one borrow the concept of others? Furthermore, are there any restrictions on borrowing the concept of others? The two features mentioned above do not answer these questions. This study therefore tries to examine whether these two features are valid and

answers the above new questions in terms of the two previous studies that clearly state the appropriation features.

The first study describes three characteristics of the appropriation process (Nunokawa & Kuwayama, 2003) based on a case study as follows:

> (a) In the process of appropriation, the student created kinds of hybrids between the old and new ideas.
>
> (b) When the new idea was presented by the others, the student attempted to interpret it in the framework of the old method he had used up to then.
>
> (c) There were long time-lags even before the student began to incorporate some aspects of the new idea into the method he had used up to then. (Nunokawa & Kuwayama, 2003, pp. 303–304)

Characteristic (a) is the same as the dynamic composition of an individual's appropriation but refers to the quality of the object to be composed. The object created through appropriation is not completely new but based on the previous object. New ideas are thus constrained by old ideas. Characteristics (b) and (c) need to be recombined from the old framework to formulate the new object to be created in the new framework, but this takes time. These points relate to appropriation and resistance (Wertsch, 1998). Appropriation is carried out on the basis of existing individual and learning community conventions, but these existing conventions historically and culturally constrain appropriation.

The second study examined the appropriation process (Carlsen, 2010) and clarified the following five appropriation processes in mathematics learning:

1. Be involved in joint activity.
2. Establish a shared focus of attention with others. Students have to develop some kind of working consensus of what to pay attention to in a mathematical task.
3. Develop shared meanings of words and concepts, i.e., meanings in accordance with the mathematics community through participating in joint decision-making processes.
4. Be involved in the activity of transforming, a process where the students appropriate actions and utterances by fellow students in the collaborative problem-solving context and use them in ongoing activities.
5. Attend to the problem of the relationship between sense and meaning by identifying the relations between pre-existing established mathematical knowledge in the classroom and students' joint activity in the small group. (Carlsen, 2010, p.99)

Like these five processes, from the standpoint of a sociocultural approach, a concept is a way of using signs. The use of signs for mathematics as concepts in dialogue with others is appropriated by the acceptance and criticism of the learning community. This is a process of collaborative creation based on existing concepts and experiences, involving sharing with the learning community. At the same time, individual learners themselves try to form a concept. This process is highly consistent with the extended model in Fig. 10.1 and can be considered to have features of both dynamic and mutual composition. In particular, note that descriptions that are conscious of the two-way concept formation can be found between the learner and other learners. Also, note the objectification (e.g., Radford, 2003; Roth & Radford, 2011) of the learning objects in this paper. Objectification in the learning community is "embedded in socio-psycho-semiotic meaning-making processes framed by cultural modes of knowing that encourage and legitimize particular forms of sign

and tool use" (Radford, 2003, p.44). Objectification is the process and result of creating a method for using signs as a concept, with focus on the connection between the learning community and individual learners, and it has much in common with appropriation. Carlsen (2010) often mentions objectification in his study. A comparison of these two concepts shows that appropriation is better to express the polyphony of Bakhtin (1981) and objectification is better to overcome dualism by emphasizing the connection with the concept of subjectification. However, the significance of these concepts will be a subject for future research. Moreover, objectification focuses on the reflection/refraction of individual learners in the process (Roth & Radford, 2011). It shows the importance of reflective thinking and the gap in thinking with the learning community when deepening individual thinking based on the connection with the learning community. The reflection/refraction viewpoint is also very important when deepening the concept of appropriation.

> From the previous two research models and a comparison of the two features of the dynamic and mutual composition, we see no reason to deny the two features. And additional factor found is the historical and cultural restrictions of the appropriation as existing conventions interfere with the appropriation. Note that reflective thinking and deviations occur when connecting with the thinking of the learning community.

So far, we considered appropriation based on Bakhtin's philosophy of language. Another source of appropriation is Vygotsky (1978), but this source uses the term internalization rather than appropriation for the internal reconstruction of external operations. Vygotsky's disciple Leont'ev replaced Piaget's notion of assimilation with appropriation (Leontyev, 1981). In assimilation, a learner takes information from the outside world in the framework of the learner's individual knowledge without change. This transformation from assimilation to appropriation can be due to the change in focus from biological ontogeny to a socio-historical perspective. Leont'ev (1974) also emphasized on activity and thought that the mediation of artifacts in activity would connect learners, objects, and others within the learning community. However, some researchers have pointed out that the concept of internalization has the following problems:

> 1: It easily leads to dualism between individual and social. This assumes that internalization occurs solely through personal influence (Wertsch, 1998).
>
> 2: It is misunderstood to be a concept of passively copying information from the outside world to an individual (Cazden, 2001).
>
> 3: The mechanism of internalization is not clear (Brown et al., 1993).

