Assessment of the Strain-Stress Distribution in the Vicinity Conceding Mountainside's Scarp Using Mathematical Modeling

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Abstract

This paper investigates the strain-stress distribution in the vicinity conceding mountainside, in which the interface zones between the scarps and the mountainside, the height of the scarp, and the effect of bulk and surface loads were considered in a single analytical model. When evaluating the strain-stress distribution near the scarp, the position and linear dimensions of the mountainside's scarp were realized by mapping such a semi-infinite area, for which the Kolosov-Muskhelishvili functions were defined as general solutions for calculating stresses from the loads set at the scarp. Quantitative and qualitative regularities of stress distribution in the vicinity of a one-sided or two-sided mountainside in the variation of the combination of forces action are obtained, which are considered in the constructed analytical solutions. The analysis of the stress distribution revealed the following. In the interface zone with the ridge crest, the horizontal component of stress has the highest concentration, which is 1.7 times higher than the vertical component due to gravity and horizontal tectonic compression. The general law of distribution of vertical stresses corresponds to the change in the shape of the mountainside; the quantitative values are somewhat less than the value of sliding stresses. The maximum values of tangential stresses are observed in the interface zone of the mountainside and scarp; the distribution pattern of stresses σx and σ2 is almost the same.

Keywords

Mountainside · Strain-stress distribution · Massifs · Distribution of stress fields · Displaying function · Mathematical model

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1 Introduction

Scarps are formed by nature or created by man (i.e., human-made), which conjugate to the mountainside at different angles of inclination and at different heights. To ensure the stability and preserve structures located in the side zone from landslides, it is necessary to carefully study the strain-stress state of the massifs when the scarps and the adjacent mountainsides work as a single unit. It is also necessary to create such an analytical model and program for calculating the strain-stress distribution of the mountainside's scarp, in which it is possible to distribute the load originating from the weight of loaded transport, transmission line supports, and the weight of buildings and structures. The studied patterns of stress distribution and their concentration zones allow us to use them to predict the possible zones of the destruction of mountainsides when conducting engineering activities to strengthen the grade of a highway and expert assessment of the strength and stability of the bases of structures and communications of transmission points located on mountainsides.

2 Methodology

In mountain areas, the strain-stress distribution of mountainsides is studied to support various human economic activities. The condition of mountainsides, used at least for road bases in mountain regions, is studied insufficiently. For this problem, the traditionally applied methods for estimation of the strain-stress distribution of mountainsides, such as the unloading method, the finite element (FEM) (Dolezalova, 1968; Goldin & Troitsky, 1971; Kurlenya & Popov, 1983; Usmanov, 2009; Yerzhanov & Kerimbaev, 1975), the finite

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difference method (FDM) (Leeman, 1968; Nikitin, 1959), analytical, and other methods, have not found wide application due to some technical difficulties. The unloading method (Aitmatov, 2011; Dolezalova, 1968; Hast, 1969; Kutepov, 1966; Mambetov, 1988; Muskhelishvili, 1976) can be carried out in the presence of underground workings in the area where the scarp and the mountainside meet. A computational model is created when using FEM and FDM for the mountainside's scarp area. In this model, one side (the day surface of the scarp) is modeled precisely, and in the other three sides of the computational model (two side surfaces and one horizontal side), the boundary conditions are set approximately according to the subjective perception of the researcher.

In this case, we consider the application of new technology of conformal mapping (Goldstein & Kalinin, 1969; Kurdin, 1971) to simulate the mutual influence of the scarp with the mountainside, as well as the creation of an analytical model of the massif in the area of the mountainside's scarp (Bayalieva & Zhumabaev, 2016; Muskhelishvili, 1976; Ter-Martirosyan & Akhpatelov, 1976) that considers the joint or separate action of surface distributed loads, gravity, and tectonic compression and the method of calculation of the strain-stress distribution of the mountainside's scarp using the MATHCAD software package (Kiryanov, 2007). Patterns of distribution of stress and strain fields in the vicinity of the scarp of the mountainside are also established.

