On Developing Extreme Rainfall Intensity–Duration–Frequency Relations for Canada: A Comparative Study of Different Estimation Methods

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Abstract Extreme rainfall intensity–duration–frequency (IDF) relations are commonly used for estimating the design storm for the design of various urban hydraulic structures. Traditionally, these IDF relations were obtained by fitting the two-parameter Gumbel distribution to the annual maximum (AM) rainfalls for each rainfall duration independently using the method of moments (MOM). However, it has been widely known that this Gumbel/MOM-based traditional approach may not produce accurate estimates of extreme rainfalls as compared to those given by, for instance, the generalized extreme value (GEV)/L-Moment method as suggested in some recent studies. Consequently, there are several new IDF estimation procedures and products that are recently developed in Canada in an attempt to provide some improvements in the estimation of design rainfalls. This study proposed therefore new approaches for developing IDF relations based on the scale-invariance behaviour of extreme rainfall processes using the GEV distribution. A detailed comparative study was then carried out to compare the performance of traditional IDF estimation methods and the proposed new approaches using the available IDF data from 39 stations located across Canada with at least 50 years of record. Results of this comparative study have indicated that the new scale-invariance GEV approaches can provide the most accurate and most robust estimates of design rainfalls for all locations in Canada as compared to existing traditional methods.

Keywords Extreme rainfall intensity · Duration · Frequency relations

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1 Introduction

Information on the spatio-temporal variability of extreme rainfall characteristics is of critical importance for planning, design, and management of various water systems [1]. In particular, for urban and small rural watersheds that are generally characterized by fast response, the design of hydraulic structures such as small dams, culverts, storm sewers, detention basins, and so on, requires extreme rainfall input for very short time durations (e.g. few minutes or hours) for runoff simulation models. More specifically, this extreme rainfall information is extracted from the "intensity–duration–frequency" (IDF) relations for various durations and return periods at a given site of interest [1–3].

In current engineering practice, the IDF relations are commonly derived based on statistical frequency analyses of annual maximum (AM) rainfall series for different durations. In Canada, Environment and Climate Change Canada (ECC) provides short-duration extreme rainfall data for nine different rainfall durations ($D = 5$, 10, 15, 30, 60, 120, 360, 720, and 1440 min) and the IDF relations for approximately 650 stations across Canada with at least 10-year rainfall record [4]. Traditionally, the extreme rainfalls for six different return periods ($T = 2$, 5, 10, 25, 50, and 100 years) were computed by fitting the two-parameter Gumbel distribution to the AM series for each rainfall duration independently using the method of moments (MOM). However, it has been widely known that this Gumbel/MOM-based traditional approach may not produce accurate and robust extreme rainfall estimates as compared to those given by, for instance, the generalized extreme value (GEV)/L-Moment method [5]. Consequently, there are several recently developed IDF products in Canada (as summarized in Table 1) to provide some improvements in the estimation of extreme rainfalls for both gaged or ungaged locations. Hence, there is an urgent need to carry out a critical review of existing rainfall estimation methods and to perform a detailed comparative study to assess their performance in order to identify the best estimation method for deriving IDF relations for Canada.

2 Extreme Rainfall Estimation Methods

As mentioned previously, Table 1 provides a summary of existing methods for extreme rainfall estimation and for developing the IDF relations for Canada. These include the traditional EC-IDF product by ECC [4], the Metro Vancouver IDF tool [6]; the IDF design values for Atlantic Provinces by Shephard [7], the Ontario Ministry of Transportation (MTO-IDF) tool by Soulis et al. [8], the IDF-CC tool by Simonovic et al. [3], and the SMExRain tool by Nguyen and Nguyen [2]. The key differences between these tools are: (i) the different probability model that was selected for describing the distribution of extreme rainfalls; (ii) the different estimation method that was used for estimating the probability model parameters; (iii) the different regression model that was chosen to represent the IDF curves; (iv) the different

