A Covariance Matching-Based Adaptive EKF for Nanosatellite Attitude Estimation

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Nomenclature

ADCS	Attitude determination and control subsystem
COTS	Commercial off the shelf
EKF	Extended Kalman filter
FFs	Fading factors
MSFs	Multiple scaling factors
SSF	Single scaling factor

1 Introduction

The small satellite industry is developing at a faster pace day by day and is conducive to more people and companies to conduct work in the field of space technologies. Although they attract a lot of attention due to their low cost and short development time, small satellites also bring some engineering challenges with them. Cheap and commercial off the shelf (COTS) components used to reduce the cost increase the risk of a system malfunction, and attitude determination and control subsystem (ADCS) is one of subsystems that suffers most from this risk (Tafazoli, 2008). Any fault in attitude-related sensors can reduce the accuracy of the attitude estimation algorithm and, as a result, may cause incorrect control actions to be taken, which can lead to fatal consequences. Therefore, this study specifically addresses the problem of inaccurate attitude estimation in case of sensor faults. This problem

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becomes more important especially for nanosatellites, because being strictly constrained in size and mass makes conventional solutions, such as hardware redundancy impractical.

Attitude estimation methods for a nanosatellite can be divided into two main categories: single-frame and filtering algorithms (Hajiyev & Soken, 2021). Single-frame algorithms use vectors measured by attitude sensors (sun sensors, magnetometers, etc.) in the body frame and corresponding reference vectors in the reference frame (sun direction, magnetic field, etc.). The goal is finding the transformation (attitude) matrix between these two frames. Many different single-frame algorithms have been developed to date (TRIAD, q-method, QUEST, SVD, etc.), and extensive studies explaining and comparing these methods can be found in the literature (Cilden & Hajiyev, 2014). On the other hand, filtering methods use satellite's mathematical model in addition to measurements. Thus, even if there is no measurement available, attitude still can be estimated. Filtering algorithms, especially Kalman filtering, have been used in satellite attitude estimation for many years and it is possible to find many studies in the literature on this subject (Lefferts et al., 1982). However, since satellite's mathematical model and some attitude sensor measurement models (e.g. magnetometer) are nonlinear, the attitude estimation process requires nonlinear filtering which inherently increases the computational load. This is particularly undesirable for nanosatellites. In order to cope with this problem, filtering algorithms can be integrated with single-frame algorithms (Hajiyev & Bahar, 2003). These integrated algorithms make the measurement model for the filter linear and, thus, reduce the computational load. Integrated algorithms also give better estimations than the estimations given by the individual algorithms.

Kalman filters can give poor estimations in case of uncertainties and malfunctions. These possibilities should be considered in the design process and the designed filter should be robust against the changing conditions and compensate for the faults. In the literature, these type of Kalman filters are known as adaptive Kalman filters, and there are different techniques to make a Kalman filter adaptive such as multiple model adaptive estimation, join state and covariance estimation, autocorrelation, and covariance matching techniques (Hajiyev & Soken, 2021). Covariance matching techniques are one of the most widely used techniques where the main idea is scaling the Q- or R-noise covariance matrix depending on the source of the fault using a single scaling factor (SSF) or multiple scaling factors (MSFs).

In this study, an integrated adaptive TRIAD/R-adaptive extended Kalman filter (EKF) attitude estimation algorithm is presented where a single-frame algorithm (TRIAD) and a filtering algorithm (EKF) are integrated in order to take advantage of the good aspects of both. In the first step of the algorithm, TRIAD produces an initial attitude estimation. In the second step, this estimation is given to the EKF as input, and after the filtering process, final estimation is obtained. The proposed integrated algorithm is made adaptive via two different covariance matching techniques, using SSF and MSFs, and they are compared with a simulation.

2 Adaptive Kalman Filter via Covariance Matching

The TRIAD (Shuster & Oh, 1981) and conventional EKF algorithms (Lefferts et al., 1982) are well explained in other studies and will not be repeated here for the sake of brevity. However, adaptive EKF algorithms via covariance matching techniques differ from the traditional EKF at several points. Following subsections explain the R-adaptive EKF routines using SSF and MSFs.

