

Chapter 2

Combining Nontraditional Response Variables with Acceleration Data for Experimental Modal Analysis



P. M. Vinze, R. J. Allemang, A. W. Phillips, and R. N. Coppelino

Abstract Accelerometer data is the most commonly used data for experimental modal analysis of structures. Together with measuring applied force, it provides the basis for FRF estimation and subsequent modal parameter estimation and validation. As discussed in the paper by Dr. Coppelino (Experimental modal analysis using non-traditional response variables. In: IMAC Proceedings, 2021), there are situations where test analysis cross orthogonality is difficult to determine on inaccessible key regions of a test article. In that chapter, it is contended that it is in theory possible to augment data from accelerometers with data from other sensor sources at these key regions that have a proportionality to acceleration or displacement. This is important as strain and pressure have been shown to be useful measurements for modal analysis (Zienkiewicz et al., *The finite element method: its basis and fundamentals*, 6th edn. Butterworth-Heinemann, Oxford, p 563–584, 2005; Kranjc et al., *J Sound Vib* 332:6968, 2013; Kranjc et al., *J Vib Control* 22(2):371–381, 2016; Dos Santos et al., Strain-based experimental modal analysis: new concepts and practical aspects. In: *Proceedings of ISMA. IEEE*, Piscataway, p 2263–2277, 2016; Dos Santos et al., An overview of experimental strain-based modal analysis methods. In: *Proceedings of the international conference on noise and vibration engineering (ISMA)*, Leuven, p 2453–2468, 2014). But they have not been used in augmentation with acceleration. Two specific examples discussed are fluid pressure and strain. Experimentally, this presents several problems. For example, in the most simple structures it is expected to have maximum acceleration at locations of 0 strain and vice versa. This makes it difficult to relate the modal information contained in acceleration variable to the strain variable at the location of maximum acceleration. Given that the FRF information will have to be uniform in units, this is another cause of concern when combining pressure, strain, and acceleration. Use of strain, pressure, and acceleration data all together for modal analysis purposes would reduce the need to place accelerometers in locations that are difficult to access. This chapter aims to present experimental results of strain and pressure FRF-based modal analysis on a rectangular steel plate and attempts to propose ways to combine these variables in the modal parameter estimation process.

Keywords Experimental modal analysis · Strain · Pressure · Augmentation · Non traditional variables

2.1 Introduction

In this research, two experiments were conducted on a rectangular steel place. The first experiment was an impact test with accelerometer and strain gauges as response measurements. The second experiment was another impact test with accelerometer, strain gauges, and microphones as the response measurements. Different ways of augmenting partial accelerometer data/results from partial accelerometer data were tried and MAC [2] values with full experimental data were checked. FRF synthesis from the modal vectors of the augmented data was also done and compared with measured acceleration FRF. Strain and pressure have been shown to be useful measurements for modal analysis [3–6] and [7]. But they have not been used in augmentation with acceleration. The aim of the MAC values and FRF synthesis comparisons was to show that augmenting accelerometer, pressure and strain data can give results that are close to results from accelerometer data.

P. M. Vinze (✉) · R. J. Allemang · A. W. Phillips

Department of Mechanical Engineering, College of Engineering and Sciences, University of Cincinnati, Cincinnati, OH, USA
e-mail: vinzepam@mail.uc.edu

R. N. Coppelino

Measurement Analysis Corporation, Torrance, CA, USA

In the paper by Coppolino [1], the case that was considered was that of a rod that is modeled as a MDOF mass spring system. One of the major changes from that model to a system like a rectangular steel plate is that the strain (for all the modes in the frequency range of interest) is no longer due to extension and compression but due to bending. Another difference that is important to point out is that strain cannot be modeled as the ratio of difference of displacement at two neighboring points of response measurements and the length. The reason this does not work is that the plate is a continuous system, and the experimental strain measured will be the local measurement at the given point. In this way if measurements are made on n points, there will be n strain measurements, whereas in the theoretical model there will be $n - 1$ strains. These two factors are a big deviation from the model discussed in the referred paper but the general idea that measurements like strained pressure should give similar modal results is what served as motivation to attempt this work

2.2 Experimental Setup

The rectangular plate is rested on four rubber ball supports 6–8 inches inside of the four corners that approximate a free-free condition. The setup for the second experiment is shown in Fig. 2.1. For the first experiment, the setup was without the microphones and microphone stands. The strain gauges are placed next to the accelerometers and aluminum cubes were placed very close to provide a surface for X direction (along the long edge) impact (along the long edge on the plate). Figure 2.2 shows an image of the sensor placement. The numbers and circles etched on the plate are 160 equally spaced impact locations, a subset (40) of which have been used.