In view of these points, Wertsch (1998) classified internalization into two types, mastery and appropriation. Internalization as mastery allows the use of cultural signs as an intermediary, whereas internalization as appropriation is the process of taking something belonging to others and making it your own (Wertsch, 1998). However, this appropriation, which is as an elaboration of internalization in Wertsch (1998), lacks the viewpoint of mutual composition. As mentioned above, a confusion exists with regard to various other terms on appropriation because of the limited number of studies. In the next section, we show that appropriation can overcome the above- mentioned problems of internalization.

10.4 Appropriation as a Concept to Overcome the Problems of Internalization

In this section, we show that appropriation can overcome the three problems of internalization mentioned in the previous section. First, problems 1 and 2 can be overcome with the features of the dynamic and mutual composition. The sociocultural approach in psychology defines internalization as the reconstruction of an individual's knowledge through interaction with others (Vygotsky, 1978). This internalization was shown as the entire process of reconstructing the knowledge of an individual, triggered by interaction with others in the activity. Later, this was developed as the reconstruction of knowledge in social processes between individuals (Leont'ev, 1974). In both of these processes, the learner's existing knowledge and experience contribute to the reconstruction of new knowledge. To overcome problems 1 and 2 of internalization, we need to emphasize that appropriation is an active concept with an aspect of bidirectional concept formation between individuals and learning communities. Appropriation allows learners of all ages, expertise levels, and interests to return to the learning community the ideas and knowledge they have dedicated to their desires and the zones of proximal development of the learning that they are working on (Brown et al., 1993), with the individuals and learning communities influencing each other. That is, appropriation is a concept with dynamic and mutual composition. Therefore, it can be seen as a concept to overcome problems 1 and 2 of internalization.

Second, we examine whether appropriation can overcome problem 3 of internalization. Both dynamic and mutual composition only outline the mechanism of appropriation. Therefore, we refer to the extended model in Fig. 10.1 and examine the mechanism in detail. Consider the subject in Fig. 10.1. The learning community can be small groups of two to four people or have the size of a whole class. In a learning community of any size, multiple people continue to speak in turn. When learner *A* speaks, *A* is the only one speaking, with the others listening. Next, learner *B* speaks, representing the learning community responding to *A*'s utterance. *B*'s utterance represents that of the learning community, but it corresponds to *A*'s utterance. If we consider a certain utterance as the starting point of a dialogue, we need to note that the individual speakers change one after another, but the moment of dialogue is an individual-to-individual dialogue. Dialogue in a learning community can be considered the accumulation of individual-to-individual dialogue in the learning community. Therefore, basically two learners form the structure of the social interaction of dialogue in the learning community. However, Fig. 10.1 shows the model of three learners as an ellipse. Of the three learners, two are real learners and the third is the learning community as a virtual learner. If we consider the learning community as a virtual learner with some information about the learning target, the model in Fig. 10.1 illustrates a three-party dialogue model. Although we need not increase the elements of consideration when examining the learner's concept formation, we need to include the learning community as an element of consideration from sociocultural perspective, because, from the standpoint of it, learners

need to be included in the history and culture of the learning community, implicitly restricting appropriation. Furthermore, the existence of appropriation may become clear when the constraints of the learning community are also considered. This is because the difference in concepts that learners *A* and *B* have appropriated and the difference in concepts that the learning community has as seen by learners *A* and *B*, are clarified. These deviations are unique to the learners because they are constrained by the existing conventions and experiences of learners *A* and *B* (Newman et al., 1989). This difference in concepts of the learning community from the perspectives of learners *A* and *B*, or the difference in the concept that they appropriate, will be useful to interpret the process of deepening mathematics learning and develop lesson designs that would be easy for children to make sense.

Next, we consider the specific appropriation process shown in Fig. 10.1. This is to show how the process of appropriation differs between the appropriation of the speaker and the listener. Here, we use a simplified symbol to clarify the process of concept formation. Let $A(x)$ show that learner *A* is in the state of concept x about the learning object and $B(\alpha)$ show that learner *B* is in the state of concept α about the learning object. The state of concept x shared by the learning community from the perspective of learner *A* is shown as $C_A(x)$, and the partial transformation of the state of learner *A*'s concept x into state x_1 is shown as $A(x_1)$. Here, we assume that learner *A* begins to talk about concept x and learner *B* is just listening.

