Critical Section for Shear Strength of Prestressed Concrete Transfer Beams

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Abstract. Prestressed concrete is widely used for the construction of long-span structures subjected to large loads. Shear strength estimations of prestressed concrete members have been the interest of many researchers. In this study, the effect of tendon profile on the shear capacity of transfer girders is studied. A parametric study is carried out to determine the critical section for the shear design of prestressed transfer girders with draped tendons. Results show that in continuous post-tensioned transfer girders, shear failure occurs close to the concentrated load transferred within the exterior span and the variations in the tendon profile affect the shear capacity of sections around the supports.

Keywords: critical section · *fib* Model Code · prestressed transfer girders · shear strength · shear design

1 Introduction

Prestressed transfer girders offer efficient solutions to support vertical members that do not continue at lower levels of high-rise buildings. During the last decade, advances in post-tensioning techniques and materials have increased the use of post-tensioned members by structural engineers. By utilizing the load balancing concept of prestressing, it is possible to design slender transfer girders. Control of creep deflections and cracking are some of the other reasons to choose post-tensioned girders when transferring high-rise columns.

Figure 1a shows a two-span transfer girder, with a profiled tendon, which supports column loads at midspans. Ideally, the prestressing profile of a post-tensioned girder transferring a column should be a harped profile to fully balance the loading from the column load. However, harped tendons with sharp deviations are not suitable for field installation especially when the girder is continuous [1]. Therefore, post-tensioned transfer girders generally have tendons with parabolic profiles.

As seen from Fig. 1a, the structural analysis system of multi-span post-tensioned girders is generally idealized as a continuous beam with pinned supports and the centre of gravity of tendons at the ends of the member is arranged such that it coincides with the centroid of the gross cross-section in order not to transmit any moments to the supporting columns. It must be noted that since continuous post-tensioned transfer girders are statically indeterminate, significant restraint actions occur due to prestressing and must be

accounted for when calculating the shear capacity of such members according to the *fib Model Code 2010* [2]. If the post-tensioned girder is transferring many stories, the column loads to balance may be very large and therefore a large effective prestressing force, F_p, is needed. Consequently, restraint actions due to prestressing become substantial if the girder is transferring a high-rise column.
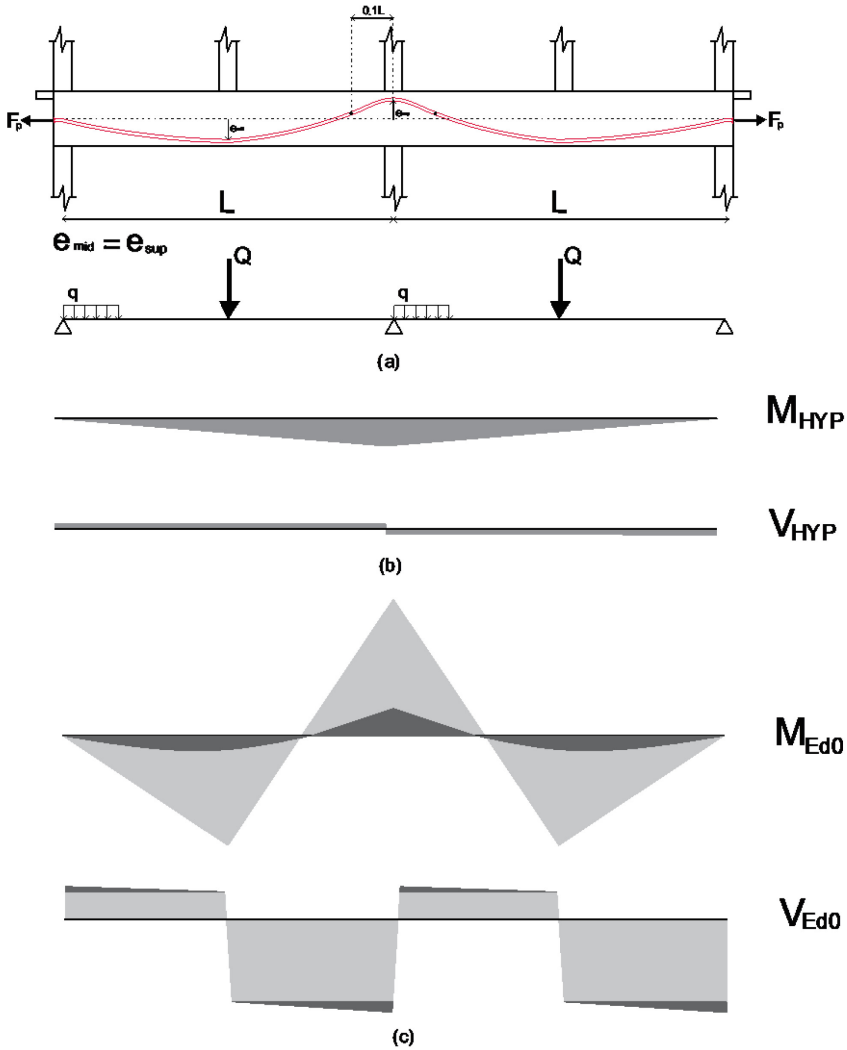

