Measuring Corporate Gender Diversity and Inclusion with UW-TOPSIS and Linguistic Intervals

Vicente Liern and Blanca Pérez-Gladish

Abstract The objective of this work is to propose fuzzy adequacy indicators to measure the degree of gender diversity in firms. The construction of these indicators will be based on an extension of the Unweighted Technique for Order of Preference by Similarity to Ideal Solution (UW-TOPSIS). This multiple criteria decision-making (MCDM) method simultaneously minimizes the distance to a positive ideal solution and maximizes distance to a negative ideal solution. The positive ideal solution is composed of the best value of each criterion, and the negative ideal solution is composed of the worst values of the decision criteria. The method provides a cardinal ranking of alternatives based on a relative proximity index to the positive ideal solution. In our proposal, the relative importance of the diversity and inclusion decision criteria will be described by means of linguistic labels which will be transformed into intervals on the real line. The main features and advantages of this approach will be illustrated with a real problem where a set of Finnish companies will be assessed based on their degree of adequacy in terms of gender inclusion and diversity.

Keywords MCDM · UW-TOPSIS · Linguistic intervals · Weights · Diversity · Inclusion · Ranking

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1 Introduction

A recently published review of academic research on the impact of diversity and inclusion (D&I) in the workplace has analyzed its relationship with business performance, risk management, and conduct outcomes (Whiting, 2021). The author, after a revision of 169 studies published by academic researchers, consultancies, government, and trade associations, concludes that gender diversity in senior leadership can be associated with positive financial performance, especially when there is a "critical mass", at least 30%, of women on board. However, there is no clear evidence of a causal relation. The evidence is stronger when the relation and causality are analyzed for board gender diversity and risk management. Almost all the 21 studies analyzed by Whiting (2021) find a positive relation between board gender diversity and risk management although only nine of them demonstrate a direct causality. Some of the studies argue that women are more risk averse than men and tend to be found on the boards of less risky firms. However, other authors argue that "this stereotyping does not hold for women who embark on a managerial career, especially in the case of financial services" (Whiting, 2021).

Firms with gender-diverse boards tend to be also more creative, innovative, and able to better solve problems (Torchia et al., 2011; Vafaei et al., 2020) causing a positive impact on their financial performance (Deszö & Ross, 2012; Richard et al., 2003; Whiting, 2021). Gender diversity seems also to have a positive impact on board meeting attendance and financial information transparency and disclosure (Whiting, 2021).

Few studies can be found trying to analyze the impact of gender inclusion on business performance. Although often used interchangeably with diversity, inclusion is a different concept. Diversity in the workplace means that firms employ a diverse team of workers reflecting the society in which the firm exists and operates. Inclusion goes beyond diversity, being defined by the Society for Human Resource Management as "(…) the achievement of a work environment in which all individuals are treated fairly and respectfully, have equal access to opportunities and resources, and can contribute fully to the organization's success" (SHRM, 2022). Due to the nature of this definition, measuring inclusion represents an important challenge. The literature review conducted by Whiting (2021) reveals a lack of consistent measurement data. This author concludes that most of the academic researchers use existing secondary diversity and inclusion data over conducting primary research. The data in most of the analyzed studies are rarely complete because some important variables and characteristics cannot be easily collected, meaning they cannot be included as control variables when analyzing the impact of diversity on performance. The demonstration of causality also requires long times data series which, for diversity and inclusion, are not available.

In addition, the existing diversity and inclusion indicators suffer from some methodological problems common to any overall performance measure. The construction of composite indicators implies several problems concerning collection of data, selection of criteria and individual indicators, normalization of the data, determination of the relative importance (weighting) of criteria and indicators, and aggregation and comparison of overall performance of the alternatives or options. In this work, we will focus on the problematic related to the determination of the criteria weights. Weights can be determined objectively or subjectively, depending on the characteristics of the real decision problem to be solved. Ouenniche et al. (2018) present a review of both types of weighting schemes highlighting the advantages and disadvantages of objective and subjective approaches. In general, the use of subjective weighting schemes is more controversial although is common in the context of TOPSIS-based approaches where decision-makers determine the relative importance of decision criteria based on their own experiences, knowledge, and perception of the problem. Several works including interesting reviews of subjective weighting methods are Barron and Barrett (1996) and Hobbs (1980) and more recently, Alemi-Ardakani et al. (2016), Eshlaghy and Radfar (2006) and Németh et al. (2019). The review of the literature shows that sometimes, the decision-maker cannot give consistent judgments under different weighting schemes and the weighting process itself is essentially context dependent (Watröbski et al., 2019). Therefore, determining reliable subjective weights is a difficult problem and can affect final decisions (Deng et al., 2000). The proposed method in this paper will show how it is possible to obtain similar results to those obtained by a well-known rating agency with a more general weighting scheme without the necessity of the a priori exact numerical establishment of the relative importance of the decision criteria. With this, we will avoid one of the most controversial questions in the construction of global or synthetic indicators.