First, learner *A* (speaker) publishes his/her sign use with regard to concept x for the first time in the public/individual domain. This is publication 1. This utterance gives the learning community's consent 1 or criticism 1 in the public/collective domain. Learner *A*'s first appropriation in response to consent 1 and criticism 1 occurs in the private/collective domain and the private/individual domain. This is appropriation 1. Following appropriation 1, the cycle proceeds to new publication 2. Here, we need to note the content of consent 1 or critique 1. If the learning community agrees to the publication of learner *A*, learner *A*'s concept remains $A(x)$ and the appropriation dynamic composition does not work. Then, the learning community from the viewpoint of learner *A* becomes $C_A(x)$ owing to the appropriation's mutual composition. However, when the use of learner *A*'s sign is criticized, learner *A* would transform the concept into a partially transformed version x_1 or completely different version y, that is, $A(x_1)$ or $A(y)$. This is a transformation in concept due to the appropriation dynamic composition. The concept of learning community from the viewpoint of learner *A* owing to mutual composition is $C_A(x_1)$ or $C_A(y)$. Table 10.1 shows learner *A*'s appropriation 1 process as a speaker. The transformation of learner *A*'s concepts in this way indicates the transformation of learner *A*'s method of using signs.

Table 10.1 Learner *A*'s appropriation 1 process as a speaker

Initial state of concept	Consent or criticism from the learning community	Appropriation 1	Learner *A* concept	Learner *A*'s concept of learning community
$A(x)$	Consent 1	$A(x)$	$A(x)$	$C_A(x)$
	Criticism 1	$A(x_1)$ or $A(y)$	$A(x_1)$ or $A(y)$	$C_A(x_1)$ or $C_A(y)$

Next, we consider learner B's appropriation 1 process as a dialogue listener. Let $B(\alpha)$ be the initial state of learner B's concept as a listener. Publication of $A(x)$ is not the same from the perspective of learner B and A because it is subject to historical and cultural restrictions based on learner B's existing conventions and experiences, interpreted as a partially transformed x_2. This is expressed as $A_B(x_2)$. This $A_B(x_2)$ indicates the state of learner A's concept as interpreted by learner B. To analyze learner B's appropriation process as a listener, we need to classify appropriation situations. First, we have three cases: the learning community agrees with, partially denies, and completely denies $A_B(x_2)$. At the same time, after obtaining consent, partial negation, and complete denial of the learning community with regard to $C_B(x_2)$, learner B needs to agree with his/her own concept α, partially deny it, or completely deny it, representing the three cases. Therefore, this will be divided into nine cases, that is, 3 × 3. The second is the classification based on the relationship between the concept x_2 and the concept α. Here, we have four cases: concept x_2 and concept α are independent, they have partial intersection, concept α contains concept x_2, and concept x_2 contains concept α. Depending on these combinations, the classification results in 9 × 4 = 36 cases. Strict case classification requires more detailed case classification, for example, whether the partial negation part is at the intersection of the two concepts, to result in 36 or more cases. However, the purpose of making this table is to show how appropriation modifies the original concept, rather than analyze its detailed processes. The paper therefore discusses appropriation considering only a part, that is, 24 of the basic 36 cases. Table 10.2 shows the process analysis of learner B's appropriation 1 as a listener using the above simplified symbols and case classification.

The symbols in Table 10.2 are described in a supplementary explanation. In column No. 1 appropriation 1, "$A_B(x_2)$, $B(\alpha)$" indicates that the state of concept of learner B includes two kinds of concepts, $A_B(x_2)$ and $B(\alpha)$. In column No. 2, "$A_B(x_2)$ + $B(\alpha)$" indicates that learner B appropriates the concept combining the two concepts of $A_B(x_2)$ and $B(\alpha)$. Thus, column No. 2 shows that the state of concept of learner B transforms into the $B(\alpha_1)$, which is an extension of $B(\alpha)$. In column No. 3, "$A_B(x_2) \supset B(\alpha)$" indicates that concept x_2 contains concept α. In column No. 6, the partially denied part of learner B's concept α is expressed as concept $B_{\text{sub}}(\alpha)$. The concept excluding the partially denied $B_{\text{sub}}(\alpha)$ from $B(\alpha)$ is "$B(\alpha) - B_{\text{sub}}(\alpha)$". Learner B then appropriates the idea of adding the concept to $A_B(x_2)$ and shows that the concept of learner B has been transformed into $B(\alpha_3)$. In column No. 9 appropriation 1, "$\neg B(\alpha)$" indicates the complete denial of concept α.