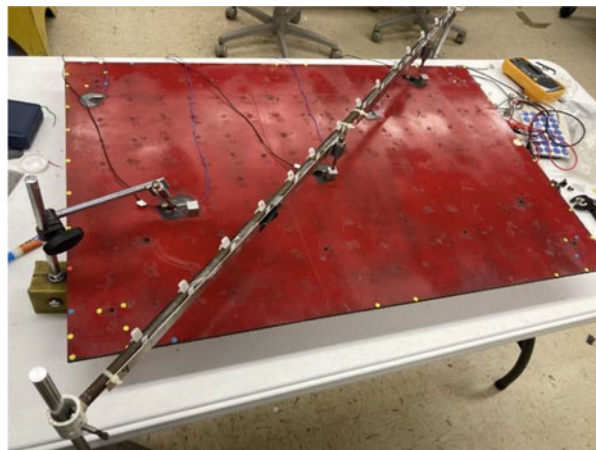


Fig. 2.1 Test setup

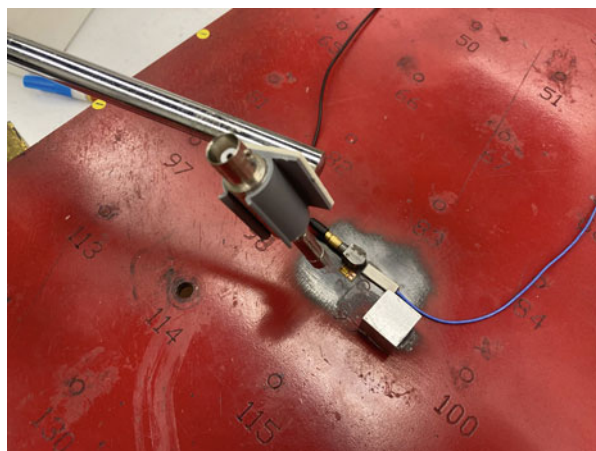


Fig. 2.2 Sensors on a plate

2.3 Analysis

A subset of the data gathered was used to get modal parameters. The idea was to work with several combinations of subsets to simulate a situation where different combinations of sensors on the structure could be worked with. This sieving of the full 9×40 FRF matrix was done in two ways:

1. Sieving by references only. This could be done before modal parameter estimation to create a new 3×40 FRF matrix that would contain one or two accelerometer reference points and the remaining reference points could be taken from the strain or pressure data for that reference points. When including pressure, it was important to scale the pressure in some way to a similar level of strain and acceleration.
2. Sieving by response as well as references. This means a subset of response points were chosen for acceleration references and another subset of response points for strain and in one case another subset of response points for pressure references. The choices are always made such that none of the 3 references or 40 response locations are completely missed out. Also some (3–5) response points are kept in common for all 3 reference locations. A composite modal vector is created by scaling the strain and pressure modal vectors based on the common response locations on the modal vectors and combining the scaled response at the locations that are not common between all three sensors.

These two methods were followed in different combination of selections and checked with a full set of accelerometer FRF-based results.

2.4 Sieving by References Only

The MACs were evaluated for strain and acoustic pressure-based modal vectors against the accelerometer-based modal vectors. The two MACs are shown in Figs. 2.3 and 2.4. This confirmed that pressure and strain data had the same modal information as acceleration data. Table 2.1 shows the modal frequencies and damping results for strain gauge and accelerometer-based modal analysis results and the percent difference between them. As can be seen the results agree expect for one damping result.

