

# **Integrating Geometric Metamodel-Assisted Process Assurance into Topology Optimization of Low-Pressure Die Castings**

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**Abstract.** Integrating complex process knowledge into structural optimization of casting parts enables design proposals to exploit manufacturing processes' full potential. However, a significant bottleneck for integrating process knowledge is the computational effort necessary for process simulations. In this article, we focused on low-pressure die casting. We used the medial axis transform and the shortest path algorithm to describe geometry-related features that we used as input data for a neural network metamodel, which replaced the casting process simulation. This allowed us to reduce the time for process simulation from multiple hours to a few seconds and, thus, incorporate the metamodel into the topology optimization framework. To reconstruct the geometry, we used an implicit modeling approach in which the modifed geometry was built from volume lattices fltered afterward to obtain solid volumes. The approach was tested on two application examples and proved that the metamodel-based results are equivalent to the results obtained using casting process simulations.

**Keywords:** Topology Optimization · Process Assurance · Medial Axis Transform · Neural Networks · Implicit Modelling

# **1 Introduction**

Two key pillars for developing new products are climate neutrality and energy efficiency. Casting processes have the capability to combine both pillars advantageously by fabricating complex structures at high volumes  $[1, 2]$  $[1, 2]$  $[1, 2]$  $[1, 2]$ . Accordingly, we can derive

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two main objectives, on the one hand lightweight structures that minimize material usage, respectively energy consumption during the product life cycle. On the other hand, efficient and error-free manufacturable geometries, which minimize the scraprate. However, the search for a "perfect part" that combines both objectives is difficult. A major bottleneck is the integration of time-consuming process assurance simulation into a topology optimization framework. In this article, we integrate the process assurance via a geometric feature-based artifcial neural network metamodel and modify the geometries using implicit modelling.

#### **1.1 Background**

Topology optimization is a widely used method for numerical structural optimization with the purpose of identifying the best material distribution in a given design space [\[3](#page-8-2)[–6](#page-9-0)]. Nevertheless, optimized design proposals are often non-manufacturable. Therefore, manufacturing constraints are commonly used, for casting processes, these are, among others, minimum length scale, symmetry, extrusion, or parting lines [[7–](#page-9-1)[9\]](#page-9-2).

These manufacturing constraints represent a simplifcation of the unconstrained optimization problem and cannot describe the limits of stable casting processes [[10\]](#page-9-3). Since the major bottleneck for the efficient integration of process assurance is the time-consuming process assurance simulation, the use of computationally cheap metamodels can leverage the quality of casting design proposals signifcantly.

Such metamodels are often built upon machine learning models, inspired by the idea that in particular artifcial neural networks represent universal approximators [[11\]](#page-9-4). Accordingly, machine learning is widely used in manufacturing related topics as energy consumption, wear modelling or tool wear prediction [\[12–](#page-9-5)[14\]](#page-9-6). In the context of casting processes, metamodels were applied to predict the results of 2D-casting simulations [\[15\]](#page-9-7), properties such as hardness based on process parameters [\[16\]](#page-9-8), or for increasing the product quality by optimizing initial temperature and wall temperature [[17\]](#page-9-9).

All the named examples apply machine learning algorithms to optimize process parameters or conditions but are not used within the context of adapting the part's geometry. Accordingly, for the objective of combining topology optimization with process assurance, well suited machine learning models are not yet identifed. The task of searching a well-suited machine learning model for a new problem is diffcult, since selecting algorithms and adjacent hyperparameters can be extremely costly. Automated machine learning systemizes the searching or selecting procedure [\[18](#page-9-10), [19\]](#page-9-11). An efficient method for a neural architecture search system provides the work of Jin et al., where the search space is explored via morphing the neural network architectures guided by a Bayesian optimization algorithm [\[19\]](#page-9-11). Automated machine learning approaches do not guarantee to outperform human guided modelling, but they can provide a helpful assistance tool in the modelling process.

#### **1.2 Approach**

The objective of this article is to use a geometric feature-based metamodel to replace the casting simulation within a topology optimization process assurance framework, as we show in Fig. [1.](#page-2-0)



<span id="page-2-0"></span>**Fig. 1.** Metamodel assisted framework for combined topology optimization and process assurance based on the workflow presented in  $[10]$  $[10]$ . A topology optimization with an initial Volume *V*<sub>0</sub> and a target Volume *V*<sub>t</sub> =  $\mu$ *V*<sub>0</sub> with volume constraint  $\mu$  and step length  $\lambda$  is conducted parallel to a casting simulation for process assurance. The latter is replaced by the metamodel (shaded in grey).

