Individual Student Internal Contexts and Considerations for Mathematics Teaching and Learning

S. Megan Che and T. Evan Baker

1 Introduction

School classrooms are enormously intricate, complex, dynamic, and, to an extent, unpredictable spaces where diverse humans meet together to deepen and expand their intellectual prowess. As the introductory chapter to this volume more fully explicates, the field of mathematics education research has devoted much intensive time and effort to improving our understandings of the myriad factors pertaining to teaching and learning in mathematics classrooms. Specifically, the chapters in this volume are connected in our shared endeavors to elaborate on one of the contexts adapted from Medley's (1987) framework, which include internal and external student and teacher contexts as well as characteristics and qualities of students, teachers, and learning environments.

Within this larger project, this chapter presents, discusses, and problematizes the progression of our field's understandings of individual student internal contexts through considerations of (1) meanings of internal/external (subject/object) dichotomies, (2) individual student cognitive processes, (3) individual student affective processes, (4) how these individual student cognitive and affective experiences connect with (are informed by and inform) each other as well as broader communities such as mathematics classroom learning environments and home environments, and (5) implications for teachers and teacher educators.

Specifically, we situate the notion of an individual student as an entity in continuous dialectic with environmental influences, to the point that—at one scale, it

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becomes meaningless to distinguish between individual and environment. As we begin this chapter, we explicate the scale we choose to use for our chapter: the scale at which individuals are distinguishable from each other and their environments but also inseparable from each other and their environments. In so doing, we elaborate a connected perspective of student identity. We then similarly situate the experiences of cognition and affect prior to deeply considering the progression of our understandings of students' cognitive and affective processes. We proceed to articulate and examine ways these individual experiences connect with multi-individual communities like home environments and mathematics classrooms. Our chapter ends with a consideration, from the lens of social justice, of implications of these understandings for teachers and teacher educators.

2 Context

The title of this chapter presents a few boundaries and restrictions on our field of view in an attempt to focus the reader's attention on what we (the authors) would like for you to be attending to. Aware of the risk of reductively simplifying and nonchalantly utilizing heavy-handed attention-directing tactics, we here attempt to openly articulate our meanings for these boundaries vis-à-vis the purposes of this chapter. The first of these boundaries is the concept of an "individual". As von Glasersfeld (2013) points out, humans dialectically and continually construct and reconstruct themselves based on one's analysis (however (un)aware one may be of this analysis) of how one's peers view one. That is, I continually (re)construct my notion of myself as an individual based, to a practically meaningful extent, on who I think people around me think I am. Thus, my individuality is inseparable from the constructions of people around me, on which I (at least partially, and however unconsciously) base my own notion of who I am; identity construction is an intertwined, reflexive process of "understanding who I am and whom you see" (Walshaw, 2010, p. 490). For teachers, this means that the self-fulfilling prophecy is at best incomplete (Wineburg, 1987); students do not simply live up to the expectations of teachers because teachers' expectations are not transmitted directly and unfiltered to students. Students construct their ideas about teachers' expectations and perspectives of them for themselves, based on their experiences with teachers. To the extent that teachers' constructions of students influence those students' identity constructions of themselves as students, these students respond to what they (the students) *think* their teacher's expectations are of them. In constructing ourselves, we do so by playing at shadows, mirror images, or doubles (Žižek, 1989; Turner & Oronato, 1999). A potential implication for teachers of reflexive, complex, and non-linear processes of identity construction is that our beliefs are not imparted in a direct one-to-one fashion onto students' psyches (Walshaw, 2010). As a teacher, for instance, if I fervently believe that all students are capable of doing important mathematics, that belief risks residing only within myself since—unless my students realize/internalize that

I am constructing them as capable, talented, and smart—they may not (re)construct themselves as such in our class.