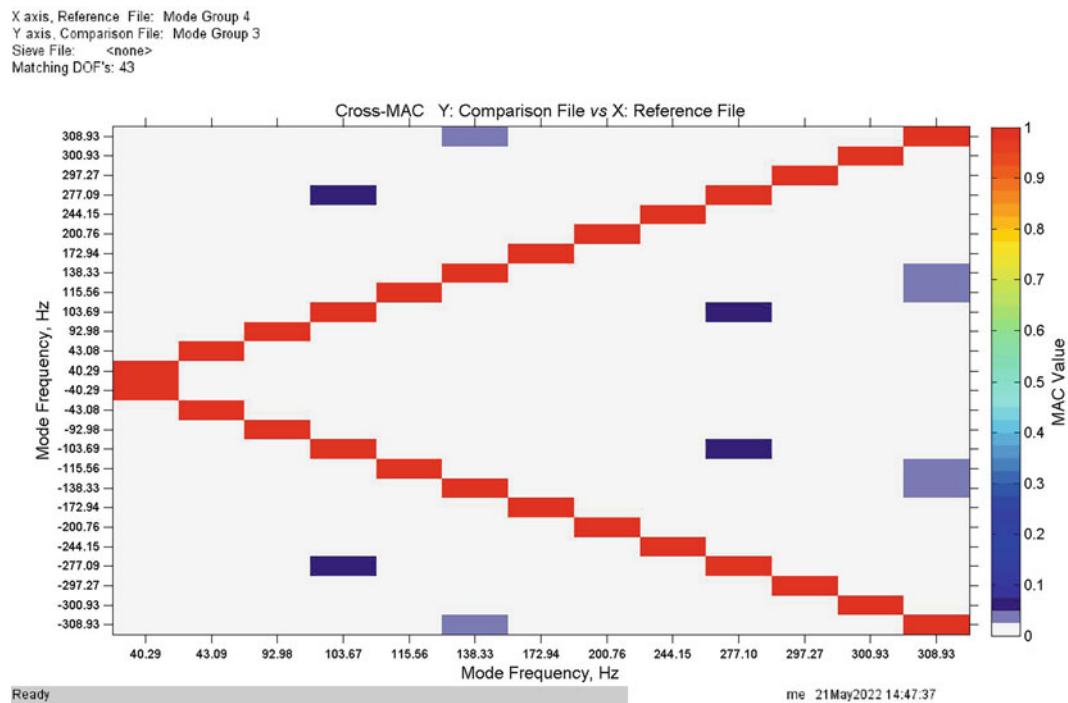


Fig. 2.3 Accelerometer-based versus strain-based modal vectors MACs

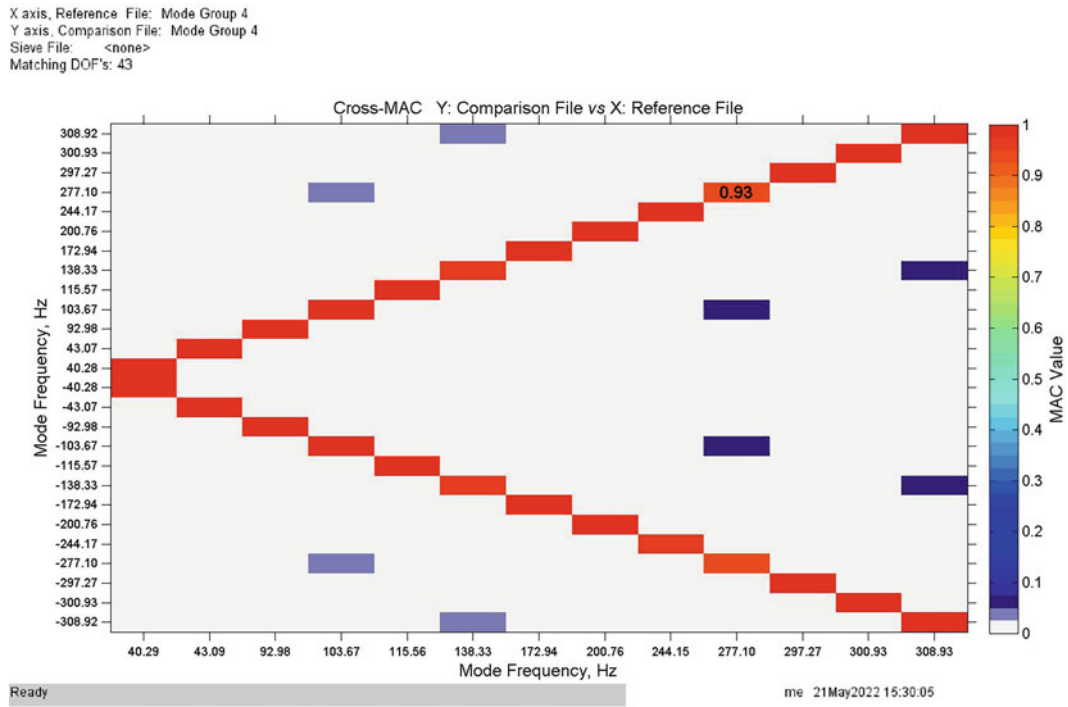


Fig. 2.4 Accelerometer-based versus microphone-based modal vectors MACs

Table 2.1 Accelerometer-based versus strain-based modal frequencies and damping results

S.No.	Strain frequency (Hz)	Strain damping (%)	Accelerometer frequency (Hz)	Accelerometer damping (%)	% diff frequency	% diff damping
1	40.2441	0.2488	40.255	0.2563	0.027077382	2.926258291
2	-40.2441	0.2488	-40.255	0.2563	0.027077382	2.926258291
3	43.0117	0.1258	43.0198	0.1401	0.018828539	10.206995
4	-43.0117	0.1258	-43.0198	0.1401	0.018828539	10.206995
5	92.931	0.0983	92.931	0.098	0	0.306122449
6	-92.931	0.0983	-92.931	0.098	0	0.306122449
7	103.679	0.029	103.6783	0.0284	0.000675165	2.112676056
8	-103.679	0.029	-103.6783	0.0284	0.000675165	2.112676056
9	115.5573	0.0274	115.5576	0.0275	0.000259611	0.363636364
10	-115.5573	0.0274	-115.5576	0.0275	0.000259611	0.363636364
11	138.3034	0.0331	138.2977	0.031	0.004121544	6.774193548
12	-138.3034	0.0331	-138.2977	0.031	0.004121544	6.774193548
13	172.9429	0.0206	172.9427	0.0199	0.000115645	3.51758794
14	-172.9429	0.0206	-172.9427	0.0199	0.000115645	3.51758794
15	200.731	0.0143	200.7539	0.0114	0.011407001	25.43859649
16	-200.731	0.0143	-200.7539	0.0114	0.011407001	25.43859649
17	244.1445	0.0264	244.1497	0.0246	0.002129841	7.317073171
18	-244.1445	0.0264	-244.1497	0.0246	0.002129841	7.317073171
19	277.0993	0.0257	277.0931	0.0265	0.002237515	3.018867925
20	-277.0933	0.0257	-277.0931	0.0265	7.21779E-05	3.018867925
21	297.2717	0.0353	297.2719	0.0351	6.72785E-05	0.56980057
22	-297.2717	0.0353	-297.2719	0.0351	6.72785E-05	0.56980057
23	300.9256	0.0148	300.9265	0.016	0.000299076	7.5
24	-300.9256	0.0148	-300.9265	0.016	0.000299076	7.5
25	308.9535	0.0356	308.9542	0.0351	0.000226571	1.424501425
26	-308.9535	0.0356	-308.9542	0.0351	0.000226571	1.424501425