From Table 10.1, concept $A(x)$ of learner A may be of three types, $A(x)$, $A(x_1)$, and $A(y)$, through an appropriation. Moreover, from the perspective of learner A, the concept of the learning community may be of three types, $C_A(x)$, $C_A(x_1)$, and $C_A(y)$, through an appropriation.

In Table 10.2, concept $B(\alpha)$ of learner B may become $B(\alpha)$, $B(\alpha_1)$, $B(\alpha_2)$, $B(\alpha_3)$, $B(\alpha_4)$, $B(\alpha_5)$, $B(\alpha_6)$, $B(\alpha_7)$, $B(x_2)$, $B(x_3)$, and $B(x_4)$ through an appropriation. The concept of the learning community can also be of two types, $C_B(x_2)$ and C_B (x_3), from the perspective of learner B. Table 10.2 is an extract of a part of learner B's appropriation 1 process. Therefore, at least 11 types of concept transformations and two

Table 10.2 Learner B's appropriation process as a listener (partial excerpt)

No.	State of the concept of $A_B(x_2)$ and (α)	Relationship between the concepts of $A_B(x_2)$ and $B(\alpha)$	Appropriation 1	Learner B's concept	Learner B's concept of learning community
1	$A_B(x_2)$: Agree $B(\alpha)$: Agree	Independence	$A_B(x_2)$, $B(\alpha)$	$B(x_2)$, $B(\alpha)$	$C_B(x_2)$
2		Have a partial intersection	$A_B(x_2) + B(\alpha)$	$B(\alpha_1)$	$C_B(x_2)$
3		$A_B(x_2)$ contains $B(\alpha)$	$A_B(x_2) \supset B(\alpha)$	$B(x_2)$	$C_B(x_2)$
4		$B(\alpha)$ contains $A_B(x_2)$	$B(\alpha) \supset A_B(x_2)$	$B(\alpha)$	$C_B(x_2)$
5	$A_B(x_2)$: agree $B(\alpha)$: Partial negation	Independence	$A_B(x_2)$, $B(\alpha) - B_{\text{sub}}(\alpha)$	$B(x_2)$, $B(\alpha_2)$	$C_B(x_2)$
6		Have a partial intersection	$A_B(x_2) + \{B(\alpha) - B_{\text{sub}}(\alpha)\}$	$B(\alpha_3)$	$C_B(x_2)$
7		$A_B(x_2)$ contains $B(\alpha)$	None	None	None
8		$B(\alpha)$ contains $A_B(x_2)$	$\{B(\alpha) - B_{\text{sub}}(\alpha)\} \supset A_B(x_2)$	$B(\alpha_4)$	$C_B(x_2)$
9	$A_B(x_2)$: agree $B(\alpha)$: Complete denial	Independence	$A_B(x_2)$, $\neg B(\alpha)$	$B(x_2)$	$C_B(x_2)$
10		Have a partial intersection	None	None	None
11		$A_B(x_2)$ contains $B(\alpha)$	None	None	None
12		$B(\alpha)$ contains $A_B(x_2)$	None	None	None
13	$A_B(x_2)$ Partial negation $B(\alpha)$: Agree	Independence	$A_B(x_2) - A_{B\text{sub}}(x_2)$, $B(\alpha)$	$B(x_3)$, $B(\alpha)$	$C_B(x_3)$
14		Have a partial intersection	$\{A_B(x_2) - A_{B\text{sub}}(x_2)\} + B(\alpha)$	$B(\alpha_5)$	$C_B(x_3)$
15		$A_B(x_2)$ contains $B(\alpha)$	$\{A_B(x_2) - A_{B\text{sub}}(x_2)\} \supset B(\alpha)$	$B(x_3)$	$C_B(x_3)$
16		$B(\alpha)$ contains $A_B(x_2)$	$B(\alpha) \supset \{A_B(x_2) - A_{B\text{sub}}(x_2)\}$	$B(\alpha)$	$C_B(x_3)$
17	$A_B(x_2)$: Partial negation $B(\alpha)$: Partial negation	Independence	$A_B(x_2) - A_{B\text{sub}}(x_2)$, $B(\alpha) - B_{\text{sub}}(\alpha)$	$B(x_3)$, $B(\alpha_6)$	$C_{B(x3)}$
18		Have a partial intersection	$\{A_B(x_2) - A_{B\text{sub}}(x_2)\} + \{B(\alpha) - B_{\text{sub}}(\alpha)\}$	$B(\alpha_7)$	$C_B(x_3)$
19		$A_B(x_2)$ contains $B(\alpha)$	$\{A_B(x_2) - A_{B\text{sub}}(x_2)\} \supset B(\alpha)$	$B(x_3)$	$C_B(x_3)$
20		$B(\alpha)$ contains $A_B(x_2)$	$\{B(\alpha) - B_{\text{sub}}(\alpha)\} \supset \{A_B(x_2) - A_{B\text{sub}}(x_2)\}$	$B(\alpha_6)$	$C_B(x_3)$
21	$A_B(x_2)$: Partial negation $B(\alpha)$: Complete denial	Independence	$A_B(x_2) - A_{B\text{sub}}(x_2)$, $\neg B(\alpha)$	$B(x_3)$	$C_B(x_3)$
22		Have a partial intersection	$\{A_B(x_2) - A_{B\text{sub}}(x_2)\} - B(\alpha)$	$B(x_4)$	$C_B(x_3)$
23		$A_B(x_2)$ contains $B(\alpha)$	$\{A_B(x_2) - A_{B\text{sub}}(x_2)\} - B(\alpha)$	$B(x_4)$	$C_B(x_3)$
24		$B(\alpha)$ contains $A_B(x_2)$	None	None	None