The mathematical model of the mountain relief is modeled with a mapping function, as in the works of B. Zhumabaev and K. Dzh. Ismailova (Ismailova & Zhumabaev, 2009; Zhumabaev, 1988; Zhumabaev & Ismailova, 2005), N. S. Kurdin (1971), and Zh. A. Bayalieva (Bayalieva, 2013; Bayalieva & Zhumabaev, 2008, 2015, 2016; Zhumabaev & Bayalieva, 2015):

$$z = w(\varsigma) = \alpha\varsigma + \sum_{k=1}^{m} \frac{\alpha_k}{\varsigma + t_{0k} - i} \tag{1}$$

where m—the number of bends within the half-plane $y \leq 0$; α and t_{ok}—real constants, k—complex constants.

Performing a detailed analysis of the parametric equations of the contour lines $x(t)$ u $y(t)$, B. Zh. Zhumabaev (1988) established that at ζ= t, $\eta = 0$, it follows that:

$$x(t) = at + \frac{a_1 t}{t^2 + 1} \quad y_1(t) = \frac{a_1}{t^2 + 1} \quad y_2(t) = \frac{a_2 t}{t^2 + 1} \tag{2}$$

In the early works of scientists (Kurdin, 1971; Zhumabaev, 1988) investigating the strain-stress distribution of rock massifs, the following three circumstances were considered:

1. The parameters were chosen so that the value of α for two neighboring values was five or more times higher than ak, resulting in the interaction of two neighboring protrusions.
2. If $t_{ok} = 0$, then all summands are summed as a_1—one constant—and there is only one summand instead of the sum in Formula (1).
3. If $(t_{ok} - t_{ok-1}) > 5$, then, in the vicinity of each scarp, the ordinates are described by a parametric equation of the type a_k with the same law of change of the abscissa $x(t_k)$ for all terms of the sum in Formula (1).

This research considers the case where $(t_{ok} - t_{ok-1}) \leq a_k$ in Formula (1).

This method of constructing a mapping function as a sum of simple fractional functions (Bayalieva, 2016; Kurdin, 1971; Zhumabaev, 1988) was proposed by N. S. Kurdin (1971), and B. Zhumabaaev (1988). Modeling methods and analytical descriptions of stresses and deformations of scarps are described in the work of Zh. A. Bayalieva (Bayalieva, 2016; Bayalieva & Zhumabaev, 2016).

The relationships found for the complex potentials Φ(ζ) and Ψ(ζ) are used to determine, insatiate, and lgorithmize the stress components in the mapping function (1) in MATHCAD symbols (Kiryanov, 2007).

These ratios are as follows:

$$\Phi(\varsigma) * \omega^{'}(\varsigma) + G(\varsigma) = A(\varsigma)$$

$$\psi(\varsigma) * \omega^{'}(\varsigma) + \omega(\bar{\varsigma}) + \Phi^{'}(\varsigma) * \omega^{'}(\bar{\varsigma}) - G(\varsigma) = B(\varsigma) \tag{3}$$

$$A(\varsigma) = -\frac{1}{2\pi t}\int_{-\infty}^{\infty} \frac{(N + iT)\omega^{'}(t)}{t - \varsigma} dt$$

$$B(\varsigma) = -\frac{1}{2\pi t}\int_{-\infty}^{\infty} \frac{(N - iT)\omega^{'}(t)}{t - \varsigma} dt \tag{4}$$

Using function (1), having selected values t0k (k = 1, 2, … n), and using this method, we can simulate the presence of m edges on the scarp (Bayalieva & Zhumabaev, 2015).

Parametric equations of the scarp's contour with values should be defined for signs Re [ω (ξ, 0)] and Im [ω (ξ, 0)] at z = ω (t, 0), where the real part is the parametric equation x(t), the imaginary part of which is y(t).