Fig. 1. Two-span post-tensioned transfer girder: (a) Tendon profile and statical system; (b) moments and shear forces resulting from statical indeterminacy; (c) moments and shear forces resulting from applied column loads, floor loads and self-weight of the girder.

fib Model Code 2010 [2] defines the control section for shear design at a location d away from the face of the supports and from discontinuities of geometry or applied loads, where d, the effective depth, is the distance from the centroid of the tensile reinforcing steel and tendons to the extreme compressive fibre.

Post-tensioned beams generally have non-prestressed reinforcement, as well and usually the control section for shear design is taken at a distance d_s away from the face of the concentrated load or supports, where d_s is the effective depth of the mild reinforcement [3]. In post-tensioned transfer girders, the effective depth changes along the span and around the transferred columns since the tendons are draped to effectively balance the applied column loads. Moreover, in post-tensioned girders the primary forces due to prestressing change with the draping of tendons and must be included in determining the design forces. It is well known that the LoA III sectional shear design procedures in the Model Code 2010 [2] depend on the section's stress conditions which in turn depend on the magnitude of the design forces. It is therefore questionable whether sections other than d_s may be critical in post-tensioned transfer girders with draped tendons. In this study, the shear capacities of post-tensioned transfer girders are investigated to confirm the control section for shear design.

2   Shear Strength Calculations According to *fib* Model Code

fib Model Code 2010 [2] offers three levels of approximations for members with shear reinforcement. Herein, LoA III is used to estimate the shear strength of statically indeterminate post-tensioned girders with parabolic tendon profiles. In level III approximation, the shear resistance provided by concrete, $V_{Rd,c}$, with a compressive strength of f_{ck} and the shear resistance provided by stirrups, $V_{Rd,s}$, with a design yield strength of f_{ywd}, must be greater that the design shear force V_{Ed} as given below.

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} \geq V_{Ed}$$

$$V_{Rd} = k_v \frac{\sqrt{f_{ck}}}{\gamma_c} b_w z + \frac{A_{sw}}{s_w} z\, f_{ywd} \cot\theta$$

$$(1)$$

In Eq. (1) A_{sw} is the cross-sectional area of shear reinforcement and s_w is the spacing of shear reinforcement. For a prestressed member, the effective shear depth z is calculated as given below.

$$z = \frac{z_s^2 A_s + z_p^2 A_p}{z_s A_s + z_p A_p} \tag{2}$$

The k_v term, which is used to calculate the shear strength provided by concrete is given by Eq. (3).

$$k_v(III) = \frac{0.4}{1 + 1500 e_x}\left(1 - \frac{V_{Ed}}{V_{Rd,max}(\theta_{min})}\right) \tag{3}$$

In Eq. (3), $V_{Rd,max}(\theta_{min})$ corresponds to the crushing of the struts and is calculated as below.

$$V_{Rd,max}(\theta_{min}) = k_c \frac{f_{ck}}{\gamma_c} b_w z \, \sin\theta_{min} \, \cos\theta_{min} \tag{4}$$