types of learning community concepts may occur in an appropriation from the listener learner's perspective. What is important here is that the appropriation as both speaker and listener changes the concept of both the learner and learning community through the appropriation process. The learner and learning community are thus connected.

In addition, note the difference between the speaker's and the listener's appropriation variations. When the learning community agrees on concept x of speaker A, the concept of the speaker becomes $A(x)$, the concept of the learning community from the speaker A's perspective becomes $C_A(x)$, and the concept of the learning community from the listener B's perspective becomes $C_B(x_2)$. The three concepts are fixed but are not exactly equal. However, if $A_B(x_2)$ is agreed upon when the concept of listener B is α, the concept of listener B may become one of six types, $B(x_2)$, $B(\alpha)$, $B(\alpha_1)$, $B(\alpha_2)$, $B(\alpha_3)$, and $B(\alpha_4)$, and not exactly match $A(x)$, $C_A(x)$, and $C_B(x_2)$. In particular, when the listener B's concept is not $B(x_2)$, the variation of difference between learner B and others has many possibilities. This difference is present in almost all cases, whether or not the speaker's concept is partially or completely denied. Even if the speaker and listener participate in the same dialogue, their appropriations may differ. This process is likely to lead to a different concept for the listener rather than speaker. Even in case of slight difference between the two concepts due to appropriation, if the listener only listens to the dialogue continues without speaking, the difference in concept with others may widen as the dialogue progresses.

Whether the learner is a speaker or listener, the appropriation process described above transforms the concept of the individual by triggering the publication of the concept, and the concept of the learning community also transforms accordingly. Thus, appropriation can completely solve internalization problems 1 and 2. The appropriation process analyzed so far reveals a certain degree of the process. However, it is difficult to say that problem 3 internalization, that is, how to know the thoughts and intentions of others, has been clarified. In the next section, we consider the process of appropriation based on intersubjectivity and try to solve internalization problem 3.

10.5 Relationship Between Appropriation Process and Intersubjectivity

How can people know the thoughts and intentions of others? This is an issue at the starting point of appropriation also. We consider this from the perspective of intersubjectivity. Lerman (1996) argues that intersubjectivity has three aspects: aspects that become a subject through social practice, aspects of cognition contextualized in practice, and aspects of mathematics as cultural knowledge (Lerman, 1996, pp.142–147). Understanding others through intersubjectivity in practice leads to self-construction. Historical and cultural restrictions affect the connection and these restrictions are applied in mathematics learning. In other words, intersubjectivity

refers to the ability to understand the thoughts of others in practice. Intersubjectivity in mathematics learning is the starting point of mathematics practice for the learning community. In the learning community, self and others are not separate but connected in practice (Roth & Radford, 2011).

How is intersubjectivity possible while understanding the thoughts of others? Steffe and Thompson (2000) argue that intersubjectivity can be built through dialogue and interaction with others. However, Lerman (2000, 2001) shows that intersubjectivity occurs in an individual's mind before the actual interaction takes place with others. The timing for intersubjectivity to occur could differ.