The notion of individuality is further quickly complicated when one considers that not all aspects of an individual, or not all of one's identities, may be equally prone to this process of reflexive reconstruction. Additionally, not every "other" may hold equal sway over one's reconstruction of themselves. Certain, perhaps more peripheral characteristics or personality traits may vary relatively widely over the course of a year, a month, a week, or even within a day (Markus & Kunda, 1986; Turner & Onorato, 1999). Other, perhaps more central, identity aspects tend to be more stable, though still dynamic (Markus & Kunda, 1986; Turner & Onorato, 1999). With time, experience, and attention teachers can develop an awareness of which aspects of their students' individualities they might have the potential to influence and which aspects are more deeply ingrained. Further, as teachers understand from their own experience, the quality and quantity of leverage we hold over the individual constructions of others are variable from person to person and also across time; we may be able to more firmly convince a student one day of our belief in their potentialities than we are on another day, and we may be less successful in our convincing than a different teacher of that same student.

Because of the dynamic nature of at least some aspects of identity, it is important for us to understand that one individual student may exhibit, enact, and construct different student identities in different academic disciplines (Aydeniz & Hodge, 2011). Even within one discipline (mathematics, for instance), an individual student's identity may be changing and changeable. To encapsulate these dynamic processes and complexities, we see (mathematics) identity as a fluid construct that dialectically shapes and is shaped by social context; for our chapter, this context is a complex range of individual, cultural, and social influences in a learning environment that are often in states of tension between conflicting roles and relationships that are activated at any moment in a mathematics classroom (McAdams, 2001; McCaslin, 2009; Nasir, 2002).

In the following sections, we focus closely on students' construction of mathematics and students' psychosocial construction of themselves as students of and doers of mathematics. In this opening section, however, we seek to foreground the interdependency of our individualities as we simultaneously acknowledge that, at some scale (for instance, the scale of visible physical humans in a classroom) we exist as sets of seemingly separate individuals brought together in community. We posit that, from a different perspective (for instance, the not-directly-visible cosmos composed of strands of relationship, influence, power, and control) and—for many of the purposes of mathematics teaching and learning—we function more as interconnected beings, inextricable from the perceptions of our surrounding environment, which we ourselves also influence. As such, we (co)construct as we are (co)constructed by the webs that enmesh us; the ontological status of our identities is unclear and perhaps unknowable (Turner & Onorato, 1999; von Glasersfeld, 2001; Walshaw, 2010; Walkerdine, 2003; Žižek, 1989, 1998). For us, the implications of the uncertain ontology of identity are that we expect (and, at times, seek) the unexpected. In the indeterminacy of identity, for us, resides the potential for curiosity and wonder not just about our natural worlds but about our 'own' selves and those selves of our students. This affords us opportunities for expansiveness and responsiveness.

This complex indeterminacy of identity also poses research challenges, not the least of which is a variety of theories of identity. In the preceding discussion, we have implicitly placed ourselves in a more poststructural view of identity than psychological or socio-cultural (Grootenboer et al., 2006) because of our emphasis on dynamic process (becoming) and on the relative nature of identity, but even that placement is murky since our perspective shares many salient features with a socio-cultural perspective, including the embodied and connected nature of identity. The experience of not being quite able to provide a static, definitive definition for a critically important aspect of student mathematical reality can be frustrating as well as confusing; in our work, we are becoming more comfortable with uncertainty and more cognizant of the value in process rather than product. That is, we see the *process* of thinking through and with these various perspectives to be increasingly important to our research methods and methodologies. Our distance from a definitive "answer" to the nature of identity is becoming less of a challenge for us to navigate because we see that distance as providing space for (re)thinking and (re)envisioning what we think we know.