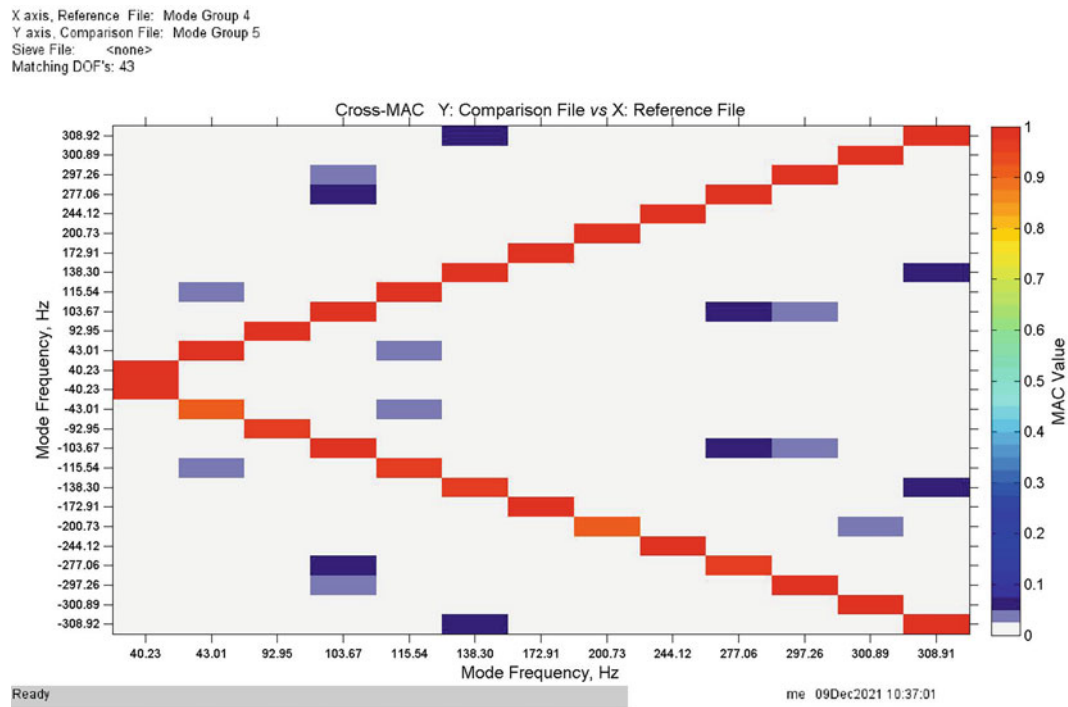


Fig. 2.5 MAC between 2 accelerometer and 1 strain versus 3 accelerometer

2.5 Accelerometers and One Strain Gauge

Modal vectors for data with two acceleration and one strain reference were compared with three acceleration modal vectors. The MAC comes out completely diagonal indicating that the modes are similar. MAC is shown in Fig. 2.5.

2.6 Accelerometer and Two Strain Gauges

Modal vectors for data with one acceleration and two strain references were compared with three acceleration modal vectors. The MAC comes out completely diagonal indicating that the modes are similar. MAC is shown in Figs. 2.6 and 2.7 for the two different acceleration references.

Another comparison that was done was to compare the density of pole estimates. There were differences in the density of the scatter plots, but it was different for different modes. There was not one set of data that gave better clusters of damping ratio values for all modes. This suggested that there was not any improvement in damping results from replacing accelerometers with strain gauges at some measurement locations.

2.7 Accelerometer and Microphones

To combine accelerometer data with microphone data, it was important to consider the fact that there was a large difference in their magnitudes. The microphone data was observed to be around 6 orders of magnitude larger. An equation was set up such that modified pressure FRF data was expressed as

$$H_{pm} = R3 * H_p + R2/\omega^2 + R1$$

where H_{pm} is the modified pressure data, H_p is measured pressure FRF, and $R1$, $R2$ and $R3$ are the terms determined in the least square sense by comparing H_{pm} to the displacement FRF on the same location. The average of all $R3$ values for all

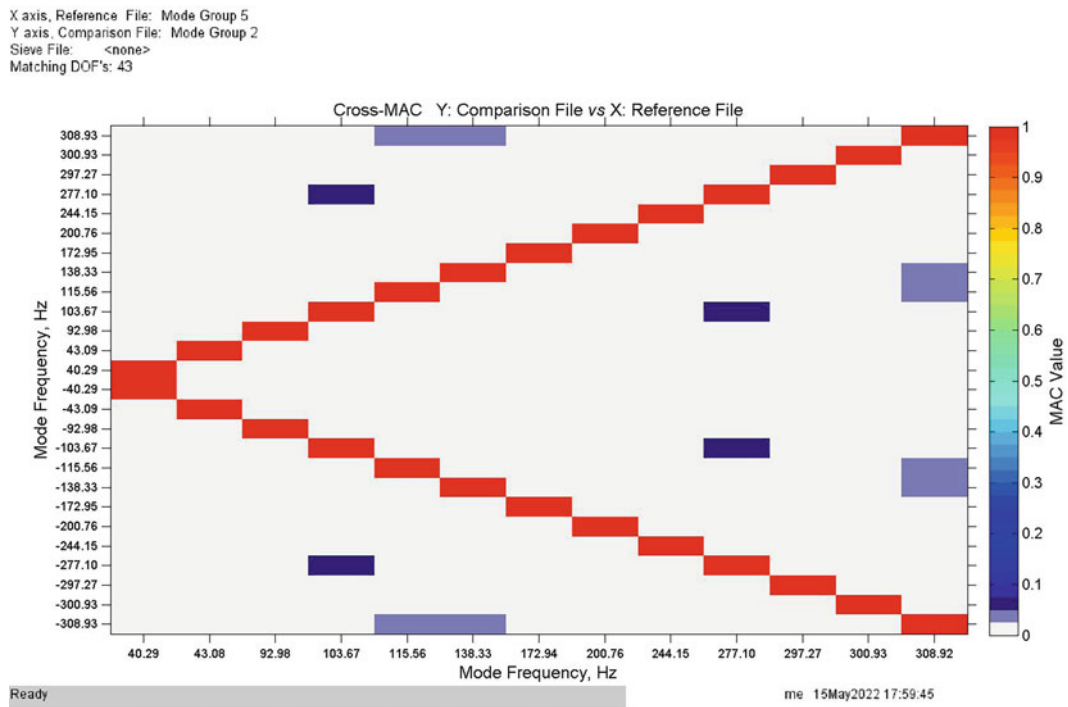


Fig. 2.6 MAC between 1 accelerometer (location14) and 2 strain versus 3 accelerometer