Let us examine this difference from the perspective of developmental psychology. In developmental psychology, intersubjectivity is defined generally as a "process in which mental activity - including motives and emotions - is transferred between minds" (Legerstee, 2009, p. 3). This concept of intersubjectivity is an important factor in the development of the theory of mind and is divided into two types, primary intersubjectivity and secondary intersubjectivity, for discussion.

Primary intersubjectivity is the "innate or early-developing sensory-motor capacities that bring us into relation with others and allow us to interact with them" (Gallagher, 2013, p.60). When a two- or three-month-old baby tries to convey his/her subjective mood through certain expression, the caregiver, for example, the mother, generates expressions that follow the baby's expressions. Infants have then been reported to pay attention to their caregiver's expressions and imitate them (Trevarthen, 1979). The caregiver's expression is also a specialized form tailored to the baby and complements the baby's expressions (Trevarthen, 1979). This period of primary intersubjectivity is the binary relational period when the interaction between the baby-other and baby-objects becomes conspicuous (Legerstee, 2005).

Secondary intersubjectivity develops around 9 to 12 months after birth, connecting infants- objects-others and helping them gain new awareness (Trevarthen & Hubley, 1978). It is based on primary intersubjectivity. For example, assume that an infant is playing with blocks in front of his mother. If he happens to pile up the blocks well, his mother will exaggerate the act, and smile for him. His attention will then shift from the building blocks to his mother, intuitively noticing his mother's joy and praise, and again shifting his attention to the building blocks in an uplifting mood. This shift of infant-objects-mother's attention and the accompanying intuitive understanding of intention can help the infant gradually become aware of the triad relationship with self, objects, and others. "In secondary intersubjectivity, interaction is shaped by joint attention and the surrounding environment" (Gallagher, 2013, p.64).

In developmental psychology discussions of intersubjectivity, human beings are considered to have the following two abilities: the ability to intuitively see the intentions of others within the binary relationship of baby-caregiver and baby-object through innate or early-developing primary intersubjectivity, and the ability to intuitively know the intentions of others and be aware of the objects in the interaction of the triad relationship with others regarding the objects around the first year of life. This discussion of intersubjectivity in developmental psychology clarifies that intersubjectivity should not be regarded as consent through children's dialogue. As

intersubjectivity occurs in infants without language speech, it is the ability to intuitively notice the intentions of others even when interacting with others without language. Thus, intersubjectivity should not be viewed as taken-as-shared (Cobb, 1999) through interaction in the learning community, including language and reasoning (Lerman, 2000). We will next consider the relationship between appropriation and intersubjectivity.

According to the extended model in Fig. 10.1, sign use is conventionalized in the public/collective domain and can be appropriated in the private/collective domain. Appropriation will be further advanced in the private/individual domain. The conventionalized sign is used in the private/collective domain as a mere imitation without reflection. In other words, it is the stage where the learner intuitively grasps the meaning of signs as conventionalized by the learning community and begins to use them in the same way. This is the intersubjective understanding of the sign use conventionalized in the learning community. In developmental psychology, intuitive awareness without language use is called intersubjectivity, but in normal mathematical learning, learning progresses through language. Therefore, the intersubjectivity of the learner at school can be reconsidered as intuitively becoming aware of the meaning of the learning object in the interaction using language. In the private/individual domain, the meaning captured intersubjectively is reconstructed through reflective thinking with connecting learners' own knowledge and experiences, and the method of sign use is reconsidered. Historical and cultural constraints can influence reflective thinking. The human-specific ability of intersubjectivity allows us to understand the intentions of others and deepen the meaning of the individual based on them. This is in line with the sociocultural approach principle of prioritizing the social aspects of the development of meaning (Vygotsky, 1978).

So far, we discussed appropriation as a concept to overcome internalization problem 3. Thus, appropriation was shown to involve two processes: intersubjectivity and reflective thinking. Correspondingly in Figure 10.1, it can be said that the first appropriation from the public/collective domain to the private/collective domain is made mainly by intersubjectivity, and the second appropriation to the private/individual domain is made mainly by reflective thinking. As the appropriation process has been clarified to some extent, the problem of internalization can be overcome. Then, by clarifying the relationship between appropriation and intersubjectivity, a connection can be created between individual learners and the learning community, as in participatory appropriation (Rogoff, 1995). This connection will allow us to continue to create new meanings dynamically and mutually.

10.6 Two Meanings of Deviation in Appropriation

In this section, we focus on the deviation in appropriation and discuss it from the perspective of the weaknesses and strengths of appropriation. First, we discuss deviation as a weakness. From the discussion so far, it is clear that the concept of speaker and community may easily deviate from the concept of listener in the process of appropriation. This can be observed from Tables 10.1 and 10.2 too. These

deviations may become even greater as the dialogue progresses and could be a major problem.