Another boundary we establish in our title is that of focusing on internal rather than external contexts. What might we mean by that? Encircled and embedded as we are within dialectical, dynamic, non-linear and non-deterministic connection, all reverberating within material and historical situations, experiences, and narratives, on what level does it make sense to distinguish between internal and external? The demarcation of internal contexts serves, for the purposes of this chapter, to establish an operationalization of what the *object* (see Deleuze & Guiattari, 1994; Gallagher, 2000; Russell, 2001; Wittgenstein, 1969 for a sampling of philosophical considerations of self, subject, and object) of this chapter is—mathematics students' psychosocial processes of identity and content construction in school mathematics. For much of this chapter, we will examine students' constructions of themselves and their mathematical contexts with an aim of better understanding how students come to understand themselves as mathematics students as well as how students understand mathematics. This understanding of students' understandings can, hopefully, further our reflexive (re)construction of ourselves as mathematics teachers and as mathematics teacher educators. Therefore, our meaning for the word "internal" in our title signifies that the primary basis of understanding for this chapter derives from insights students construct within themselves about themselves as mathematics students as well as the perceptions they have (co)constructed for/within themselves about the discipline of school mathematics.

Understanding that these student insights about themselves and about mathematics are situated within (that is, formed by and also forming) threads and strands of material and historical circumstance, community (defined at various scales), and family (expansively conceived), we want to reiterate that we are not attempting to pretend away those strands but rather, for the moments of the reading and writing of much of this chapter, to foreground the students in the strands rather than the strands. In the penultimate section of this chapter, we (re)focus on the ways in which students interact and relate with (that is, how they are formed by and how they form) the threads and strands connecting them with events and circumstances at broader scales than an individual. Additionally, situating these insights in historical circumstance, community (e.g., family, school, classroom, peer, biosystem) commits us to (co)construct the student as an active, self-cognizing agent (Davis & Sumara, 1997; Newell, 2008) rather than (co)constructing the student reductively as a product of context and environment.

One last boundary we have built as we define what this chapter might mean for us and for you is the demarcation of which aspects of individual identity we intend to include; for our purposes of articulating our field's progression of understanding of student internal context in school mathematics, we will consider student cognitive (psychological) and (with) affective (social) contexts. Because of the importance of both knowing and feeling in mathematics classrooms, we feel that giving emphasis to these aspects of identity, for this moment, might afford us a clarity of insight relevant to mathematics teaching and learning that could be potentially obscured if we attempted a more distributed examination of student identity. Medley (1987) framed these internal contexts as variables that affect student response to mathematics teacher behavior. While we concur that student internal context often influences how a student responds to activities in their mathematics classrooms (activities which Medley framed as Type C, interactive mathematics teacher activities, and Type B, student mathematics learning activities), our dialectic and indeterministic perspective of student internal context is informed also by critical, postmodern, and constructivist insights that have largely emerged in the decades since Medley established his framework.

As we traverse the unfolding of our field's understandings of student internal cognitive contexts, we will be seeking insight about how students come to know themselves—with the obvious implication that this knowing is ever incomplete and is continually ongoing, that this knowing is much more a process than a product (Davis, 2004). Particularly, we will discuss many of our field's contributions to questions of how students know themselves as mathematics students, what knowing means for mathematics students, and what students know they know (or not) about mathematics. As important and interesting as it is for mathematics teachers and mathematics teacher educators to attain deep insight, from students, about how mathematics students come to know about themselves and about mathematics in school, it is also interesting and important for us to understand students' affective and sociological realities and identities in school mathematical environments. In the next section, we will articulate trends and patterns in our field's progression of understanding about how students feel in school mathematics classrooms—how students feel about themselves as mathematics learners, and how students feel about mathematics (or school) mathematics.

3 Students' Internal Contexts

In this section, we detail our field's insights of student processes of generating mathematical understandings, focusing on cognitive, sociological, and psychosocial perspectives of student mathematical experiences. Additionally, in the second subsection, we discuss mathematics students' perceptions of themselves as mathematics learners, including our understandings of mathematics students' identities, self-concepts, self-perception, and self-beliefs. In the third subsection, we discuss students' internal contexts vis-a-vis the subject of mathematics itself, so we investigate students' perceptions and processes of forming attitudes about mathematics content. Throughout, we emphasize those many places where the insights in these subsections connect and overlap with those that we have highlighted in other subsections; these are not mutually exclusive categories we are setting up but rather focal points in connection with each other.