3 Results

The stress state in the vicinity of the mountain is simulated by constructing the stress fields σ_x, σ_y, τ_{xy}, which arise from the combined action of gravitational, uniform-seismic forces and horizontal tectonic compression, as well as surface forces that are applied in the final section of the scarp. The strain-stress distribution model is defined as the sum of the four stresses:

$$\begin{cases} \sigma_x = \sigma_x^{\pi} + \sigma_x^{r} + \sigma_x^{T} + \sigma_x^{N} \\ \sigma_y = \sigma_y^{\pi} + \sigma_y^{r} + \sigma_y^{T} + \sigma_y^{N} \\ \tau_{xy} = \tau_{xy}^{\pi} + \sigma\tau_{xy}^{r} + \tau_{xy}^{T} + \sigma\tau_{xy}^{N} \end{cases} \quad (5)$$

where $\sigma_x^{\pi}, \sigma_y^{\pi}, \tau_{xy}^{\pi}$—the complex stresses for the half-plane that can be written in the following form:

$$\sigma_x^{\pi} = \lambda * p_y * y * T_x,$$

$$\sigma_y^{\pi} = p_y * y,$$

$$\tau_{xy}^{\pi} = p_x * y, \quad (6)$$

where $p_x = K_c\gamma$, $\sin\delta$ is the horizontal part and $y = \gamma(1 - K_c cos\delta$ is the vertical part of the volumetric force. The components given in Formula (6), when summed with the stress components $\sigma_x^{r}, \sigma_y^{r}, \tau_{xy}^{r}$, give a stress state, which is called the initial stress state (Fig. 1). In this case, the integrals A (ζ) and B (ζ) in Formula (4) are calculated from imaginary loads N and T, which have the following form (Muskhelishvili, 1976):

$$\begin{cases} N + iT = \dfrac{\sigma_x^{\pi} + \sigma_y^{\pi}}{2} - \dfrac{\sigma_{xy}^{\pi} - \sigma_x^{\pi} + 2i\tau^{\pi}xy}{2} e^{2ia} \\ N - iT = \dfrac{\sigma_x^{\pi} + \sigma_y^{\pi}}{2} - \dfrac{\sigma_{xy}^{\pi} - \sigma_x^{\pi} - 2i\tau^{\pi}xy}{2} e^{-2ia} \end{cases} \quad (7)$$

We express the integrals (4) from the boundary conditions (7) through AG(ζ) and BG(ζ) (Bayalieva et al., 2018):

$$AG(\varsigma) = \left[\frac{T_3 \sum_{k=1}^{n} a_k}{(\varsigma + t_{ok} - i)}\right] * \left[\frac{\sum_{j=1}^{n} a_j}{(\varsigma + t_{ok} - i)^2}\right] + \sum_{j=1}^{n} a_j \left[\frac{(a - T_{1j})(A_3 + iA_2)}{2(\varsigma + t_{oj} - i)} + \frac{T_3 T_{2j}}{(\varsigma + t_{oj} - i)^2}\right],$$

$$BG(\varsigma) = \left[\frac{\bar{T}_4 \sum_{r=k=1}^{n} a_k}{(\varsigma + t_{ok} - i)}\right] * \left[\frac{\sum_{j=1}^{n} a_j}{(\varsigma + t_{oj} - i)^2}\right] + \sum_{j=1}^{n} a_j \left[\frac{(a - T_{1j})(A_3 + iA_2)}{2(\varsigma + t_{oj} - i)} + \frac{T_3 T_{2j}}{(\varsigma + t_{oj} - i)^2}\right]; \quad (8)$$

The total stress fields obtained from integrals (5) satisfy the conditions T (ξ) = 0 and N (ξ) = 0 in the mountainside contour.

Horizontal tectonic forces Tx, leading to the stress distribution, satisfy the following boundary conditions:

$$N^n + iT^n = \frac{T_x}{2}\left[1 - e^{2ia}\right], \quad N^n - iT^n = -\frac{T_x}{2}\left[1 - e^{-2ia}\right], \quad (9)$$

The values of integrals A(ζ) and B(ζ) at point (4), calculated from the boundary conditions (9), are denoted as AT(ζ) and BT(ζ) and represented in the following form:

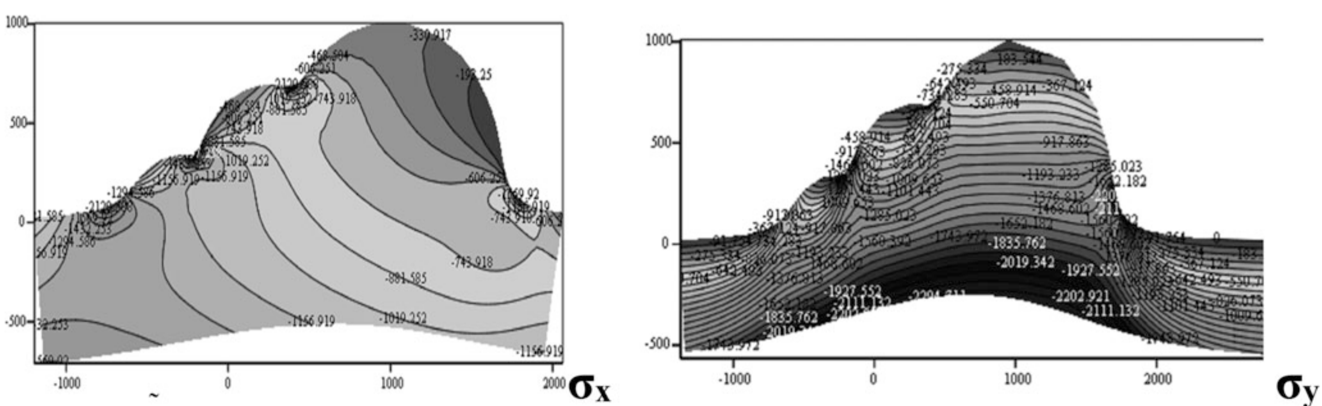

Fig. 1 Strain-stress distribution on a two-sided scarp under the action of gravity. *Source*: Developed by the authors using the compiled algorithm and the capabilities of the MATHCAD software system

$$AT(\varsigma) = T_1 * \frac{\sum_{j=1}^{n} a_j}{\left(\varsigma + t_{oj} - i\right)^2}, BT(\varsigma) = -T_1 * \frac{\sum_{j=1}^{n} a_j}{\left(\varsigma + t_{oj} - i\right)^2}, \quad T_1 = \frac{T_\infty}{2} \tag{10}$$

The results of these calculations are given for the area of the mountainside conjugation and their quantitative values in relation to the height of the mountainside H.

An example would be its deformed position (Bayalieva & Zhumabaev, 2016). The height of the mountainside H = 1000 m. The vertical forces N vary over a wide range. If there is a traffic lane on the scarp, the value of the weight of the loaded car or wagon train is N. When calculating the deformation at the edges of the scarp, we set the conditional values of the loads. The following parameters are adopted: Young's modulus E = 2.2 · 104 MPa; Poisson's ratio μ = 0.3; lateral expansion of rocks λ = 0.4; tectonic compression Tx = −50 MPa; vertical external load N = −200 MPa. Hooke's law is used in the following form:

$$\gamma_{xy}(x, y) = \frac{2 * (1 + v)}{2.2 * 10^4} * \tau_{xy}(x, y)\varepsilon(x, y) = \frac{1}{2.2 * 10^4} \times \left(\sigma_x(x, y) - v * \sigma_y(x, y)\right),$$

$$\varepsilon_y(x, y) = \frac{1}{2.2 * 10^4}\left(\sigma_y(x, y) - v * \sigma_x(x, y)\right).$$

A similar problem was solved, where (Table 1) the values of all stress components are given for the contour points when the load N = −50 MPa is applied on the top scarp (Fig. 2).

4 Conclusion

This paper considers a mathematical method for modeling a polyhedral scarp consisting of a mapping function and a derivative relation for two Kolosov-Muskhelishvili functions calculated for two integrals of boundary conditions. The developed analytical model is used to create the distribution of stresses and strains under the action of forces on the massif (gravitational, inertial, and surface distributions).

The analysis of the stress distribution reveals the following:

- The horizontal component has the greatest concentration in the zone of conjugation with the ridge wall, which exceeds the effect of gravity and horizontal tectonic compression by a factor of 1.7;
- The law of distribution of vertical stresses corresponds to the change in the shape of the mountainside; the numerical values are slightly lower than the values of stresses;
- The maximum values of tangential stresses reach the conjugate zone of the mountainside and wall, the stress distribution graphs σx and σ2 practically coincide, and the same stress is observed between σc and the wall.

Deformation of the two-sided mountainside under the action of gravity indicates stretching in the vertical direction in the vertical part and compression in the area of the ridge base.