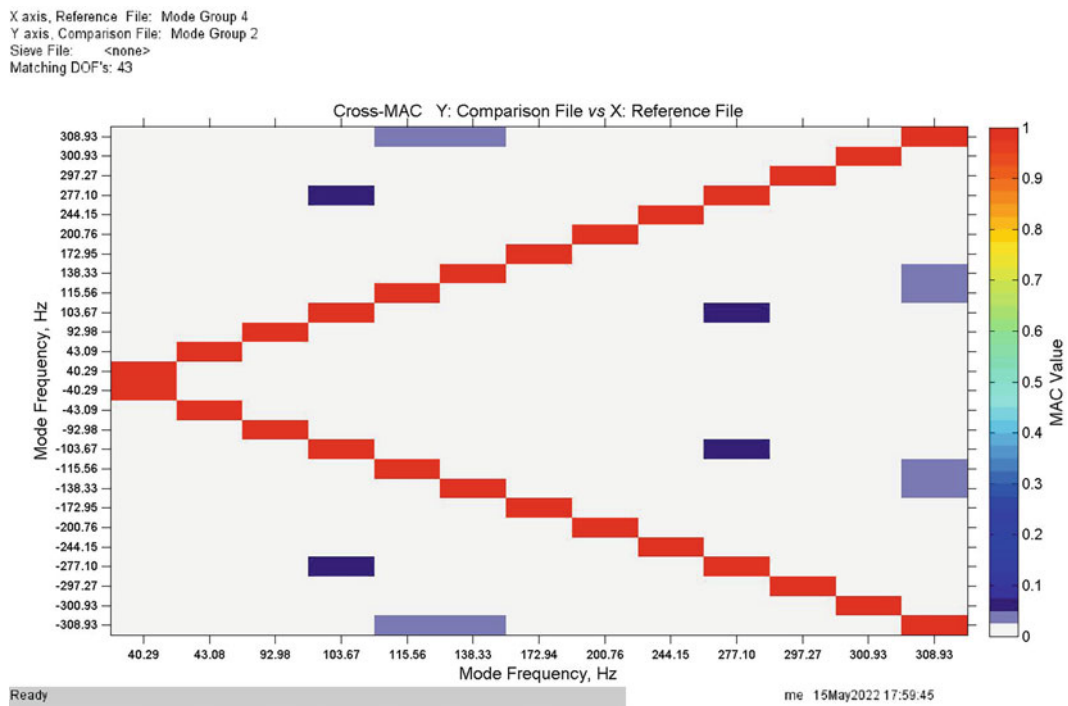


Fig. 2.7 MAC between 1 accelerometer (location99) and 2 strain versus 3 accelerometer

FRFs is calculated and used as the multiplier. The combining of pressure and displacement FRFs is only possible with the modified pressure because of the significant difference of scale of pressure and displacement data. Figures 2.8 and 2.9 show that the modal vectors obtained from combining pressure with acceleration measurements result in diagonal MAC values.