In this article, we replace the time-consuming casting simulation by a geometric feature-based metamodel for process assurance. Subsequently, we use implicit modelling to modify the design proposals based on the given evaluation criterion along with the topology optimization results. We apply this approach to a cantilever beam and a traverse link, shown in Fig. [2,](#page-2-1) and compare the modifed design proposals with simulation-based geometry modifcation.



<span id="page-2-1"></span>**Fig. 2.** Overview of the design space (green) and optimized design proposals without integrated process assurance (blue) for the two test examples. (a) Cantilever beam; (b) Traverse link. The load case is sketched in red on the design spaces, while the ingate position is marked orange on the design proposals.

# **2 Materials and Methods**

#### **2.1 Metamodel Architecture**

In this article, we used artifcial neural networks as metamodels. For preparing the data, building, optimizing and evaluating the metamodel, we used the python libraries numpy [[20\]](#page-10-0), seaborn [\[21](#page-10-1)], pandas [[22,](#page-10-2) [23\]](#page-10-3), matplotlib [\[24](#page-10-4), [25](#page-10-5)], scikit-learn [\[26](#page-10-6)], scikit-optimize [[27\]](#page-10-7), TensorFlow[[28,](#page-10-8) [29\]](#page-10-9), Keras [\[30](#page-10-10), [31\]](#page-10-11) and Auto-Keras [[19\]](#page-9-11). Figure [3](#page-3-0) shows the general architecture of the artifcial neural network.



<span id="page-3-0"></span>**Fig. 3.** Architecture of the baseline feed forward multilayer perceptron. Two geometric input features are used to estimate the solidifcation time in low-pressure die castings.

The baseline model was a dense feed forward multilayer perceptron neural network. We optimized the architecture and the hyperparameters using a two-step automated machine learning approach. The output parameter is the solidifcation time, which we obtained from previously made casting simulations. As geometric input features, we used shortest path distances, which provide the shortest distance between the ingate and any point in the geometry, and the radii of the medial axis points obtained by the medial axis transform, which represent the part thickness along the solidifcation path. The calculation of both geometric input features is shortly explained in the following.

**Medial Axis Transform.** The medial axis transform is a surface skeletonization technique for three dimensional objects, which is calculated from the Voronoi diagram. Every point on the medial axis is associated to an internal ball that touches the part's surface but does not penetrate it [[32\]](#page-10-12). The radii of the balls provide therefore the part thickness for every point on the medial axis. We used the implementation pre-sented in our previous work [[32\]](#page-10-12).

**DIJkstra's Shortest Path.** Dijkstra's shortest Path algorithm begins at a starting node and gradually selects the currently most favorable paths via the nodes on a graph that can be reached next [\[33](#page-10-13)]. We used the shortest path to calculate the shortest fow distances between the ingate and all points on the medial axis, thus providing qualitative information how fast a point can possibly be reached by the melt.

**Data Preparation.** For training and testing the models, we used a dataset containing 220,000 pairs of shortest path distance, radius of medial axis point, and solidifcation time. The data was split into 200,000 training and 20,000 test data.

**Automated Machine Learning.**The two-step automated machine learning approach consists of Bayesian optimization supported search for an well suited architecture and a subsequent hyperparameter optimization using the hyperband algorithm [\[34](#page-10-14)]. The objective for both optimization tasks was to minimize the mean squared error. During both optimizations, a 5-fold cross-validation was used on the training data. Table [1](#page-4-0) shows the selected parameters summarized.

<span id="page-4-0"></span>**Table 1.** Summary of the chosen settings for two-step automated machine learning. The objective function was in both cases the mean squared error (MSE).