Equileap is one of the leading EU gender diversity data providers. They research and rank more than 3500 public companies all over the world. Equileap evaluates firms based on 19 diversity and inclusion criteria organized into four main dimensions: gender balance in leadership and workforce, fair remuneration, policies promoting gender equality, and commitment, transparency, and accountability. This organization ranks firms based on a global diversity and inclusion score. Equileap does not provide public information about the relative importance given to the individual indicators and dimensions used to globally score the companies.

Our evaluation framework will rely on a multiple criteria decision analysis approach, Unweighted Technique for Order Preference by Similarity to Ideal Solution, and UW-TOPSIS developed by Liern and Pérez-Gladish (2022). This method allows us to consider the complex multidimensional character of decision problems avoiding some of the difficulties related to the determination of the relative importance of these multiple dimensions. The novel contribution of this paper is related to the type of required information regarding the importance of the decision criteria in the aggregation process leading to the ranking of the firms. In the UW-TOPSIS framework, weights are treated as unknown variables in the optimization problem which determines the worst and best possible relative proximity of each decision alternative to the positive ideal solution (PIS). In this work, we give the decision-maker the opportunity of assessing the importance of the decision criteria using linguistic terms that are transformed into numerical intervals included in the optimization problem.

In what follows we will present the main characteristics of the classical TOPSIS approach followed by a description of the UW-TOPSIS algorithm developed by Liern and Pérez-Gladish (2022). Once the methodological framework has been described, we will propose a fuzzy treatment of the weights expressing the relative importance of the decision criteria and we will incorporate this treatment into the UW-TOPSIS algorithm. In Sect. 3, a real case study will be presented. We will illustrate the proposed approach ranking a sample of Finnish companies based on their gender equality degree. Finally, in Sect. 4, the main conclusions of the work will be presented.

2 Unweighted TOPSIS with Linguistic Intervals

Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) proposed by Hwang and Yoon (1981) provides a ranking of decision alternatives simultaneously minimizing distance to a positive ideal solution (PIS) and maximizing distance to a negative ideal solution (NIS). The positive ideal solution is composed of the best value of each criterion, and the negative ideal solution is composed of the worst values of the decision criteria. The method provides a cardinal ranking of alternatives, and it is widely used due to its simplicity and nice properties allowing total linear compensation using a single criterion aggregation approach (Behzadian et al., 2012; Chen & Hwang, 1992; Yoon & Hwang, 1995).

As mentioned in the previous section, weights of the criteria in TOPSIS-based approaches are given by the decision-makers a priori in the first steps of the algorithm and maybe objective or subjective depending on the characteristics of the decision problem (Ouenniche et al., 2018; Watröbski et al., 2019). Numerical subjective weights, usually directly established by the decision-maker based on expert knowledge or subjective preferences, are difficult to be uphold, especially in public decision-making. In this work, we propose to handle subjective weights using linguistic terms. In what follows we will illustrate the selected procedure.

2.1 *Fuzzy Treatment of Decision Criteria Weights*

Let us define a linguistic evaluation scale as the following set

$$l_1 = \{s_\alpha : \alpha \in \{0, 1, \ldots, H\}\}, \tag{1}$$

verifying the following conditions (see Herrera & Martínez, 2000, 2001; Xu, 2004, 2012; Yager, 1995):

i. The set is ordered: $s_\alpha > s_\beta$ if $\alpha > \beta$;
ii. There is a negation operator: $\text{neg}(s_\alpha) = s_\beta$ such that $\beta = H + 1 - \alpha$;

iii. There are max and min operators: $\max(s_\alpha, s_\beta) = s_\alpha$ if $\alpha \geq \beta$, and $\min(s_\alpha, s_\beta) = s_\alpha$ if $\alpha \leq \beta$.

Let us consider the following collection of elements from l_1, $\{s_0, s_2, \ldots, s_p\}$, where $s_0 \leq s_2 \leq \ldots \leq s_p$. Then, following Xu (2004), it is possible to express the collection as an interval $[s_0, s_p]$.

Definition 1 (Xu, 2004). A linguistic interval evaluation scale can be defined as

$$l_2 = \left\{\tilde{s} = [s_\alpha, s_\beta] : \alpha \leq \beta,\ \ \alpha, \beta \in \{0, 1, \ldots, H\}\right\}. \tag{2}$$

The extension to a continuous scale of the previous sets is as follows (Herrera & Martínez, 2000, 2001; Xu, 2004):

$$L_1 = \{s_\alpha : \alpha \in [0, H]\}, \tag{3}$$

$$L_2 = \left\{\tilde{s} = [s_\alpha, s_\beta] : \alpha, \beta \in [0, H], \alpha \leq \beta\right\}. \tag{4}$$

Let us now consider a partition of the interval $[0, H]$ into H disjoint subintervals, $\overline{r}_\alpha,\ \alpha = 1, 2, \ldots, H$, such that $\sup \overline{r}_\alpha \leq \inf \overline{r}_{\alpha+1}, \alpha = 0, 1, \ldots, H-1$. If we make

$$T(s_\alpha) = \overline{r}_\alpha, \quad \alpha = 0, 1, \ldots, H, \tag{5}$$

we can transform each term of l_1 into an interval contained in $[0, H]$.