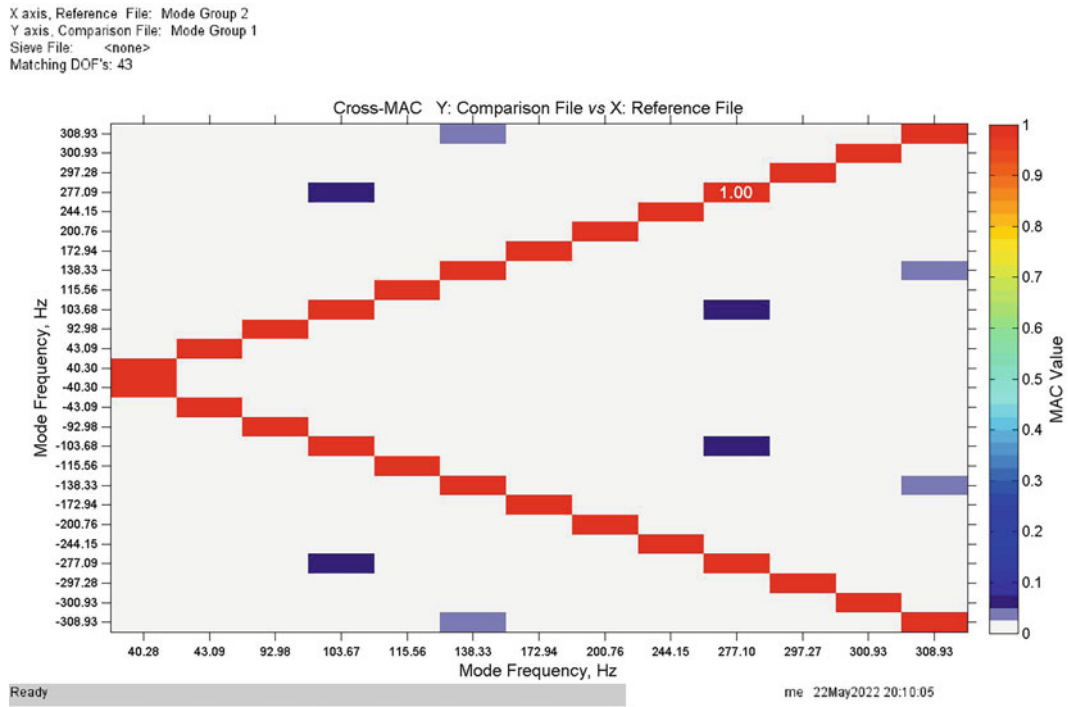


Fig. 2.8 MAC for 2 accelerometer 1 pressure modes versus 3 accelerometer modes

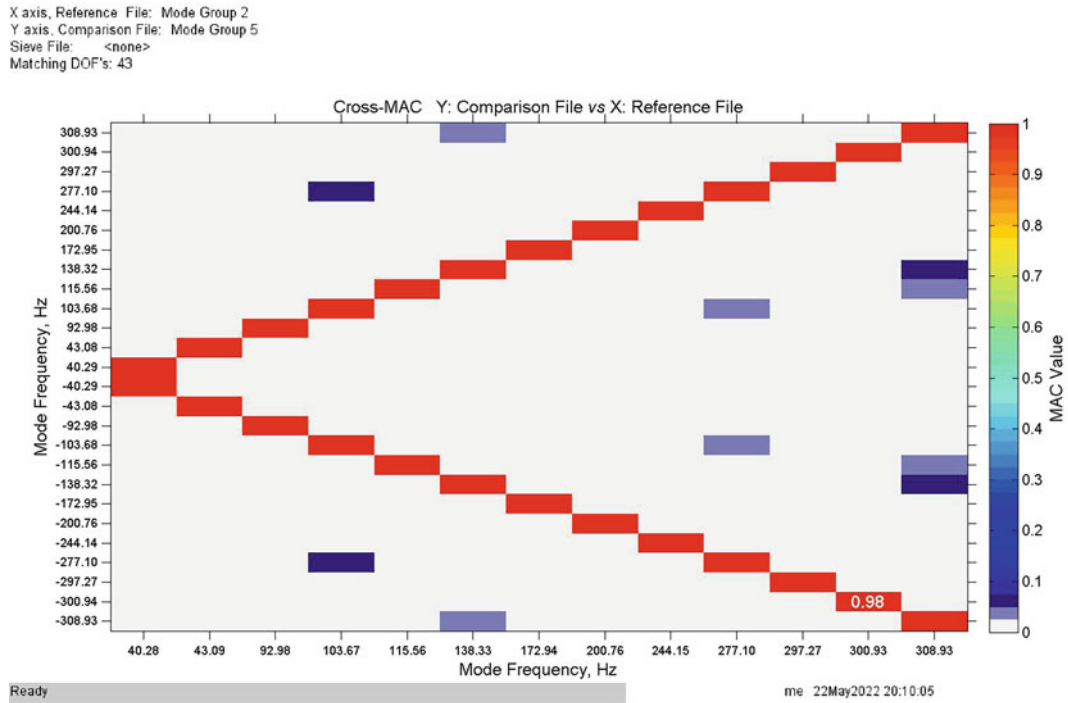


Fig. 2.9 MAC for 1 accelerometer 2 pressure modes versus 3 accelerometer modes

2.8 Sieving by References and Responses

2.8.1 Acceleration and Strain

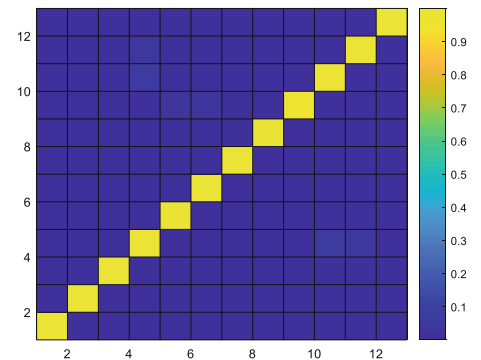
Two different modal parameter estimation processes were carried out on two different sets of sieved FRF data. Seventeen response locations with 3 acceleration references were selected and modal vectors were extracted for first 13 modes. Twenty-eight response locations with 3 strain references were selected and modal vectors were extracted for first 13 modes. The 28 locations selected were such that there were exactly 5 locations in common with the 13 points selected for accelerometer data. Three out of these 5 locations on the two modal vectors (strain based and accelerometer based) were used to scale the strain modal vector upto the accelerometer modal vector. This was done by taking the point-by-point ratio of each of the 3 locations for the 13 modal vectors and averaging this value across the 3 ratio values. This yielded one multiplier for each strain based modal vector. Except for the 2 entries (corresponding to the 5th mode) in column 2, the ratio values are consistent across the three columns.

The MAC between the composite vectors and modal vectors found from 40×3 accelerometer data also comes diagonal showing that the sets of vectors match. The crossMAC is shown in Fig. 2.10. Residuals were calculated for these modal vectors based on the Modal A values obtained from X-Modal and synthesized displacement FRFs were compared to measured FRFs. Some example comparisons are shown in Fig. 2.11 in magnitude and phase format. It can be seen that the FRFs compare well. Another variation tried was when doing the acceleration MPE, and one of the references were removed. The FRF data for this reference was then synthesized from the composite modal vector. The results obtained were also consistent with the measured acceleration FRF of the selected reference.