Students' cognitive processes as mathematics learners

In the decades prior to the accessibility of translations of Piaget (especially Piaget, 1972) and Vygotsky (see Vygotsky, 1978a, 1978b) in the U.S. in the 1970s, the educational research community's understandings of students' epistemological processes were influenced by a behaviorist management perspective, which emphasized connections between repetition, routinization, and skill performance (Doll & Broussard, 2002); though there existed progressive counter-narratives to the reductionist and utilitarian outcroppings of behaviorism, particularly from Dewey, one of the first philosophers to emphasize students' active roles in learning (Dewey, 1933/1998; Bruner, 1990). From a behaviorist epistemology, students learn mathematics by repeatedly performing small chunks of mathematical operations through steps provided by a teacher, and fluent performance as well as immediate recall of procedure are prioritized (Doll & Broussard, 2002). Constructivist epistemological paradigms have, for the past several decades, expanded and complicated educational scholars' understandings of student processes of making sense of mathematics by illuminating the active role of student cognition in learning processes (Doolittle, 2014). In mathematics education, our understandings of constructivist cognition have been bolstered in no small measure by our long-standing research connections with cognitive psychologists:

> Cognitive psychologists have provided the concept of 'well-organized' schemata to explain how people impose order on experiential information. Assimilation, accommodation, and mode of functioning in response to new information are important in the enterprise of schooling […] Schema use must be a dynamic, constructive process, for people do not have a schema stored to fit every conceivable situation. In this view, acquisition of knowledge implies changes in schemata, not just the aggregation of information. (Romberg, 1992, p. 62)

Cognitive constructivism espoused by Piaget and the radical constructivism of Von Glasersfeld (2013) are focused on how individuals internally, actively *construct* knowledge as they seek to make sense of lived experiences (Bruner, 1990); the emphasis is understanding people's internal cognitive structures and processes rather than on an imposition or interaction of external 'knowledge' (Schunk, 2020). Social constructionism (Vygotsky, 1978a, 1978b) tends to highlight the social nature of knowledge construction and orients us to the importance of interaction in processes of knowledge construction. Connecting these two orientations is the premise that, rather than environmental stimuli producing knowledge (or adaptations), it is an individual's active processing of stimuli in relationship to that individual's cognitive structures that brings about knowledge (Huitt, 2003). Doolittle and Hicks (2003) distill constructivist epistemology into four tenets:

1. Knowledge is not passively accumulated, but rather, is the result of active cognizing by the individual.
2. Cognition is an adaptive process that functions to make an individual's cognition and behavior more viable given a particular environment or goal.
3. Cognition organizes and makes sense of one's experience, and is not a process to render an accurate representation of an external reality.
4. Knowing has its roots in both biological/neurological construction and in social, cultural, and language-based interactions (pp. 77–78).

Given these understandings of individual student cognitive processes of learning, several potential insights and implications for teachers emerge, including the indirect nature of teaching (Ackermann, 2001). That is, the content that teachers may try to impart into students is not transmitted in a direct, unfiltered manner. Instead, students actively respond to content from teachers (stimuli) by connecting it to (and connecting to it) their pre-existing cognitive structures. Doolittle and Hicks (2003) discuss several additional learning principles which can inform constructivist pedagogy, including:

- The construction of knowledge and the making of meaning are individually and social active processes
- The construction of knowledge involves social mediation within cultural contexts
- The construction of knowledge takes place within the framework of the learner's prior knowledge and experience
- The construction of knowledge is integrated more deeply by engaging in multiple perspective and representations of content, skills, and social realms
- The construction of knowledge is fostered by students becoming self-regulated, self-mediated, and self-aware (Doolittle, 2014, pp. 498–490).