Based on Xu (2004), we can define in L_2 the following operation:

$$\lambda \otimes \tilde{s} = \lambda \otimes \left[s_\alpha, s_\beta\right] = \left[s_{\lambda\alpha}, s_{\lambda\beta}\right], \quad \lambda \in 0, 1, \tag{6}$$

and by its own construction,

$$\lambda \oplus \tilde{s} \in L_2, \quad \lambda \in [0, 1]. \tag{7}$$

2.2 *UW-TOPSIS with Fuzzy Weights*

In what follows we will present the steps of the new algorithm proposed in this paper which does not require the introduction of a priori precise weights. Figure 1 displays the steps in the classical TOPSIS model.

In the UW-TOPSIS approach, the PIS and NIS solutions are determined without consideration of the relative importance of the criteria. In Liern and Pérez-Gladish (2022), weights are introduced as unknown variables in Step 4 when separation

STEP 1. DETERMINE THE DECISION MATRIX D, where the number of criteria is m and the number of alternatives is n, $D = [x_{ij}]_{n \times m}$.

STEP 2. CONSTRUCT THE NORMALIZED DECISION MATRIX. Criteria are expressed in different scaling and therefore a normalizing procedure is necessary to facilitate comparison. Hwang and Yoon (1981) propose a vector normalization,

$$r_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{n} (x_{ij})^2}} \in [0,1], \qquad 1 \le i \le n, \qquad 1 \le j \le m. \tag{8}$$

STEP 3. DETERMINE THE WEIGHTED NORMALIZED DECISION MATRIX. It is well known that the weights of the criteria in decision making problems do not have the same mean and not all of them have the same importance. The weighted normalized value v_{ij} is calculated as:

$$v_{ij} = w_j r_{ij}, \qquad 1 \le i \le n, \qquad 1 \le j \le m, \tag{9}$$

where w_j is the weight associated to each criterion.

STEP 4. DETERMINE THE POSITIVE IDEAL (PIS) AND NEGATIVE IDEAL SOLUTIONS (NIS). The positive ideal solution, $A^+ = (v_1^+, \dots, v_m^+)$, and the negative ideal solution, $A^- = (v_1^-, \dots, v_m^-)$, are determined as follows:

$$v_j^+ = w_j r_j^+ = \begin{cases} \max\limits_{1 \le i \le n} v_{ij}, & j \in J \\ \min\limits_{1 \le i \le n} v_{ij}, & j \in J' \end{cases} \qquad 1 \le j \le m, \tag{10}$$

$$v_j^- = w_j r_j^- = \begin{cases} \min\limits_{1 \le i \le n} v_{ij}, & j \in J \\ \max\limits_{1 \le i \le n} v_{ij}, & j \in J' \end{cases} \qquad 1 \le j \le m, \tag{11}$$

where J is associated with the criteria that indicate profits or benefits and J' is associated with the criteria that indicate costs or losses.

STEP 5. CALCULATE THE SEPARATION MEASURES. Calculation of the separation of each alternative with respect to the PIS and NIS, respectively:

$$d_i^+ = \left(\sum_{j=1}^{m} (v_{ij} - v_j^+)^2 \right)^{1/2}, \qquad d_i^- = \left(\sum_{j=1}^{m} (v_{ij} - v_j^-)^2 \right)^{1/2}, \quad 1 \le i \le n. \tag{12}$$

STEP 6. CALCULATE THE RELATIVE PROXIMITY TO THE IDEAL SOLUTION. Calculation of the relative proximity of each alternative to the PIS and NIS using the proximity index.

$$R_i = \frac{d_i^-}{d_i^+ + d_i^-}, \qquad 1 \le i \le n. \tag{13}$$

Th R_i value lies between 0 and 1. If $R_i = 1$, then $A_i = A^+$ and if $R_i = 0$, then $A_i = A^-$. The closer the R_i value is to 1 the higher the priority of the *i-th* alternative.

STEP 7. RANK THE PREFERENCE ORDER. Rank the best alternatives according to R_i in descending order.

Fig. 1 Classical TOPSIS

measures from the PIS and NIS are calculated. Their values are determined in Step 5 solving two groups of mathematical programming problems which maximize and minimize the separation of each alternative to the PIS and NIS, respectively, considering different constraints referred to the values of the weights. These constraints include the classical constraint in TOPSIS approaches which ensures all the weights are positive and sum up one and other constraints imposing lower and upper bounds on the weights. The resulting mathematical programming problems are, due to the nature of their objective, fractional mathematical programming problems. Figure 2 displays the UW-TOPSIS algorithm with weights being unknown variables.