Table 1 Existing IDF products and tools in Canada

Product	Provider	Latest version (Year)	Region	Brief Description	Reference
EC-IDF	Environment Canada	EC_IDF v-3.1 (2020)	Canada	IDF graphs and tables for 651 stations across Canada	[4]
IDF-CC tool	University of Western Ontario	IDF_CC Tool 4.5 (2021)	Canada	Web-based application, IDF graphs and tables for gaged and ungaged stations using (EC_IDF v3.1 data)	[3]
SMExRain tool	McGill University	SMExRain 1.0 (2019)	Canada	Standalone software, IDF graphs and tables for any site with IDF data	[2]
MTO IDF Curves Finder	Ontario Ministry of Transportation	MTO IDF 3.0 (2016)	Ontario	Web-based application for gaged (using EC_IDF v2.3 data) and ungaged stations	[8]
Metro Vancouver	Metro Vancouver	IDF curves (2009)	Vancouver	IDF graphs and tables for different homogeneous zones	[6]
IDF climate design values	Atlantic Climate Adaptation Solutions Association	IDF curves (2011)	Atlantic Provinces	IDF curves for different Atlantic Provinces	[7]

estimation method that was used for estimating the regression model parameters; (v) the different spatial interpolation technique that was used for transferring IDF information from gaged sites to the target ungaged location; and (vi) the different consideration of the scale-invariance property of the extreme rainfall processes for different rainfall durations.

More specifically, for the whole Canada, only the EC-IDF and IDF-CC are readily available, and the data are maintained up to date. The MTO IDF tool is mainly developed for the province of Ontario using data primarily from EC-IDF in combination with USGS digital elevation data to derive physiographic characteristics [8]. The Metro Vancouver IDF curves are primarily developed for the City of Vancouver and its neighbouring regions based on Hosking and Wallis [9]'s regional approach [6]. The IDF curves for Atlantic Provinces use the same approach as in the EC-IDF and have been recently integrated into the EC-IDF product [4]. The SMExRain tool is a standalone application that can be used to generate IDF curves for any location in Canada with available IDF data [2]. Therefore, only the EC-IDF product and the IDF-CC tool were selected for this comparative study. A summary of the basic features of these two products is provided in the following sections.

The EC-IDF tool provides the historical IDF relations for approximately 650 stations across Canada with at least 10-year record [4]. The extreme rainfalls for these IDF relations were computed for nine different rainfall durations ($D = 5$–1440 min) and for six different return periods ($T = 2$–100 years) using the two-parameter Gumbel distribution. In particular, the Gumbel distribution is fitted to the historical AM series for each rainfall duration independently using the method of moments (MOM). In addition, the IDF relations are described by a simple power-form relation as follows:

$$I = aD^b \tag{1}$$

in which I is the rainfall intensity; D is the rainfall duration; and a, b are coefficients that are computed for each return period by the least-square technique in the log-data space. These two coefficients are also computed for locations without data using a simple linear spatial interpolation technique [4].

The IDF-CC tool uses the generalized extreme value (GEV) distribution as the parent distribution for representing the distribution of AM rainfall series [3]. The L-moment (LMOM) estimation method is used to estimate the three parameters of the GEV model. This tool also provides the IDF curves derived from the Gumbel distribution for the purpose of comparison. The IDF curves are described by the following mathematical relation:

$$I = a(D + c)^b \tag{2}$$

in which a, b, c are coefficients that are computed for each return period using the differential evolution optimization algorithm [3] in the real data space.

In summary, these two traditional approaches are based on the regression of the computed rainfall intensities (or depths) over durations using Eq. (1) with two coefficients (QR2C method) or Eq. (2) with three coefficients (QR3C method). In the present study, a new approach is introduced based on the regression of the empirical statistical moments of observed rainfall amounts over durations. As compared to the traditional approaches, this new method can account for the observed scale-invariance property of the empirical statistical moments. In general, the proposed new method consists of the following four steps:

(i) Firstly, compute the statistical moments of the AM rainfalls for the first few orders for each rainfall duration (e.g. from 5-min to 24-h intervals) for a given location of interest. More specifically, only the first two non-central moments (NCMs) are required for the GUM/MOM model, and the first three probability weighted moments (PWMs) are necessary for the GEV/LMOM model;

(ii) Secondly, construct regression models to describe the relationships between the computed NCMs (or PWMs) of rainfall amounts and the rainfall durations. These relationships indicate the scale-invariance property of the AM processes.