2.1 *R-Adaptive EKF with a Single Scaling Factor*

The R-adaptive EKF with an SSF scales the measurement noise covariance matrix, R, in case of faults using a calculated SSF. SSF is calculated by comparing the real and theoretical innovation covariance matrices. The trace of these covariance matrices is matched such that (Hajiyev, 2007)

$$tr\left[e(k)e^T(k)\right] = tr\left[H(k)P(k|k-1)H^T(k) + \mathrm{SSF}(k-1)R(k)\right] \quad (1)$$

where $e(k)$ is the EKF innovation sequence, SSF$(k-1)$ is the introduced SSF, and $tr[\cdot]$ is the trace of the related matrix. P, H, and R matrices are the classical Kalman filter matrices and known as estimation error covariance matrix, observation matrix, and measurement noise covariance matrix, respectively. Solving the Eq. (1) for the SSF$(k-1)$, the following is obtained:

$$\mathrm{SSF}(k-1) = \frac{e^T(k)e(k) - tr[H(k)P(k|k-1)H^T(k)]}{tr[R(k)]} \quad (2)$$

Then, using the calculated SSF, the diagonal R matrix element corresponding to the faulty measurement channel is adjusted as

$$R_{\mathrm{adjusted}_{j,j}}(k) = \mathrm{SSF}(k-1)R_{j,j}(k) \quad (3)$$

2.2 *R-Adaptive EKF with Multiple Scaling Factors*

For the case of R-adaptive EKF with MSFs, the real and theoretical innovation covariance matrices are matched such that (Hajiyev & Soken, 2021)

$$\frac{1}{\xi}\sum_{k=m-\xi+1}^{m}\boldsymbol{e}(k)\boldsymbol{e}^T(k) = H(k)P(k|k-1)H^T(k) + \mathrm{MSFs}(k-1)R(k) \tag{4}$$

where ξ is the window size, namely the number of measurements that will be considered, and $MSFs(k-1)$ is the scaling matrix that contains scaling factors for each measurement channel. Solving Eq. (4) for the $\mathrm{MSFs}(k-1)$ gives

$$\mathrm{MSFs}(k-1) = \left\{\frac{1}{\xi}\sum_{k=m-\xi+1}^{m} e(k)e^T(k) - H(k)P(k|k-1)H^T(k)\right\}R^{-1}(k) \tag{5}$$

After calculating the scaling factor matrix, Kalman gain now can be adjusted as

$$K(k) = \frac{P(k|k-1)H^T(k)}{H(k)P(k|k-1)H^T(k) + MSFs(k-1)R(k)} \tag{6}$$

In the literature, it is possible to find slightly different algorithms to find the scaling factor matrix. In Kim et al. (2015), authors present an R-adaptive EKF for various types of global navigation satellite system (GNSS) faults using what they call fading factors (FFs). Although the fundamental idea is the same, in their study they calculate the scaling matrix in a slightly different way as

$$\mathrm{FFs}(k-1) = \begin{bmatrix} FF(1) \\ FF(2) \\ \vdots \\ FF(N) \end{bmatrix} = \max\left(1, \frac{\mathrm{diag}\left(\widehat{C_k}\right)}{\mathrm{diag}(C_k)}\right) \tag{7}$$

where $\max(\cdot)$ function gives the higher value inside of it and $diag(\cdot)$ gives the diagonal elements of the related matrix. The term $\widehat{C_k}$ is the real innovation covariance and given as

$$\widehat{C_k} = \frac{1}{\xi - 1}\sum_{k=m-\xi+1}^{m}\boldsymbol{e}(k)\boldsymbol{e}^T(k) \tag{8}$$

Note that, in this version, the term $\xi - 1$ is used instead of ξ. The term C_k is the theoretical innovation covariance and similar to Eq. (4), it is given as

$$C_k = H(k)P(k|k-1)H^T(k) + R(k) \tag{9}$$

After the scaling matrix FFs is calculated, Kalman gain is now updated as

$$K(k) = \frac{P(k|k-1)H^T(k)}{FFs(k-1)[H(k)P(k|k-1)H^T(k) + R(k)]} \tag{10}$$

Note that, in this version, the whole denominator of the Kalman gain is scaled using the scaling matrix rather than scaling only the R matrix.