Remark 1 According to Canós and Liern (2008), given the intervals $A = [a_1, a_2]$ and $B = [b_1, b_2]$ contained in $\mathbb{R}$, we will say that A is bigger than B, if and only if

$$A \succ B \Leftrightarrow \begin{cases} k_1a_1 + k_2a_2 > k_1b_1 + k_2b_2, & k_1a_1 + k_2a_2 \neq k_1b_1 + k_2b_2 \\ a_1 > b_1, & k_1a_1 + k_2a_2 = k_1b_1 + k_2b_2 \end{cases}$$

where k_1 and k_2 are two pre-established positive constants. In the context that concerns us, the values k_1 and k_2 inform us about the degree of confidence of the decision-maker that the alternatives are in their best position or on the contrary (Canós & Liern, 2008). When ordering the intervals $\left[R_i^L, R_i^U\right]$, $1 \leq i \leq n$, the relation k_2/k_1 informs us about the importance (or truthfulness) given to the best situation of the alternatives R_i^U regarding of the worst situation R_i^L. In the following examples, since we do not have information that makes us opt for the best or worst situation, we have chosen to give the same importance to both, that is, $k_1 = k_2 = 1$.

Let now us assume a situation in which weights are given by the decision-maker using a linguistic interval evaluation scale as in (2) which in the continuous case takes the form

$$\tilde{W} = \left\{\tilde{w} = [w_\alpha, w_\beta] : \alpha, \beta \in [0, 1],\ \alpha \leq \beta\right\}. \tag{23}$$

To be able to use weights in (23) in the UW-TOPSIS method, it is necessary to establish some conditions:

Definition 2 A vector of weights $\tilde{w} = (\tilde{w}_1, \ldots, \tilde{w}_m) \in \tilde{W}^m$, whose components are intervals with linguistic valuations, is UWL feasible, if it belongs to the following set $\tilde{\Omega}$:

$$\tilde{\Omega} = \left\{\tilde{w} = (\tilde{w}_1, \ldots, \tilde{w}_m) \in \tilde{W}^m,\ \widetilde{w_j} = \left[w_{\alpha_j}, w_{\beta_j}\right],\right.$$
$$\left. 1 \leq j \leq m,\ \sum_{j=1}^{m} \alpha_j \leq 1,\ \sum_{j=1}^{m} \beta_j \geq 1 \right\} \tag{24}$$

Given a non-null vector $\tilde{w} = (\tilde{w}_1, \ldots, \tilde{w}_m) \in \tilde{W}^m$, we can obtain a vector of weights UWL feasible if

STEP 1. DETERMINE THE DECISION MATRIX $[x_{ij}]$, $1 \le i \le n$, $1 \le j \le m$, WHERE THE NUMBER OF ALTERNATIVES IS N AND THE NUMBER OF CRITERIA IS M.

STEP 2. CONSTRUCT THE NORMALIZED DECISION MATRIX

$$[r_{ij}],\ r_{ij} \in [0,1],\quad 1 \le i \le n,\ 1 \le j \le m. \tag{14}$$

STEP 3. DETERMINE THE POSITIVE IDEAL $A^{+} = (r_1^{+}, \dots, r_m^{+})$ AND THE NEGATIVE IDEAL SOLUTIONS

$A^{-} = (r_1^{-}, \dots, r_m^{-})$, WHERE

$$r_j^{+} = \begin{cases} \max\limits_{1\le i\le n} r_{ij}, & j \in J \\ \min\limits_{1\le i\le n} r_{ij}, & j \in J' \end{cases} \qquad 1 \le j \le m, \tag{15}$$

$$r_j^{-} = \begin{cases} \min\limits_{1\le i\le n} r_{ij}, & j \in J \\ \max\limits_{1\le i\le n} r_{ij}, & j \in J' \end{cases} \qquad 1 \le j \le m, \tag{16}$$

WHERE J IS ASSOCIATED WITH "THE MORE, THE BETTER" CRITERIA AND J' IS ASSOCIATED WITH "THE LESS, THE BETTER" CRITERIA.

STEP 4. LET US CONSIDER $\Omega = \left\{ w = (w_1, \dots, w_m) \in \mathbb{R}^m,\ w_j \in [0,1],\ \sum_{j=1}^{m} w_j = 1 \right\}$. GIVEN A^{+}, A^{-}, WE DEFINE TWO SEPARATION FUNCTIONS,

$$D_i^{+}: \Omega \times \mathbb{R}^m \longrightarrow [0,1], \qquad D_i^{-}: \Omega \times \mathbb{R}^m \longrightarrow [0,1], \quad 1 \le i \le n,$$

GIVEN BY

$$D_i^{+}(w) = d\big((w_1 r_{i1}, \dots, w_m r_{im}), (w_1 r_1^{+}, \dots, w_m r_m^{+})\big), \quad 1 \le i \le n, \tag{17}$$

$$D_i^{-}(w) = d\big((w_1 r_{i1}, \dots, w_m r_{im}), (w_1 r_1^{-}, \dots, w_m r_m^{-})\big), \quad 1 \le i \le n, \tag{18}$$

WHERE D IS A DISTANCE FUNCTION IN $\mathbb{R}^m$.