For instance, some previous studies have found that extreme rainfall processes for durations ranging from a few minutes to several days could display one or two different scaling regimes [10, 11]. These rainfall-statistical-moment-based regression models are quite useful for the estimation of the NCMs (or PWMs) of sub-hourly or sub-daily rainfall amounts from those of the daily amounts;

(iii) Thirdly, estimate the parameters of a selected theoretical distribution using the method of moments (or the method of L-moments) in consideration of the regression relationships established in step (ii). More specifically, there are two different approaches are proposed in this study depending on the application of these regression relationships: (a) the rainfall-statistical-moment-based method (referred hereafter as MR method) if the parameters of the GUM/MOM or GEV/LMOM model are computed based on the direct regression of the statistical moments of AM rainfalls for different durations; and (b) the scaling-statistical-moment-based method (referred herein as MRS method) if the GUM/MOM and GEV/LMOM parameters are computed based on the scale-invariance relations between daily and sub-daily statistical moments; and

(iv) Finally, estimate the design rainfall intensity (or depth) for a given duration and return period of interest using the quantile function of the Gumbel distribution or GEV distribution [10].

In this study, two common dimensionless goodness-of-fit (GOF) criteria were selected to compare the performance of the four methods (QR2C, QR3C, MR, and MRS). These criteria include the mean absolute relative difference (MADr) and the root mean square relative error (RMSEr) as defined below:

$$\mathrm{MADr} = \frac{1}{n}\sum\left\{\frac{|x_{ij} - y_{ij}|}{x_{ij}}\right\} \tag{3}$$

$$\mathrm{RMSEr} = \left[\frac{1}{n}\sum\left\{\frac{(x_i - y_i)}{x_i}\right\}^2\right]^{1/2} \tag{4}$$

in which x_{ij} and y_{ij} are the rainfall quantiles corresponding to duration D_i ($i = 1$, 2, …, 9) and return period T_j ($j = 1, 2, \ldots, 6$) estimated based on the observed data and based on one of the four estimation methods (i.e. QR2C, QR3C, MR, and MRS), respectively; $n = i \cdot j$ is the number of design rainfall values at each station.

3 Study Sites and Data

As mentioned previously, the IDF network consists of approximately 650 raingages located across Canada [4]. For each site, ECC provides the AM series for nine different rainfall durations ranging from 5 min to 24 h. The record length for these AM series varies from 10 to 82 years, and most of the data are recorded after 1960 and updated to 2017. For this comparative study, only 39 stations with very long record (containing at least 50 years) were selected since the estimation of rainfall quantiles for high return periods (e.g. for $T = 50$ and 100 years) can be considered as reliable for these long datasets [2]. The details of the 39 chosen stations are presented in Table 2. The comparison of the performance of the two traditional (QR2C and QR3C) approaches and the two new MR and MRS methods will be carried out using the available IDF data for these 39 stations.

4 Results and Discussions

In the present study, the rainfall quantiles for each station were estimated for all six return periods ($T = 2$–100 years) and for all nine durations ($D = 5$–1440 min) using the four selected approaches (QR2C, QR3C, MR, and MRS). The IDF relations are then constructed for each station. For purposes of illustration, Fig. 1 shows the IDF relations for St Thomas WPCP Station for all methods. It can be observed that the QR2C method produced the least accurate results. As expected, the QR3C, MR, and MRS methods, considered as the general form of the QR2C method, provided more accurate results. Similar results were found for other stations.

More generally, Fig. 2 shows the comparison of these four methods based on the MADr (%) and RMSEr (%) for both Gumbel and GEV models and for all stations. The slope ratio between the first and second scaling regimes of AM series is also plotted. Notice that the first scaling regime is identified as from the 5-min duration to the breakpoint on the graph between the empirical statistical moments and the rainfall durations; the second scaling regime is defined from the breakpoint to the 24-h duration for each station. It can be seen that the QR2C method yields high error for most of stations, especially stations with large difference in the slope ratio. The QR3C method produces good results for most stations with MADr less than 5%, except the first two stations with the slope ratio less than 1 indicating the convex pattern for the IDF curves. On the other hand, both MR and MRS approaches can capture very well this convex pattern for these two stations and can also provide the MADr less than 5% for all other stations. The MADr and RMSEr results for all stations are also summarized in the form of boxplots as shown in Fig. 2. It can be seen that the QR2C is the least accurate method.