Here, we refer to the extended model in Fig. 10.1. A learner who does not speak but only listens to a dialogue does not pass through the public/individual domain in Fig. 10.1. However, appropriation begins after passing through the public/collective domain. Thus, learners can appropriate and enrich their own concepts to some extent by just listening to the utterances of others. After appropriation begins, the focus changes to whether to pass through the private/individual domain before listening to the next utterance of another person. One can go through the private/individual domain to the public/collective domain or to go to the public/collective domain without passing through that domain. In Fig. 10.2, the former learner follows the path of the "dashed line a." After appropriation begins, this process is as follows: reflect on the existing conventions and experiences, advance the appropriation, transform one's own concept, and listen to the next utterance. Clearly, this listener is thinking reflectively while listening to the utterance of others. A listener who self- regulates his/her own concept while listening to others' stories using reflective thinking is called an active listener (Kosko, 2014). An active listener fulfills the feature of dynamic composition. However, we cannot find a process that leads to the learning community from individual thoughts because this learner does not disclose his/her thoughts to others. In Fig. 10.2, no arrow connects the private/individual domain to the public/individual domain. Thus, the learning community and learner are connected not in both directions, but in only one direction, that is, from the public/collective domain. In this situation, mutual composition does not work well. In addition, because learners do not disclose their thoughts, the gap in appropriation remains. As the dialogue progresses, the gap is likely to increase. Active listeners who only listen to dialogues can deepen their learning; on the other hand, if they have differences with speakers, they are unlikely to be able to reduce differences. Further, they cannot contribute to concept formation of the learning community. This is a problem for learners who are active listeners.

Next, we consider the latter learner. This learner listens to the thoughts of others, begins to appropriate them without reflective thinking, listens to the next utterance of others, and re- appropriates them without reflective thinking. This is a cycle through the "dashed line b" in Fig. 10.2. Such learners often focus solely on problem solving and the superficial methods of others. For example, they may obtain the answer to a problem from another person and feel relieved if the answer is the same, or may ask another person how to solve the problem and follow the solution without inquiring further. If the answer is different or they have a question about how to solve the problem, reflective thinking intervenes to find a reason. In this case, the learner becomes a learner as active listener and hence becomes the former learner. The learner who does not think reflectively has little effect on the dynamic composition of the concept. Moreover, because the appropriation here is not bidirectionally connected to the learning community, it does not have a mutual composition. In other words, appropriation of learners who do not work reflective thinking just by listening to the thoughts of others becomes very superficial. This is the problem of the latter learner.

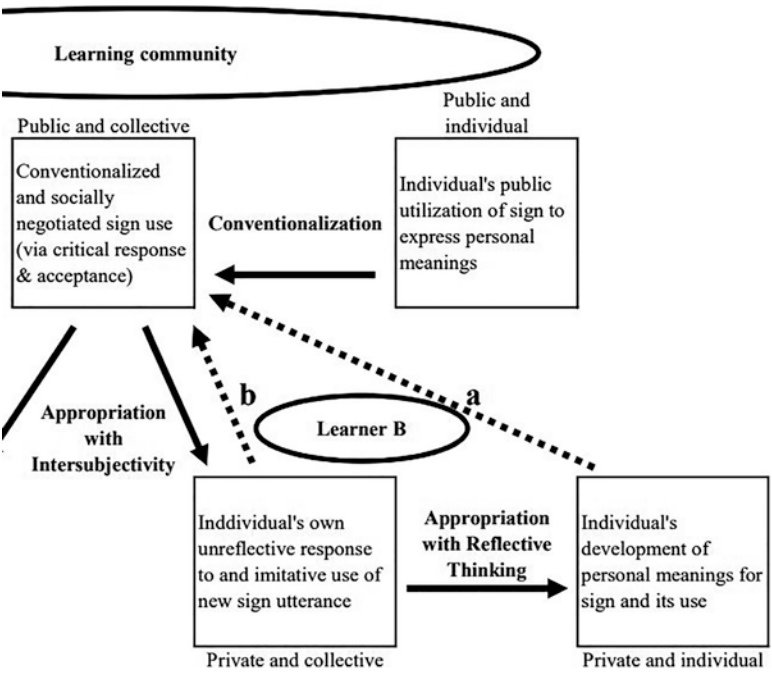

Fig. 10.2 An extended model for the learner as a non-speaking listener

Thus, we highlight the features of appropriation once again. Appropriation is the inner working where the thoughts of individuals who are connected from the public/collective domain to the private/individual domain via the private/collective domain work together. Sufficient appropriation requires two functions: intersubjectivity and reflective thinking.