STEP 5. CALCULATE THE FUNCTION OF RELATIVE PROXIMITY TO THE IDEAL SOLUTION, $R_i: \Omega \longrightarrow [0,1]$, $1 \le i \le n$, AS

$$R_i(w) = \frac{D_i^{-}(w)}{D_i^{+}(w) + D_i^{-}(w)}, \quad 1 \le i \le n. \tag{19}$$

STEP 6. FOR EACH I, $1 \le i \le n$, WE CALCULATE THE VALUES $R_i^{L}(w), R_i^{U}(w)$ SOLVING THE TWO FOLLOWING MATHEMATICAL PROGRAMMING PROBLEMS WHERE DECISION VARIABLES ARE THE CRITERIA WEIGHTS:

$$R_i^{L} = \text{Min}\left\{ R_i(w), \sum_{j=1}^{m} w_j = 1,\ l_j \le w_j \le u_j,\ 1 \le j \le m \right\},\ 1 \le i \le n, \tag{20}$$

$$R_i^{U} = \text{Max}\left\{ R_i(w), \sum_{j=1}^{m} w_j = 1,\ l_j \le w_j \le u_j,\ 1 \le j \le m \right\},\ 1 \le i \le n, \tag{21}$$

BEING $l_j, u_j \ge 0$ LOWER AND UPPER BOUNDS FOR EACH CRITERION'S WEIGHT. THEN, WE OBTAIN N RELATIVE PROXIMITY INTERVALS,

$$R_i^{I} = [R_i^{L}, R_i^{U}], \quad 1 \le i \le n. \tag{22}$$

STEP 7. WE RANK THE INTERVALS $R_1^{I}, R_2^{I}, \dots, R_n^{I}$ (SEE REMARK 1).

Fig. 2 UW-TOPSIS

Proposition 1 *Given a vector* $\tilde{w} = (\tilde{w}_1, \ldots, \tilde{w}_m) \in \tilde{W}^m$, $\widetilde{w_j} = [w_{\alpha_j}, w_{\beta_j}]$, $1 \leq j \leq m$, *with at least one* $\beta_j \neq 0$, *it is possible to construct a vector* $\tilde{w}^* = \left(\tilde{w}_1^*, \ldots, \tilde{w}_1^*\right) \in \tilde{\Omega}$, *such that*

$$\widetilde{w_j} = \left[w_{\alpha_j}, w_{\beta_j}\right] \subseteq \tilde{w}_1^* = \left[w_{\alpha_j^*}, w_{\beta_j^*}\right], \quad 1 \leq j \leq m.$$

Proof We will give a constructive demonstration considering the conditions to belong to $\tilde{\Omega}$.

(a) If $\sum_{j=1}^{m} \alpha_j > 1$, applying (6), we can make

$$w_{\alpha_j^*} = \frac{1}{M \sum_{j=1}^{m} \alpha_j} w_{\alpha_j}, \quad 1 \leq j \leq m, \; M > 1, \tag{25}$$

verifying

$$w_{\alpha_j^*} \leq w_{\alpha_j}, \quad 1 \leq j \leq m. \tag{26}$$

(b) If $\sum_{j=1}^{m} \beta_j < 1$, as by hypothesis $\beta_{j_0} \neq 0$, we make

$$w_{\beta_{j_0}^*} = \frac{1}{\beta_{j_0}} w_{\beta_{j_0}} \quad \text{and} \quad w_{\beta_j^*} = w_{\beta_j}, \; j \neq j_0, \tag{27}$$

which verify

$$w_{\beta_j} \leq w_{\alpha_j^*}, \quad 1 \leq j \leq m. \tag{28}$$

Remark 2 Of course, the construction given in (27) and (28) is not the only way to demonstrate Proposition 1. On the other hand, Por otro lado, it is worth highlighting the value $M = 1$ in expression (25) because in this case, $\sum_{j=1}^{m} \alpha_j^* = 1$ and, as we will see in the next section, this makes it so that when weights are applied to a multi-criteria method (as in the case of TOPSIS), the values $[w_{\alpha_j^*}, w_{\beta_j^*}]$, $1 \leq j \leq m$ cannot be true intervals, but the values $w_{\alpha_j^*}$, $1 \leq j \leq m$.

Steps 1–3 will remain the same than in the UW-TOPSIS algorithm. However, the remaining steps in the algorithm will be transformed as follows:

Step 4. Given a weight UWL feasible $\tilde{w} \in \tilde{\Omega}$, we construct set

$$\Omega_{\tilde{w}} = \left\{ w = (w_1, \ldots, w_m) \in W^m, \; \alpha_j \leq w_j \leq \beta_j, \; \sum_{j=1}^{m} w_j = 1, \; 1 \leq j \leq m \right\}. \tag{29}$$

Given A^+, A^-, we define two separation functions,

$$D_i^+ : \Omega_{\tilde{w}} \times \mathbb{R}^m \to [0, 1], \quad D_i^- : \Omega_{\tilde{w}} \times \mathbb{R}^m \to [0, 1], \quad 1 \le i \le n.$$

Given by

$$D_i^+(w) = d\big((w_1 r_{i1}, \dots, w_m r_{im}), \big(w_1 r_1^+, \dots, w_m r_m^+\big)\big), \quad 1 \le i \le n, \tag{30}$$

$$D_i^-(w) = d\big((w_1 r_{i1}, \dots, w_m r_{im}), \big(w_1 r_1^-, \dots, w_m r_m^-\big)\big), \quad 1 \le i \le n, \tag{31}$$

where d is a distance function in $\mathbb{R}^m$.