A more detailed investigation of the slope ratios of the first and second scaling regimes is shown in Fig. 3. These slopes are computed based on the first NCMs (or first PWMs) of rainfall depths over durations for all available IDF stations (approximately 650 stations). In general, it can be seen that the pattern of the IDF curves varies over different regions of Canada. However, for a large number of stations in the Pacific region the convex pattern (i.e. the slope ratio value is less than one) occurs more frequent. This could be due to the orographic effect in this region which causes a very different behaviour of short-duration extreme rainfall processes. Hence, the QR3C method should not be applied in this region.

Table 2 Details of the selected 39 stations, including the province (PR), station identification (ID), station name, latitude (lat, degrees), longitude (long, degrees), and record period and length (No. of years)

No.	PR	ID	Station Name	Lat (degree)	Long (degree)	Period	No. of years
1	BC	1018611	Victoria Gonzales CS	48.42	123.32	1925–2017	65
2	BC	1018621	Victoria Intl A	48.65	123.43	1965–2017	50
3	BC	1105192	Mission West Abbey	49.15	122.27	1963–2017	54
4	BC	1108395	Vancouver Intl A	49.18	123.18	1953–2017	63
5	AB	3012206	Edmonton Intl CS	53.32	113.62	1961–2017	52
6	AB	3012209	Edmonton Blatchford	53.57	113.52	1914–2017	71
7	AB	3031094	Calgary Int L CS	51.12	114.00	1947–2017	62
8	SK	4012410	Estevan	49.22	102.97	1964–2017	53
9	SK	4015322	Moose Jaw CS	50.33	105.53	1960–2017	50
10	SK	4016699	Regina R CS	50.43	104.67	1941–2017	62
11	SK	4043901	Kindersley A	51.52	109.18	1966–2016	50
12	MB	502S001	Winnipeg A CS	49.92	97.25	1944–2017	58
13	ON	6012199	Ear Falls (Aut)	50.63	93.22	1952–2017	56
14	ON	6016525	Pickle Lake (Aut)	51.45	90.22	1953–2017	51
15	ON	6042716	Geraldton A	49.78	86.93	1952–2016	54
16	ON	6048268	Thunder Bay CS	48.37	89.33	1952–2012	53
17	ON	6078285	Timmins Victor P. A	48.57	81.38	1952–2016	51
18	ON	6104175	Kingston Pumping Stn	44.23	76.48	1914–2007	63
19	ON	6105978	Ottawa Cda R CS	45.38	75.72	1905–2017	57
20	ON	6127519	Sarnia Climate	43.00	82.30	1962–2017	50
21	ON	6131983	Delhi CS	42.87	80.55	1962–2015	50
22	ON	6137362	St Thomas WPCP	42.77	81.22	1926–2016	82
23	ON	6139525	Windsor A	42.28	82.97	1946–2016	66
24	ON	6143089	Guelph Turfgrass	43.55	80.22	1954–2017	52
25	ON	6144478	London CS	43.03	81.15	1943–2017	66
26	ON	6153301	Hamilton RBG CS	43.28	79.92	1962–2017	53
27	ON	6158355	Toronto City	43.67	79.40	1940–2017	67
28	ON	6158731	Toronto Intl A	43.68	79.63	1950–2017	64
29	QC	701S001	Quebec Jean L. Intl	46.80	71.38	1961–2017	57
30	QC	7014160	L Assomption	45.82	73.43	1963–2017	55
31	QC	702S006	Montreal P.E.T. Intl	45.47	73.73	1943–2017	72
32	QC	7024280	Lennoxville	45.37	71.82	1960–2017	54
33	NB	8100885	Charlo Auto	47.98	66.33	1959–2017	55
34	NB	8103201	Moncton Intl A	46.12	64.68	1946–2016	67