	$\overline{AB}$ Algorithm	<b>Objective</b>	<i>I</i> Iterations
Architecture Search	Bayesian Optimization   MSE		100
Hyperparameter Optimization   Hyperband		<b>MSE</b>	250

#### **2.2 Evaluation of Process Assurance**

The process assurance is evaluated using the evaluation criterion developed in our previous work for low-pressure die castings [\[10](#page-9-3)]. By assuming a directional solidifcation in low-pressure die casting, the ratio of the solidification time  $(t_{sol})$  and the shortest path distance (*spd*) should be descending along the solidifcation path, which we evaluated using the following equation:

$$
QL = \log\left(\frac{t_{sol}}{spd}\right) \tag{1}
$$

#### **2.3 Modifcation of the Design Space**

To modify the design spaces, we used the software nTopology (Version 3.35, nTopology Inc., New York, NY, USA) that allows an implicit representation of geometries. Based on the density values of the topology optimization, we frst create a volume lattice for activated (high-density, material) and deactivated (low-density, holes) elements, which strut diameters are further adapted by the evaluation criterion point map of QL. We illustrate this procedure in Fig. [4.](#page-5-0)



<span id="page-5-0"></span>**Fig. 4.** Volume lattices for (a) activated elements and (b) deactivated elements. The colored point map represents the evaluation criterion's value on each cell of the design proposal.

This procedure allows further the fltering of geometric features according to the lattice density. Subsequently, the activated geometry and the deactivated geometry are fltered, smoothed, and merged, as we show in Fig. [5.](#page-5-1) The result is a modifed design proposal with increased manufacturability.



<span id="page-5-1"></span>**Fig. 5.** Illustration of the reconstruction process: (a) The fltered meshes for activated (grey) and deactivated (red) elements are merged. (b) Combined and smoothed geometry which represents the fnal design proposal.

# **3 Results and Discussion**

### **3.1 Metamodel Performance**

We show the results of the metamodel performance in Fig. [6](#page-6-0) by plotting the predicted solidification times ( $y_{predicted}$ ) against the true – or target – solidification times ( $y_{\text{tar}}$ get). For the cantilever the coefficient of determination  $(R^2)$  reached 0.79, while the



<span id="page-6-0"></span>**Fig. 6.** Results for the metamodel performance for (a) cantilever beam; (b) traverse link.

prediction for the traverse link reached 0.55. This result is plausible due to the cantilever's simpler geometry.

However, since the metamodel's objective is an acceptable estimation rather than a high-fdelity prediction, the achieved results show suffcient accuracy for both test examples.

#### **3.2 Evaluation of Modifed Design Proposals**

To evaluate the modifed design proposals, the QL was frst calculated using the predicted solidifcation times. In the next step the calculated QL modifes the geometries according to the process described in Sect. 2.3. In the following the fnal metamodel-based design proposals are compared to the simulation-based design proposals. Figure [7](#page-7-0) shows that for the cantilever beam the difference volume between metamodel and simulation design proposal is close to zero (Fig. [7b](#page-7-0)). Accordingly, the presented metamodel-based approach led to equivalent design proposals.



**Fig. 7.** Comparison of the fnal cantilever beam design proposals for (a) metamodel-based; (c) simulation-based. (b) represents the difference Volume between both design proposals.

<span id="page-7-0"></span>The comparison of the traverse link design proposals shows Fig. [8](#page-7-1). In this case deviations between simulated and predicted solidifcation were signifcantly greater, which also resulted in deviation of the modifed design proposals. In both design proposals, the reconstruction aimed to better connect the box with drill hole on the left side (Fig. [2](#page-2-1)b) to the rest of the geometry. Interestingly, the metamodel's deviations led to a design proposal, which – geometrically – looks superior the simulation-based proposal.



<span id="page-7-1"></span>**Fig. 8.** Comparison of the fnal traverse link design proposals for (a) metamodel-based; (c) simulation-based. (b) represents the difference Volume between both design proposals.

We attribute this effect to the evaluation criterion calculation on the activated elements and further extrapolation of the values on the deactivated elements. Accordingly, we conclude that also for the traverse link the metamodel resulted in an at least equivalent design proposal, though a smaller coeffcient of determination.

## **4 Summary**

In this article, we presented an incorporation of process assurance evaluation into a topology optimization framework, using geometric feature-based metamodel. While building the metamodel, we used a two-stage automated machine learning approach which helped to systematically exploit the parameter space and, thus, identifying a well-suited model architecture and hyperparameters.

The built metamodel reduced the needed time for process assurance evaluation from multiple hours to a few seconds while maintaining a suffcient prediction accuracy. The automated geometry reconstruction further mitigated weak points in the design proposal provided by the isolated topology optimization.

Therefore, our presented approach realizes an effcient incorporation of process assurance into structural optimization and fosters its further dissemination and exploitation of lightweight potentials.

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