Step 5. Calculate the function of relative proximity to the ideal solution, $R_i : \Omega \to [0, 1],\ 1 \le i \le n$, as

$$R_i(w) = \frac{D_i^-(w)}{D_i^+(w) + D_i^-(w)}, \quad 1 \le i \le n. \tag{32}$$

Step 6. For each i, $1 \le i \le n$, we calculate the values $R_i^L(w)$, $R_i^U(w)$ solving the two following mathematical programming problems where decision variables are the criteria weights:

$$R_i^L = \mathrm{Min}\{R_i(w),\ w \in \Omega_{\tilde{w}}\}, \quad R_i^U = \mathrm{Max}\{R_i(w),\ w \in \Omega_{\tilde{w}}\}. \tag{33}$$

Then, we obtain n relative proximity intervals

$$R_i^I = \big[R_i^L, R_i^U\big], \quad 1 \le i \le n. \tag{34}$$

Step 7. We rank the intervals R_1^I, R_2^I, …, R_n^I (see Remark 1).

Definition 3 We will call diversity and inclusion adequacy index (DIAI) of alternative i

$$\mathrm{DIAI}_i = \frac{R_i^L + R_i^U}{2}, \quad 1 \le i \le n. \tag{35}$$

In the next section, we will illustrate our method with a real example in a decision-maker establishing the importance of the diversity and inclusion criteria using linguistic terms. As we will see, these valuations will give rise to a linguistic interval expressing the importance of each criterion. Once these linguistic intervals are obtained, and after verification of the previously described properties, the weights will be integrated in the UW-TOPSIS algorithm, and a set of firms will be assessed in terms of their diversity and inclusion adequacy.

3 Ranking Finnish Companies Based on Their Gender Equality Degree Using UW-TOPSIS with Linguistic Variables

In order to illustrate the proposed assessment method, we will measure the degree of gender diversity and inclusion of a sample of 26 Finnish companies (see Table 1).

Following Equileap, we will assess firms using 19 gender equality criteria organized into four main dimensions: gender balance in leadership and workforce, fair remuneration, policies promoting gender equality, and commitment, transparency

Table 1 Selected Finnish companies

Firm	Sector	Group
F_1	Basic materials	Paper and forest products
F_2	Telecommunications services	Telecommunications services
F_3	Utilities	Electric utilities and IPPs
F_4	Energy	Oil and gas
F_5	Technology	Software and IT services
F_6	Basic materials	Paper and forest products
F_7	Consumer non-cyclicals	Food and drug retailing
F_8	Technology	Communications and networking
F_9	Basic materials	Chemicals
F_{10}	Telecommunications services	Telecommunications services
F_{11}	Basic materials	Paper and forest products
F_{12}	Consumer cyclicals	Automobiles and auto parts
F_{13}	Industrials	Industrial conglomerates
F_{14}	Health care	Pharmaceuticals
F_{15}	Consumer cyclicals	Household goods
F_{16}	Consumer cyclicals	Media and publishing
F_{17}	Industrials	Machinery, tools, heavy vehicles, trains, and ships
F_{18}	Basic materials	Containers and packaging
F_{19}	Basic materials	Metals and mining
F_{20}	Basic materials	Paper and forest products
F_{21}	Industrials	Machinery, tools, heavy vehicles, trains, and ships
F_{22}	Financials	Real estate operations
F_{23}	Industrials	Machinery, tools, heavy vehicles, trains, and ships
F_{24}	Industrials	Machinery, tools, heavy vehicles, trains, and ships
F_{25}	Industrials	Machinery, tools, heavy vehicles, trains, and ships
F_{26}	Financials	Insurance

Source Equileap (2019)

Table 2 Gender diversity criteria

Criteria	Description
C_1	Gender balance in leadership and workforce
C_2	Fair remuneration
C_3	Policies promoting gender equality
C_4	Commitment, transparency, and accountability

Source A detailed description of the decision criteria can be found in Liern and Pérez-Gladish (2022) and Equileap (2019)

and accountability (see Table 2). Table 3 shows the initial decision matrix, with C_i, $i = 1, \ldots, 4$ diversity criteria and $A_j, j = 1, \ldots, 26$ companies (our decision alternatives).

Let us assess the degree of diversity and inclusion of the firms using FUW-TOPSIS. First, we define a linguistic evaluation scale for the relative importance of the decision criteria as the following finite and totally ordered discrete term set composed of five possible values for the linguistic variable representing the weight of criterion i. Let us suppose all the criteria weights are described by the same linguistic evaluation scale:

$$l_1 = \{s_0, s_{0.2}, s_{0.4}, s_{0.6}, s_{0.8}, s_1\} \tag{36}$$

being

$s_0 =$ not important
$s_{0.2} =$ slightly important
$s_{0.4} =$ moderately important
$s_{0.6} =$ important
$s_{0.8} =$ very important
$s_1 =$ essential.