The minimum inclination of the struts, θ_{min}, is given in Eq. (5).

$$\theta_{min} = 20^\circ + 10000\,\epsilon_x \tag{5}$$

When calculating $V_{Rd,max}(\theta_{min})$, a strength reduction factor, $k_c = k_\epsilon\,\eta_{fc}$, is included to consider the reduction in crushing strength of concrete. This reduction factor accounts for the effects of the brittleness of the concrete for compressive strengths greater than 30 MPa, and the strain state in the web of a section, which is significant in prestressed members. The brittleness factor and the strain effect factor are given in Eq. (6).

$$\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 1 \quad (f_{ck} \text{ in MPa})$$

$$k_\epsilon = \frac{1}{1.2+55\epsilon_1} \leq 0.65 \tag{6}$$

The strain effect is based on the principal strain $\epsilon_1 = \epsilon_x + (\epsilon_x + 0.002)\cot^2\theta$, which is defined by a Mohr's circle of strain. For prestressed members, the longitudinal strain, ϵ_x, at the mid-depth of the effective shear depth is given as follows:

$$\epsilon_x = \frac{\frac{M_{Ed}}{z} + V_{Ed} + N_{Ed}\frac{z_p - e_p}{z}}{2\left(\frac{z_s}{z}E_s A_s + \frac{z_p}{z}A_p A_p\right)} \tag{7}$$

In Eq. (7) the sectional forces M_{Ed}, V_{Ed} and N_{Ed} including prestressing effects are calculated as:

$$
\begin{aligned}
M_{Ed} &= M_{Ed0} + \left(\pm F_p \cos\delta_p(e_p) + M_{P,ind}\right) \\
N_{Ed} &= N_{Ed0} - F_p \cos\delta_p \\
V_{Ed} &= V_{Ed0} - F_p \sin\delta_p
\end{aligned}
\tag{8}
$$

$M_{P,ind}$ is the secondary moment resulting from static indeterminacy and F_p is the effective prestressing force which can be assumed to be constant along the span for the sake of simplicity. The distance measured from the centroid of the gross cross-section to the centroid of prestressing steel, e_p is known as the eccentricity of prestressing and δ_p is the inclination of the tendon with respect to the horizontal axis of the girder. The secondary shear force $V_{P,ind}$ is included in determining the design shear force only if it is increasing the magnitude of V_{Ed} (Fig. 1b).

Herein, shear capacity assessments of post-tensioned girders with stirrups are performed for a given flexural and shear reinforcement configuration to locate the control section for shear design. Therefore, the inclination θ is taken equal to θ_{min} as also demonstrated by Sigrist et al. [3]. To calculate the level III shear capacity of a prestressed section with shear reinforcement, an iteration process is necessary. The load that corresponds to the concentrated load applied by the transferred column is increased such that $V_{Rd} = V_{Ed}$, where V_{Rd} is the least of $V_{Rd,max}(\theta_{min})$ given by Eq. (4) and $V_{Rd,c} + V_{Rd,s}$ given by Eq. (1).

3 Parametric Study and Results

In this study, two-span and three-span transfer girders with equal spans of 12 m are considered (Table 1). The transferred column load is increased to study the effect of large loads and corresponding large prestressing forces. The height of the girders is calculated such that the required average prestressing on gross area, F_p/A_g, to transfer the specified column loads is around 5 MPa, which is a good practical value to avoid bursting at anchorage zones and to control restraint deformations in the surrounding structural members. The width of the girders is selected as equal to the calculated height to accommodate anchor plates. Effective prestressing stress in the tendons after all losses is assumed as 1070 MPa. A minimum spacing of duct diameter is considered between the ducts and a minimum clear concrete cover of 0.8 diameter to ducts is considered.

Table 1. Properties of the transfer girders used in the parametric study.