(continued)

Table 2 (continued)

No.	PR	ID	Station Name	Lat (degree)	Long (degree)	Period	No. of years
35	NS	8204700	Sable Island	43.93	60.02	1962–2013	51
36	NS	8205092	Shearwater R CS	44.63	63.52	1955–2017	60
37	NS	8205702	Sydney CS	46.17	60.03	1961–2016	53
38	NL	8401705	Gander Airport CS	48.95	54.57	1939–2017	70
39	NL	8501900	Goose A	53.32	60.42	1961–2016	53

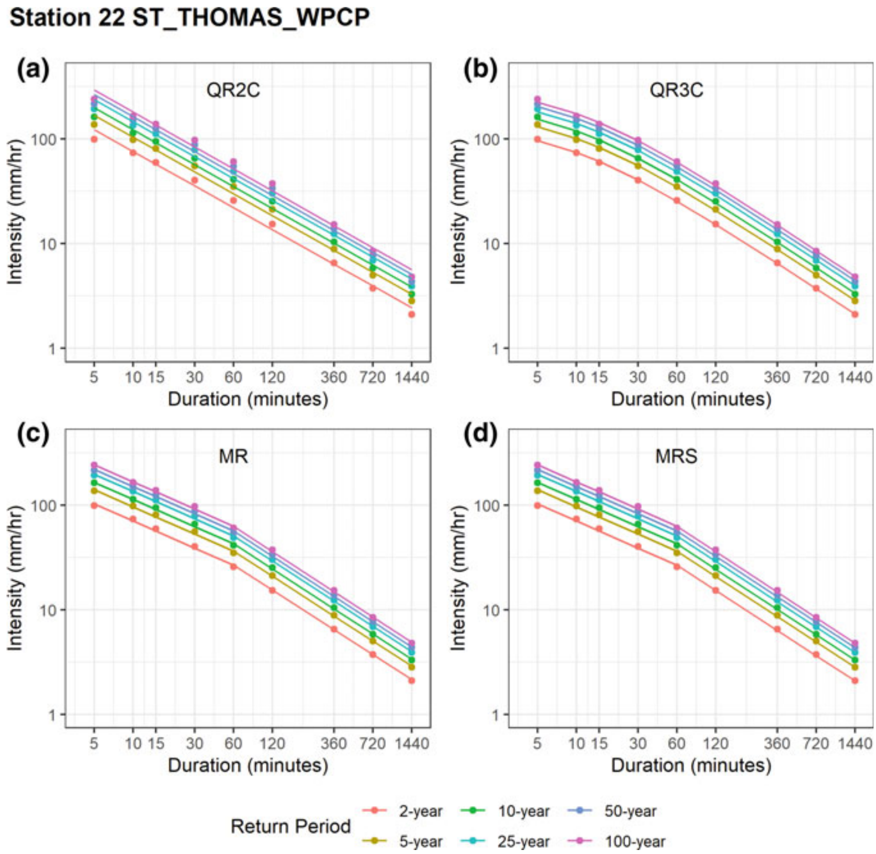

Fig. 1 IDF relations at St Thomas WPCP Station based on different methods: **a** and **b** quantile-based regression methods (QR2C and QR3C); **c** and **d** moment-based regression without and with scaling (MR and MRS). Markers represent the observed GUM/MOM rainfall quantiles