2.8.2 Acceleration, Strain, and Pressure

Three different modal parameter estimation processes were carried out on three different sets of sieved FRF data. Seventeen responses with 3 acceleration references, 17 responses with 3 strain references, and 16 responses with 3 pressure references were taken such that 5 of the response locations were common. The acceleration response from these locations was used to scale the other two modal vectors to the acceleration modal vector magnitude. Similar to the data in Table 2.1, a ratio was evaluated between the common location modal vectors elements strain and acceleration and pressure and acceleration. The ratios were found to be generally consistent across 5 locations. This ratio was averaged and used to scale the strain and pressure-based modal vectors. A composite modal vector was then assembled using the three partial modal vectors. MAC was evaluated for the composite modal vector versus a full accelerometer-based modal vector. This MAC is shown in Fig. 2.12. Synthesized FRFs from these composite modal vectors were compared with measured FRFS and were observed to compare well as shown in Fig. 2.13. Synthesis of an absent reference has not been tried in this combined method yet.

Fig. 2.10 MAC composite (acceleration and strain) modal vector versus normal modal vector



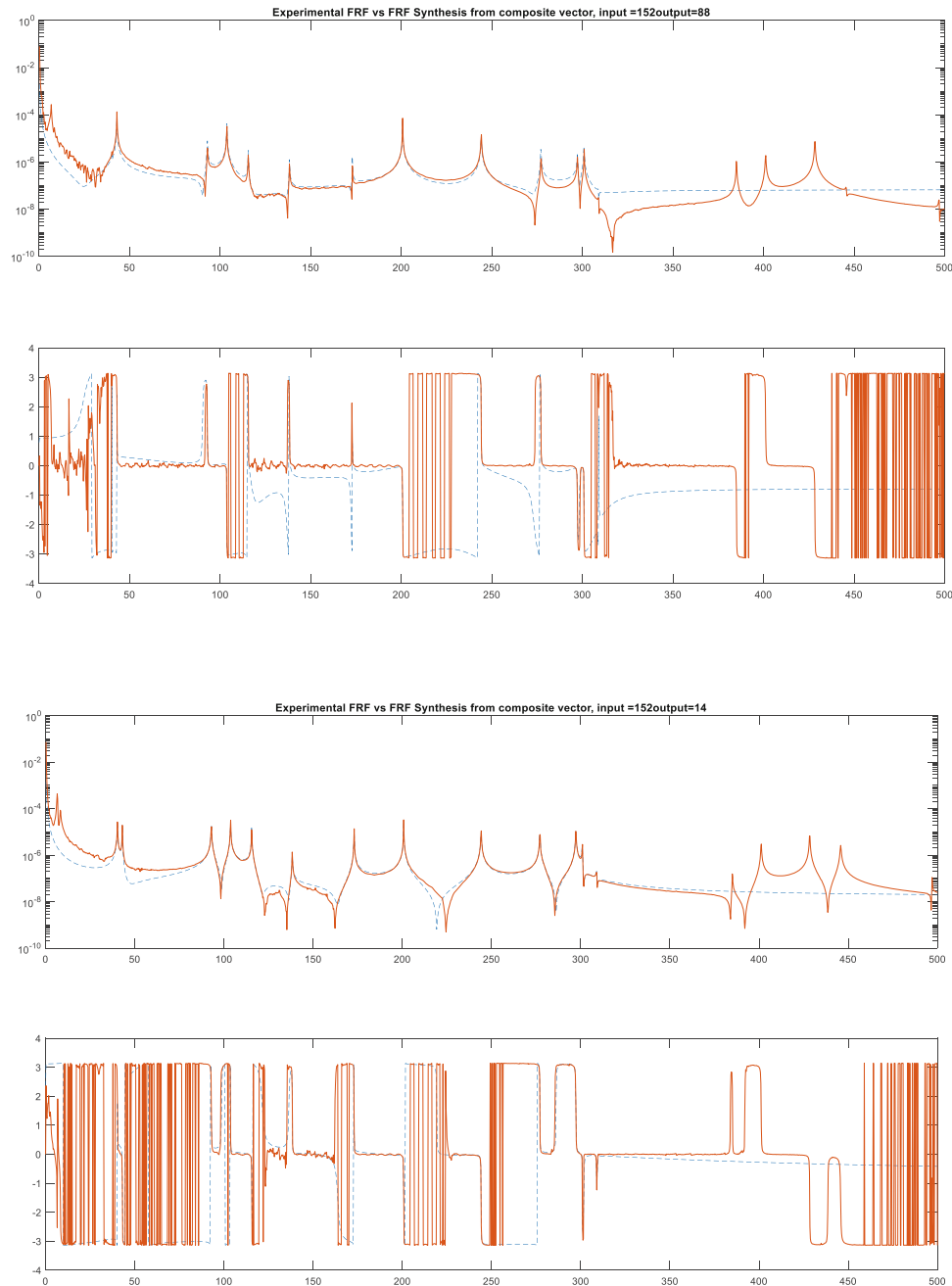


Fig. 2.11 Experimental (solid orange) versus synthesized (dotted blue) FRFs from composite vector (acceleration and strain)

2.9 Conclusion

Substituting acceleration reference with strain yielded close modal vector results as well as modal frequencies and damping. The difference in modal frequencies and damping is close to what is shown in Table 2.1 in all cases. There is variation in terms of how much of a scatter is observed in the damping estimates. But no consistent improvement due to adding strain data was observed across all modes. Substituting acceleration reference with acoustic pressure was not as simple. The pressure data had to be modified to bring it to a similar scale as the acceleration and strain data. Once that was done, the modal vector results did have a good MAC with full acceleration-based modal vectors.