So far, we considered two problems with appropriation deviation. However, the appropriation should be in the appropriate direction of the aim of learning. How can we ensure that our appropriation is proper? The answers to this question are presented in Tables 10.1 and 10.2. From a comparison of the number of concepts in Tables 10.1 and 10.2, Table 10.1 is extremely small. This indicates that if the appropriation is repeated for the speaker of the dialogue, the probability of approaching the proper use of sign is high. A more proper sign use can be achieved by repeating the appropriation only by listening, but the probability of achievement would be low because there are many types of concepts that can occur after appropriation. Repeating the appropriation as speaker of the dialogue may enhance its appropriateness. If we are asked, "Why does dialogue deepen mathematics learning?," we can answer this from the standpoint of a sociocultural approach: "If you repeat your appropriation as a speaker of dialogue, you can use the concept of mathematics appropriately because the appropriateness of appropriation increases." However, if we are asked, "Will mathematics learning deepen without dialogue?," our answer

could be, “It’s not that it does not deepen, but it’s less likely that the direction of deepening is appropriate than if you were the speaker of the dialogue.” The appropriation can be made more proper by becoming the speaker of the dialogue.

Second, we consider the deviation of appropriation as a feature. The appropriation process is referred to as a “quite general process that can account for the emergent creativity of social interactions and the growth of flexible expertise in learners” (Newman et al., 1989, p.143). The process of becoming an adaptive proficient can be explained by the chain of appropriation and the reduction in its gap by becoming the speaker of the dialogue. In this study, we focus on its creativity. Where is creativity related to the appropriation process? This is its own deviation. Table 10.2 lists some of the possibilities of a wide variety of deviations. This includes the gap between the concepts of the learners interacting with each other and between the concepts of the learning community as seen by each learner. The cause of these deviations lies in the learners’ historical and cultural constraints. By interacting according to these deviations, each learner may misunderstand what he/she is talking about. Simultaneously, the learner may create a new concept not included in the speaker’s concept. In other words, the deviation of appropriation can be the source of creativity that leads to new ideas that the speaker did not intend. Fig. 10.2 shows the possibility of creating a new concept based on the deviation from the original concept. Therefore, deviation in appropriation can be a good feature.

10.7 Reconsideration of Appropriation Features

As mentioned earlier, appropriation is a “process that has as its end result the individual’s reproduction of historically formed human properties, capacities, and modes of behavior” (Leontyev, 1981, p.422).” In this definition, the word “historical” is supposed to express the historical and cultural constraints and mutual composition. However, the phrase “as its end result” obscures the dynamic composition. In addition, in this study, we reconsidered the process of appropriation based on two features, dynamic composition and mutual composition, and the extended model in Fig. 10.1, which roughly depicts the process of appropriation. The following characteristics have been pointed out in this research so far: the possibility of conceptual deviation between the speaker, listener, and learning community; intersubjectivity and reflective thinking in appropriation; appropriateness of appropriation; and creativity of appropriation. Therefore, the features of appropriation are summarized in the following six features.

Feature 1 (dynamic composition):Gradually forming one’s concept by speaking, while borrowing the concept of others.

Feature 2 (mutual composition):The concept of learning community is formed in the process of forming the concept of self.

Feature 3 (constraints and deviations):

Deviations occur in individual concept generation owing to historical and cultural restrictions of the learning community.

Feature 4 (intersubjectivity and reflective thinking):
Appropriation is begun with the awareness of others' intentions from intersubjectivity and is transformed one's concept through own reflective thinking.
Feature 5 (appropriateness):
Becoming a speaker in dialogue enhances the appropriateness of appropriation.
Feature 6 (creativity):Deviation in appropriation creates new ideas.

10.8 Future Research

In this study, we reconsidered the concept of appropriation from a sociocultural perspective. We thus clarified the process of appropriation and presented its six features. These features will be useful when analyzing the process of appropriation. In addition, because many studies in the literature are related to the formation of individual concepts, a future research topic could be to compare and consider the process of appropriation of the learning community and individual learners. This comparative study will allow the formulation of a lesson design that would be easy for all children to understand. Objectification and subjectification are concepts of similar to appropriation that can be considered. Research on appropriation is an ongoing process.

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