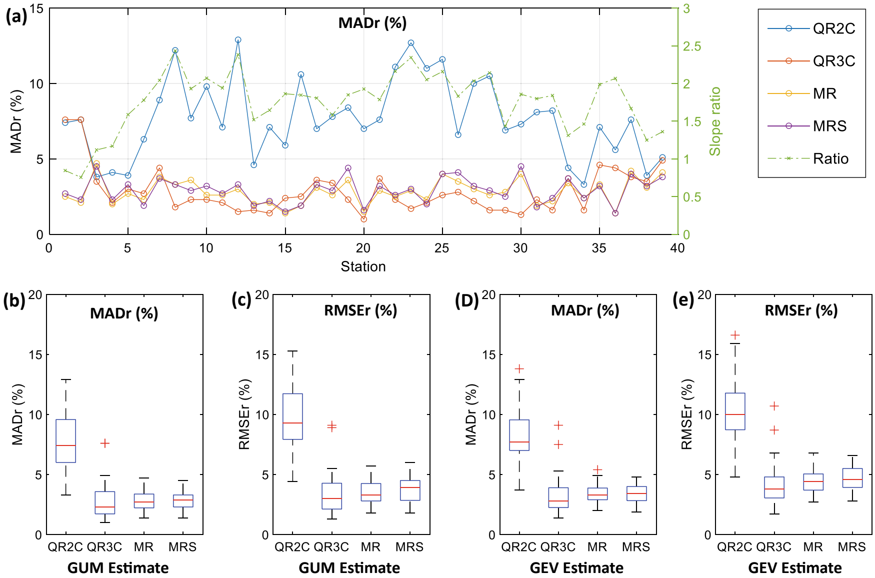

Fig. 2 Plots of the scaling slope ratio, MADr (%), and RMSEr (%) for different methods (QR2C, QR3C, MR, and MRS) for Gumbel and GEV models

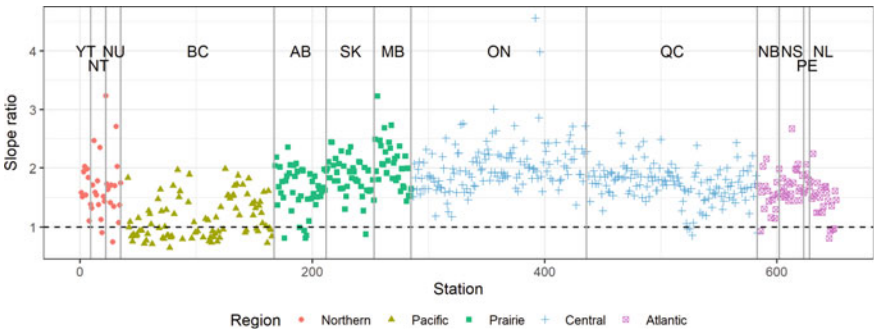

Fig. 3 Slope ratios of the first and second scaling regimes based on the first NCMs (or PWMs) of rainfall depths for different regions of Canada

5 Summary and Conclusions

In Canada, there exist several different methods for developing IDF relations. Hence, in this research, a comparative study was carried out to assess the performance of these methods in order to identify the best approach for use in practice. More specifically, two traditional approaches (QR2C and QR3C) were compared with two new proposed procedures (MR and MRS). In general, the traditional methods were based on the relationships between the extreme rainfall quantiles with the rainfall durations, while the new approaches were relied on the relationships between the statistical moments of extreme rainfalls and the rainfall durations. The comparation was performed using the available historical AM data for nine different durations (from 5 min to 24 h) for 39 stations with long record length (at least 50 years) across Canada to represent the diverse climatic conditions of Canada.

Results of this comparative study have indicated that the QR2C based on the GUM/MOM model is the least accurate method as compared with the other three methods. In addition, it was found that different patterns of the IDF curves for different locations in Canada can be linked to the scaling behaviour of the extreme AM processes. This scaling behaviour can be identified based on the relationships between the empirical statistical moments (NCMs or PWMs) of rainfall amounts and the rainfall durations. Consequently, the QR3C method can be used for IDF relations with linear and concave scaling patterns but cannot provide an accurate rainfall estimation for IDF curves with the convex pattern that is a common pattern observed in the Pacific region. On the other hand, it was found that the new MR and MRS methods proposed in this study can be used for all regions of Canada.

Furthermore, the MR method produces slightly better results than the MRS since it was relied on the best fit of the regression model to the empirical statistical moments over all different rainfall durations, while the MRS method was based on the approximate fit of the derived sub-daily rainfall statistical moments from the daily statistical moments. However, this difference is not significant (on average, less than 2% between the two methods). The MRS method, however, offers a key advantage for cases of ungaged for partially-gaged sites where the derivation of the distributions of sub-hourly and sub-daily extreme rainfalls from that of daily amounts is required.

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