#of spans - H	L (m)	HxB (mxm)	L/H	$Q_{service}$ (kN)	F_p (kN)	$e_{mid} = e_{sup}$ (mm)
2–1100	12	1.1 × 1.1	9.2	2000	6292	446
2–1400	12	1.4 × 1.4	8	4000	9437	596
2–1800	12	1.8 × 1.8	6	8000	14381	716
3–1100	12	1.1 × 1.1	9.2	2000	7190	446
3–1400	12	1.4 × 1.4	8	4000	10336	583
3–1800	12	1.8 × 1.8	6	8000	16178	716

For all girders, a concrete strength of $f_{ck} = 40$ MPa is used. Table 1 gives the service loading, prestressing and dimensions of the post-tensioned transfer girders in the parametric study. The girders' cross-sectional dimensions given in the third column of Table 1, are determined considering service stress limits and reasonable average prestressing values for the increasing transferred loads. Since the span length is kept constant, the slenderness of the girders decreases as the transferred load increases (Table 1). In addition to prestressing steel, mild reinforcement A_s is provided at the top and bottom of the girder section throughout all spans without curtailment. The area of mild reinforcement is calculated such that the moment resistance provided by only mild reinforcement is greater than the applied moment due to the self-weight of the girder and the slab within the tributary width of the girder, except for transfer girders 2–1800 and 3–1800. For these girders, the required prestressing forces are substantial and therefore it is often necessary to apply stage prestressing. It is preferable to grout ducts once stressed; however, in stage prestressing, grouting of ducts is usually carried out after all stressing stages are completed and the tendons will be temporarily unbonded before the last prestressing stage when the transfer girder is supporting, say half of the transferred floors. Herein, it is assumed that the required prestressing force is applied in two stages and the area of mild reinforcement is calculated such that the resistance provided by only mild reinforcement is sufficient to support the self-weight of floors existing at the beginning

of the final prestressing stage before the rest of the building is constructed. The shear reinforcement ratio of the girders, A_{sw}/s_w is determined using the level II approximation and considering two sections, d_s away from the face of the supporting column and the transferred column, where d_s is the effective depth of the provided mild reinforcement. The spacing of the calculated shear reinforcement is lowered to half within the distance H from the centre of the transferred and supporting column, where H is the height of the beam. The variation of the stirrup ratio can be assumed to be linear as shown in Fig. 2.
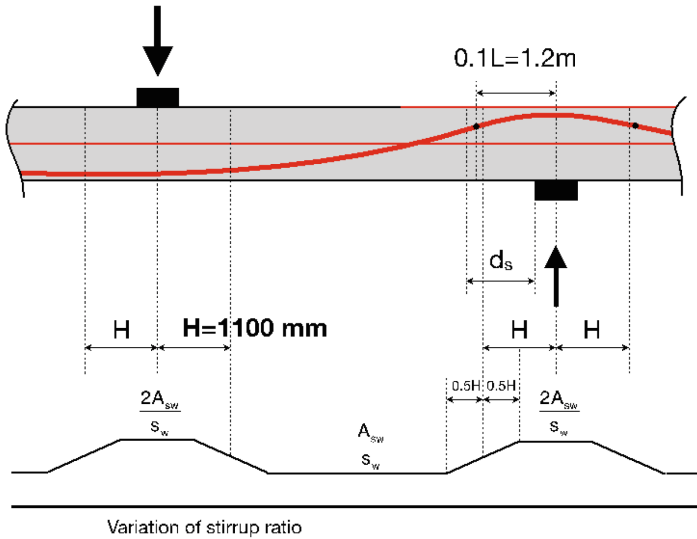

Fig. 2. Location of sections, variation of stirrup ratio and location of inflection point for reversed tendon profile at supports of transfer girders 2–1100 and 3–1100.

The hyperstatic forces due to prestressing (Fig. 1b) are determined by the method given in [4]. When calculating the tendon profile and the resulting hyperstatic actions, it is assumed that the inflection point of the tendons is arranged at 0.1L away from the centre of supports and the tendons do not have any inflection point at the ends of the girder (Figs. 1a and 2).