When data was filtered through references as well as responses, it was done post modal parameter estimation of both strain and acceleration reference subsets. The ratio of modal contributions at the common locations for strain and acceleration came

Fig. 2.12 MAC composite
(acceleration, strain and pressure)
modal vector versus normal
modal vector

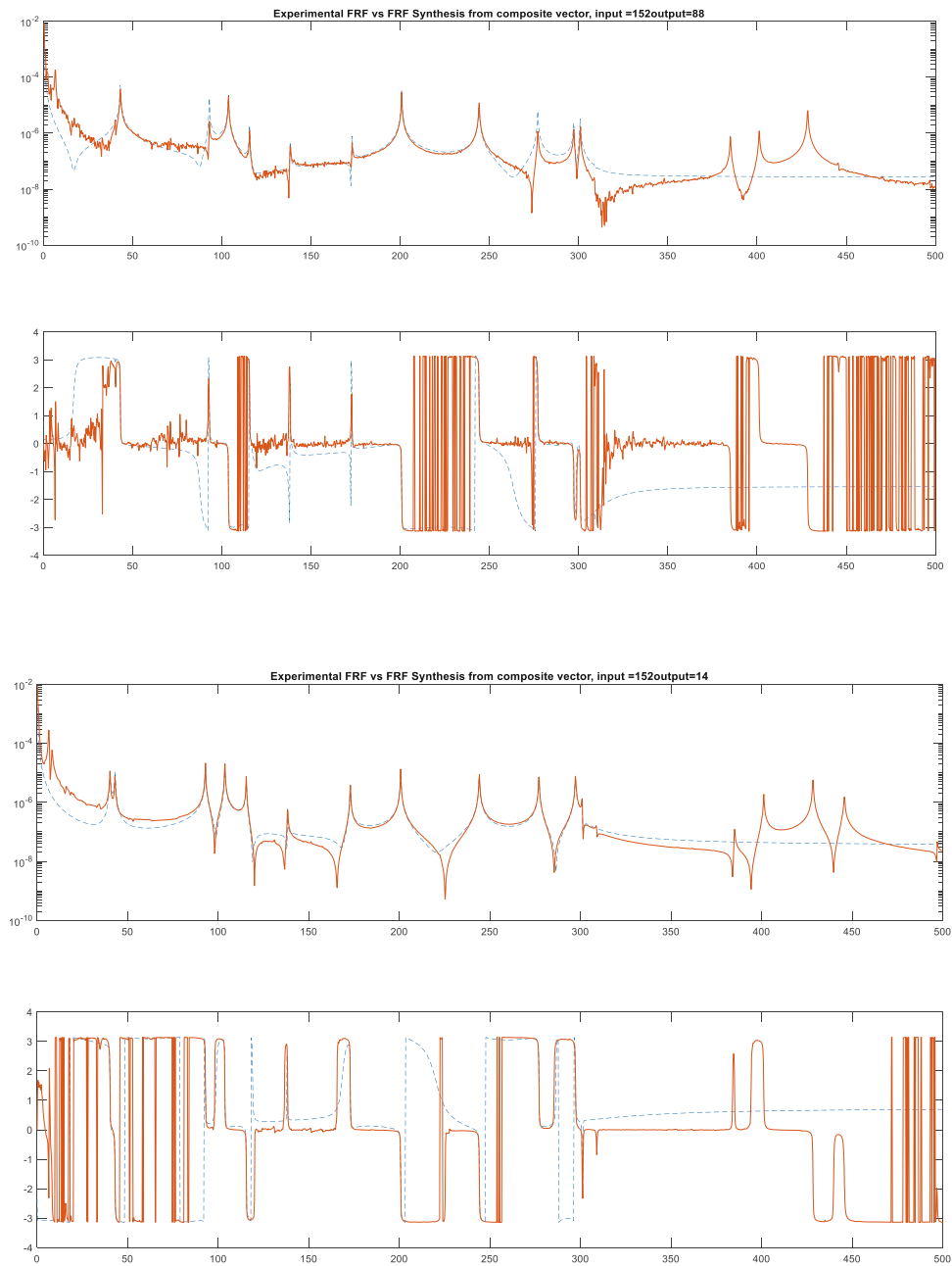
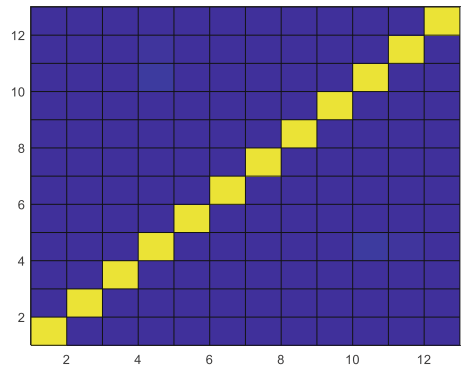


Fig. 2.13 Experimental (solid orange) versus synthesized (dotted blue) FRFs from composite vector (acceleration, strain and pressure)

out very close to -1 and 1 except for the 5th mode. The composite modal vector MAC with accelerometer-based modal vector came out diagonal. FRF data synthesized based on the composite modal vector compared generally well with the measured FRF. Composite modal vector evaluated with pressure data included with acceleration and strain also had a good MAC with accelerometer-based modal vectors. FRF synthesized compared well with acceleration-based modal vector.

The modal parameters obtained are the same or very close to what was obtained with the same amount of accelerometers, which brings up the question of whether replacing accelerometer data with other sensor data has yielded any improvements. It may be that due to its simplicity three accelerometers prove to be enough for the modal analysis of a rectangular plate. In that case, it might be useful to have a similar experiment of a structure that does not yield great results with three accelerometers. It will be worthwhile to also test a cylindrical container as originally discussed in the paper by Coppolino [1].

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