Table 1 Stress values on the contour near the scarp

0	ξ	x	y	σ_1	σ_2	N	T	τ_{max}
1	2.476	57.048	587.46	0.382	$102·10^{-13}$	$09·10^{-13}$	$52·10^{-13}$	0.12
2	2.485	59	582.14	0.394	$89·10^{-14}$	$62·10^{-14}$	$78·10^{-14}$	0.106
3	2.489	59.768	581.22	0.456	$96·10^{-13}$	$92·10^{-13}$	$09·10^{-13}$	0.113
4	2.515	62.42	582.47	−50.51	−52	−52	$45·10^{-13}$	0.347
5	2.511	63.79	586.34	−50.45	−52	−52	$2.5·10^{-13}$	0.349
6	2.57	66.83	586.27	−50.55	−52	−52	$14·10^{-13}$	0.587
7	2.576	67.49	586.13	−50.34	−52	−52	$05·10^{-15}$	0.249
8	2.548	68.58	586.42	−50.41	−52	−52	$95·10^{-13}$	0.325
9	2.537	72.43	586.27	−50.24	−52	−52	$39·10^{-14}$	0.328
10	2.564	72.82	586.36	−50.30	−52	−52	$48·10^{-14}$	0.252
11	2.605	75.48	586.48	−50.13	−52	−52	$08·10^{-14}$	0.376
12	2.614	76.15	586.25	−50.15	−52	−52	$06·10^{-14}$	0.402
13	2.581	76.84	586.19	−50.19	−52	−52	$43·10^{-14}$	0.43
14	2.583	78.55	586.43	−50.09	−52	−52	$3.5·10^{-14}$	0.459
15	2.594	81.13	586.28	−50.03	−52	−52	$06·10^{-14}$	0.49
16	22.598	83.47	586.18	−50.89	−52	−52	$64·10^{-13}$	0.523
17	2.65	85.06	586.27	1.259	$85·10^{-12}$	$88·10^{-12}$	$55·10^{-13}$	0.558

Source: Developed by the authors using the compiled algorithm and the capabilities of the MATHCAD software system

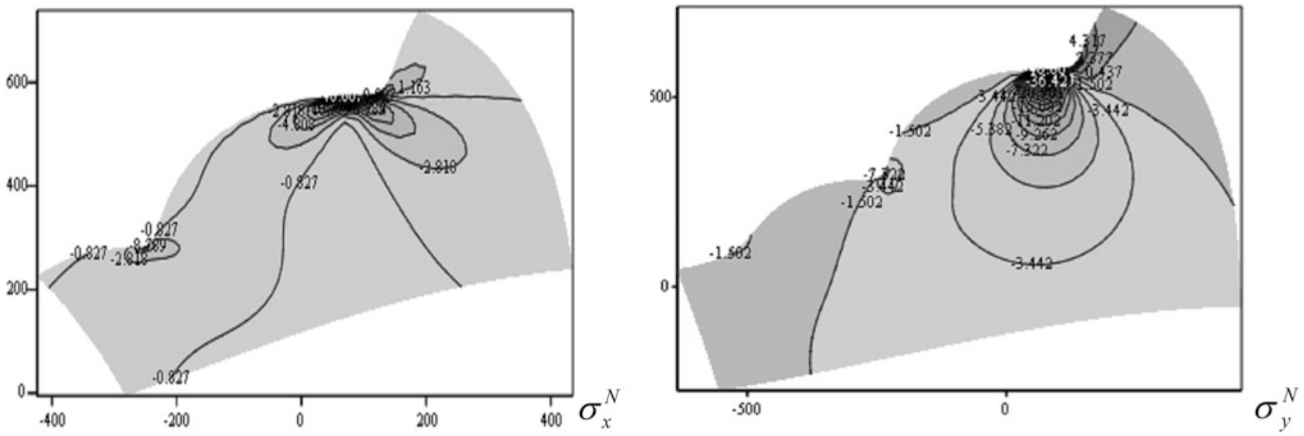

Fig. 2 Mountainside under the action of surface loads. *Source*: Developed by the authors using the compiled algorithm and the capabilities of the MATHCAD software system

Vertical relative deformations are everywhere under the action of tectonic forces, except for horizontal deformations. This method of determining and calculating the strain-stress distribution is much more economical and efficient than other methods.

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