3 Simulation Results and Discussion

In order to compare the performance of the R-adaptive EKF with SSF, MSFs, and FFs, a simulation is performed where the measurement noise of the x-axis magnetometer is increased 100 times at the 3500th second of the simulation and this effect is maintained for 100 s. Figure 1 shows the performance of the conventional EKF, R-adaptive EKF with SSF, MSFs, and FFs. As can be seen from Fig. 1, with the introduction of the noise increment, the tracking performance of the conventional EKF reduces significantly and it starts to give unreliable attitude estimations. On the other hand, R-adaptive EKFs continue to follow the ground truth. Since the results of the MSFs and FFs are very similar, they overlap with each other and MSFs results are not visible.

Apart from Fig. 1, Table 1 shows the root mean square error (RMSE) values of the conventional EKF, R-adaptive EKF with SSF, MSFs, and FFs. The performance

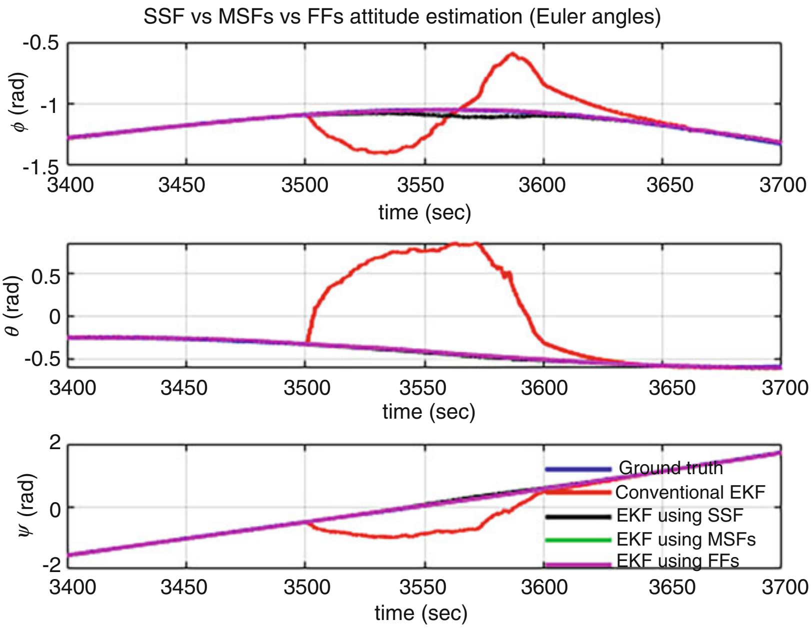

Fig. 1 Attitude estimation results in case of noise increment in the x-axis magnetometer

Table 1 RMSE values of standard and R-adaptive EKF with SSF, MSFs, and FFs during magnetometer noise increment

Euler angle (rad)	Standard EKF	SSF	MSFs	FFs
ϕ (roll)	0.274645	0.035006	0.002552	0.002567
θ (pitch)	0.997492	0.006021	0.005015	0.004972
ψ (yaw)	0.712692	0.026243	0.005962	0.005931

of each algorithm can be seen more clearly in Table 1. Regardless of the method, R-adaptive EKFs improve the results significantly. In addition, the superiority of the MSFs and FFs algorithms over the SSF algorithm is obvious. However, although the MSFs and FFs algorithms are slightly different, no visible difference is observed between these two algorithms.

4 Conclusion

In this study, three different integrated TRIAD/R-adaptive EKF algorithms are presented. TRIAD algorithm is used as the first-phase attitude estimation algorithm and obtained estimation is given to the R-adaptive EKF as input. As the adaptive method, covariance matching technique is chosen with single scaling and multiple scaling factors. Also, two different algorithms are presented for the multiple scaling approach. In order to verify the performance of the proposed algorithms, one simulation is performed where the x-axis magnetometer noise is increased 100 times. Simulation result shows that while the tracking performance of the conventional EKF algorithm decreases significantly, proposed R-adaptive EKF algorithms maintain their tracking performance and continue to give reliable attitude estimations. In addition, while it is observed that MSF and FF approaches give better results than the SSF approach, no significant difference is observed between the MSFs and FFs algorithms.

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