Reasoning as in (4) and (5), we obtain the following sets

$$L_1 = \{s_\alpha/\alpha \in [0, 1]\}, \quad L_2 = \left\{\tilde{s} = \left[s_\alpha, s_\beta\right] : \alpha, \beta \in [0, 1], \ \alpha \le \beta\right\}. \tag{37}$$

Table 4 displays the importance of the diversity decision criteria weights in linguistic terms.

For weights in Table 4 being UWL feasible (24), we apply Proposition 1, making $M = 2$ in expression (25),

$$W^* = \left\{\left[\frac{1}{2\sum_{k=1}^{T}\alpha_k} s_{\alpha_k}, s_{\beta_k}\right], \quad 1 \le k \le T\right\}. \tag{38}$$

If we apply (38) to the third column in Table 4, we obtain the set of weights expressing the relative importance of the decision criteria

$$W^* = \left\{\left[\frac{1}{3.2} s_{0.6}, s_1\right], \left[\frac{1}{3.2} s_{0.6}, s_1\right], \left[\frac{1}{3.2} s_{0.2}, s_{0.6}\right], \left[\frac{1}{3.2} s_{0.2}, s_{0.4}\right]\right\}.$$

Table 3 Decision matrix

Firm	Sector	C_1	C_2	C_3	C_4
F_1	Basic materials	29.3	16.4	17.5	0.0
F_2	Telecommunications services	32.0	10.9	17.5	2.5
F_3	Utilities	29.3	13.6	17.5	0.0
F_4	Energy	24.0	16.4	15.0	0.0
F_5	Technology	24.0	13.6	17.5	0.0
F_6	Basic materials	29.3	6.8	17.5	0.0
F_7	Consumer non-cyclicals	24.0	13.6	15.0	0.0
F_8	Technology	24.0	8.2	17.5	2.5
F_9	Basic materials	29.3	6.8	15.0	0.0
F_{10}	Telecommunications services	26.7	8.2	15.0	0.0
F_{11}	Basic materials	21.3	9.5	17.5	0.0
F_{12}	Consumer cyclicals	26.7	5.5	15.0	0.0
F_{13}	Industrials	26.7	5.5	15.0	0.0
F_{14}	Health care	21.3	8.2	15.0	0.0
F_{15}	Consumer cyclicals	21.3	8.2	15.0	0.0
F_{16}	Consumer cyclicals	21.3	8.2	12.5	0.0
F_{17}	Industrials	21.3	5.5	15.0	0.0
F_{18}	Basic materials	18.7	5.5	17.5	0.0
F_{19}	Basic materials	16.0	5.5	15.0	0.0
F_{20}	Basic materials	10.7	8.2	17.5	0.0
F_{21}	Industrials	13.3	5.5	17.5	0.0
F_{22}	Financials	16.0	6.8	12.5	0.0
F_{23}	Industrials	13.3	5.5	15.0	0.0
F_{24}	Industrials	13.3	5.5	15.0	0.0
F_{25}	Industrials	13.3	5.5	15.0	0.0
F_{26}	Financials	16.0	5.5	10.0	0.0

Source Equileap (2019)
Note For each firm, F_i, subindex i shows its position in Equileap's ranking

Table 4 Importance of the decision criteria

	Criteria	Linguistic terms	Intervals
C_1	Gender balance in leadership and workforce	[Important, very important]	$[s_{0.6}, s_1]$
C_2	Fair remuneration	[Important, very important]	$[s_{0.6}, s_1]$
C_3	Policies promoting gender equality	[Slightly important, important]	$[s_{0.2}, s_{0.6}]$
C_4	Commitment, transparency, and accountability	[Slightly important, moderately important]	$[s_{0.2}, s_{0.4}]$

Source Own

Table 5 Relative importance of the decision criteria

	Criteria	Linguistic	UW-TOPSIS bounds	
		Interval	l_j	u_j
C_1	Gender balance in leadership and workforce	$[s_{0.1875}, s_1]$	0.1875	1
C_2	Fair remuneration	$[s_{0.1875}, s_1]$	0.1875	1
C_3	Policies promoting gender equality	$[s_{0.0625}, s_{0.6}]$	0.0625	0.6
C_4	Commitment, transparency, and accountability	$[s_{0.0625}, s_{0.4}]$	0.0625	0.4

Source Own

In Table 5, we have displayed the new weights and their use as bounds in the UW-TOPSIS method.

Table 6 shows the obtained scores applying UW-TOPSIS. In the second column, we have displayed the minimum relative proximity value that each firm can obtain given the weights in Table 5. In the third column, we display the maximum possible value, and in last column, we have displayed the average value which we consider the diversity and inclusion index.

Figure 3 displays the obtained results graphically.