It can be understood from Fig. 1b that the critical section is located between the applied load and the first inner support, where the applied shear force is highest. The shear strength of the member is calculated at several sections around the applied column load and the first inner support to locate the most critical section for shear design (Fig. 2). Table 2 gives the results of this analysis. In Table 2, Q_d is the applied design load and $Q_{failure}$ is the load at which the first sectional failure occurred as per LoA III shear design procedures.

Table 2. Shear strength and sectional forces of sections at failure as per MC2010 LoA III.

#of spans - H	Q_d (kN)	$Q_{Failure}$ (kN)	M_{Ed}/V_{Ed} (kNm/kN) @d_s from face of column	V_{Rd} (kN) @ d_s from face of column	M_{Ed}/V_{Ed} (kNm/kN) @ H from support centre	V_{Rd} (kN) @ H from support centre
2–1100	2625	2916	1213/1640	2070	−2392/1954	**1955**
2–1400	5250	6454	1768/3397	**3397**	−5086/1517	2689
2–1800	10500	12987	1425/6440	**6440**	−9156/3987	7169
3–1100	2625	3256	2180/1678	**1678**	−1161/2066	2260
3–1400	5250	6272	3155/2951	**2951**	−2113/972	4592
3–1800	10500	13265	4765/5792	**5792**	−3974/3066	8071

The design bending moment and shear forces at the failure section and at a section H away from the centre of the support, where the stirrup spacing is lowered to half, are also listed in Table 2 for discussion. It is important to note that all transfer girders reached their LoA III shear capacity at a load lower than their calculated flexural failure load. It is also observed that the critical section for shear design is about d_s away from the face of the concentrated column load at the exterior span for all 2-span and 3-span continuous girders except for transfer girder 2–1100. It can be understood from Table 2 that the design shear forces, V_{Ed}, are well below the calculated shear capacities, V_{Rd}, when a failure occurs at a section d_s away from the face of the transferred column, except for transfer girder 3–1100.

As mentioned above the critical section for transfer girder 2–1100 is calculated at H = 1100 mm away from the centre of the support. For this girder, section d_s away from the face of the support is 1355 mm away from the support considering a column width of 650 mm (Fig. 2). The location of the critical section for this girder can be explained by checking the location of the inflection point of the reversed parabola with respect to the height of the girder as depicted in Fig. 2. As the tendon is reversed to avoid a kink, the inclination of the tendon with respect to the horizontal axis of the girder, δ_p reduces and consequently the favourable contribution of the tendon force as included in Eq. (8) reduces dramatically. Moreover, for 2-span transfer girders, the bending moment at the support is always greater than the bending moment at the mid-span under concentrated loads. As a result, the LoA III failure section is calculated at a distance H away from the face of the support for the transfer girder 2–1100.

It is of interest to note that although the same tendon and stirrup layout exist for the transfer girder 3–1100, section H away from the centre of the support is not determined as critical according to LoA III shear design procedures. This can be explained by observing the associated M_{Ed} values in these sections. Contrary to 2-span girders, for a 3-span continuous girder under equal concentrated loads at midspans, the bending moment at the support is always lower than the bending moment at the mid-span. Consequently, the associated bending moment value at section H away from the centre of support is

lower than that of section d_s away from the face of the transferred column and failure occurs at d_s away from the face of the transferred column according to the LoA III shear design procedures.

4   Conclusions

The shear design of prestressed members is not as straightforward as the shear design of conventionally reinforced concrete members. Post-tensioned transfer girders are generally slender members and may be heavily loaded if transferring a high-rise column. Although prestressing adds an advantage to the shear strength of these members the design must be carried out carefully since transfer girders are instrumental to the integrity of the whole structure. Herein, a parametric study is carried out to locate the critical section for the shear design of multi-span post-tensioned transfer girders using the *fib* MC2010 LoA III shear assessment procedures and to understand the variation of shear capacities versus demands of different sections along the entire length of post-tensioned transfer girders with draped tendons. It is observed that the variations in the tendon profile affect the shear capacity of sections around the supports, especially for 2-span continuous girders. Experimental studies must be carried out to develop the insight on the shear behaviour of multi-span post-tensioned transfer girders subjected to concentrated loads.

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