Figure 3 shows the worst, best, and average possible results in terms of diversity and inclusion of each firm, given the weights expressed by the decision-maker in linguistic terms using linguistic intervals. Firm F_2 ranks the first one, followed by firm F_8. However, they both present a big amplitude of their relative proximity intervals which means greater ambiguity and imprecision. Position of firm F_{22} is, for instance, more stable, as the amplitude of the relative proximity interval is small. Each position in the ranking, worst and best, has an associated set of weights given in linguistic terms that can be interpreted as weaknesses and strengths of the firms in terms of the diversity and inclusion decision criteria.

4 Conclusions

In this work, we have shown how an extension of TOPSIS can contribute to the assessment and ranking of firms in terms of their diversity adequacy degree. The proposed method allows the ranking of the decision alternatives without a priori determination of a precise weighting scheme. The main contribution of our proposal is the use of linguistic labels transformed into linguistic intervals incorporated into the UW-TOPSIS algorithm to rank a set of decision alternatives. With our method, the relative proximity to the positive ideal solution is optimized for each firm based on

Table 6 Obtained results

Firms	Min R_i	Max R_i	DIAI$_i$
F_1	0.169470863	0.787936101	0.478703482
F_2	0.549559488	0.929459363	0.739509425
F_3	0.149526912	0.676063302	0.412795107
F_4	0.153238539	0.779985161	0.466611850
F_5	0.136896660	0.668261752	0.402579206
F_6	0.100941001	0.648569355	0.374755178
F_7	0.129331318	0.668011039	0.398671179
F_8	0.343894743	0.888739909	0.616317326
F_9	0.100129071	0.648279611	0.374204341
F_{10}	0.099568687	0.602317635	0.350943161
F_{11}	0.100173194	0.445748992	0.272961093
F_{12}	0.082432026	0.577738735	0.330085381
F_{13}	0.082432026	0.577738735	0.330085381
F_{14}	0.079913502	0.434864431	0.257388966
F_{15}	0.079913502	0.434864431	0.257388966
F_{16}	0.070983113	0.434335707	0.252659410
F_{17}	0.057156154	0.417495629	0.237325892
F_{18}	0.046312094	0.450781654	0.248546874
F_{19}	0.031321757	0.340987609	0.186154683
F_{20}	0.041050744	0.451877937	0.246464341
F_{21}	0.024073463	0.432012017	0.228042740
F_{22}	0.041363915	0.224162803	0.132763359
F_{23}	0.019392000	0.330523138	0.174957569
F_{24}	0.019392000	0.330523138	0.174957569
F_{25}	0.019392000	0.330523138	0.174957569
F_{26}	0.028663844	0.218147224	0.123405534

the possible linguistic intervals expressing the criteria weights. As a result, we obtain a relative proximity interval informing the decision-maker about the worst and best possible positions of each firm in the ranking. This could provide a useful information in terms of improvement opportunities for the firms and allows the decision-maker to express certain preferences regarding the decision criteria using linguistic terms.

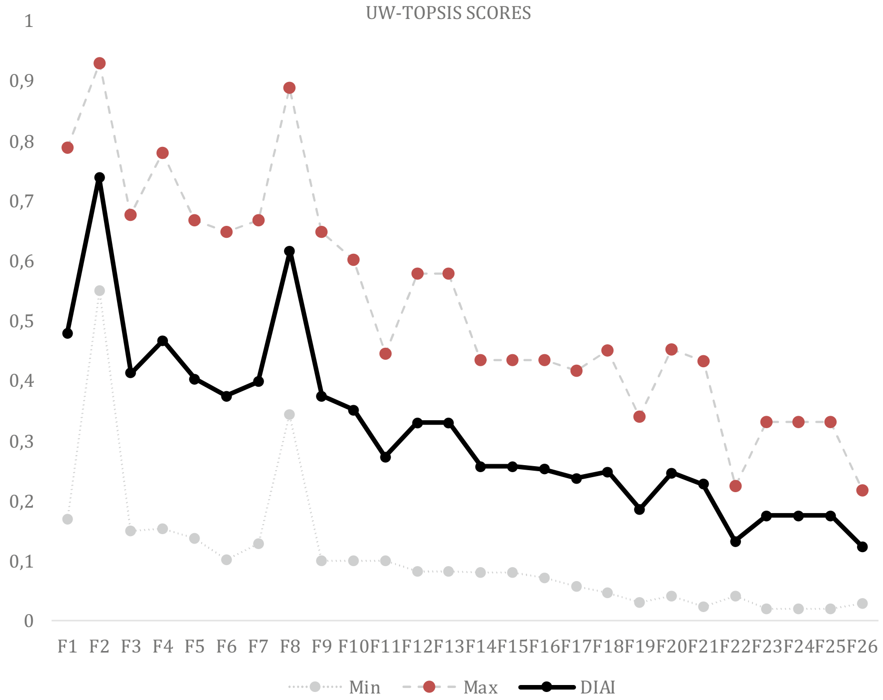

Fig. 3 Results applying UW-TOPSIS with weights displayed in Table 5

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