For mathematics teaching and learning environments, specifically, constructivism is strongly connected with ontological questions about the nature of mathematics, because one corollary of certain constructivist views (particularly from radical constructivism) is that the process of coming to know is an adaptive process grounded in experiential realities and that knowing is *not* a process of discovering external, independent, pre-existing realities (Lerman, 1989). Contrasting with an enduring popular view of mathematics (the "Romance of Mathematics" as Lakoff and Nuñez (2000) call it (p. 339)) as existing outside of the mind of a knower, as Lakoff and Nuñez point out, "Ideas do not float abstractly in the world. Ideas can be created only by, and instantiated only in, brains" (p. 33). As Lakoff and Nuñez detail precise ways in which humans have used language to develop mathematical metaphors that extend our very limited innate mathematical capabilities, they connect the ways in which human minds make sense of experiences that are external to us:

1. There are regularities in the universe independent of us.
2. We human beings have invented consistent, stable forms of mathematics (usually with unique right answers).
3. Sometimes human physicists are successful in fitting human mathematics as they conceptualize it to their human conceptualization of the regularities they observe in the physical world. But the human mathematical concepts are not out there in the physical world (pp. 345–346).

Many other constructivists in mathematics education assert, rather than claiming that mathematics does or does not map an external reality, that—because we construct our understandings from the basis of our own experiences and previous knowledge—we cannot know whether a mathematical concept exists in an objective reality (Von Glasersfeld, 1995; Steffe & Gale, 1995; Simon, 1995). Instead, our test of emergent knowledge is not an independent, objective existence or truth but the extent to which that mathematical knowledge works in our lived realities; that is, the extent to which mathematical insights are "viable" (Von Glasersfeld, 1995). As students construct their mathematical knowledge, they do so by coordinating mathematical material or mental actions into organized, goal-directed action patterns (Steffe, 1991). The goal towards which students are oriented is that of resolving the perturbation or disequilibrium that arises when students have a novel experience and "restoring coherence" to their experiential worlds (Cobb, 1994). Further, as students interact with peers and teachers in a mathematics classroom environment, they have opportunities to test and refine the viability of their mathematical conjectures, contributing to an emergence of a socioculturally-embodied mathematical knowledge (Cobb, 1994; Cobb & Yackel, 1996).

In the next subsection, we leverage these current constructivist understandings of student cognition to investigate how these cognitive processes might connect with mathematics students' perceptions of themselves as mathematics learners. In the last section of this chapter, we explicate several insights specific to mathematics teachers relating to a constructivist epistemology of student learning.

Students' identity constructions as mathematics learners

During the past several decades, mathematics educators have (re)formulated a variety of constructs to facilitate our understandings of how students view themselves as learners of mathematics, including mathematics self-concept, self-efficacy, mathematics identity, and mathematical disposition. Di Martino and Zan (2011) emphasize that, far from being disconnected with cognitive processes, the interactions between emotional and cognitive dynamics constitutes the concept of affect. This affective sphere in mathematics education, which Di Martino and Zan (2011) frame as interactions between Emotional Disposition, Perceived Competence, and Vision of Mathematics, constitutes a fundamentally important "internal representation system" (p. 1).

Medley's (1987) framework articulates these internal contexts as characteristics (for us, specifically, identity, self-concept, self-efficacy, disposition) of students that affect their response to behaviors of mathematics teachers. In this subsection, we examine our understandings of how mathematics students construct themselves as learners.

As we discussed in the opening section, for us, (mathematics) identity dynamically dialectically shapes and is shaped by a complex range of influences existing in a social context like a mathematics classroom. There is no shortage of reasons to devote energy to better understanding student mathematics identity, given its central location to mathematics learning. As Andersson et al. (2015) point out, "When considering how students' affective responses impact on their willingness to engage in learning mathematics, the notion of identity becomes particularly important because it provides ways to understand the complexity of students' decision making". Grootenboar and Zevenberguen (2008) also affirm the importance of identity: "The teachers' role is temporal, and at the end of the teaching period it is the students' mathematical identities that will endure." Boaler and Greeno (2000) and Boaler (2002) raise the point that mathematics learning environments provide stimuli for students as they construct their mathematics identities and that the range of identities students construct may be linked to the mathematical learning environments they experience. Specifically, there are indications that students learning in traditional lecture-based environments as compared to discussion-based environments may construct different mathematical identities (Boaler, 2002).

Just as we in this chapter at times highlight certain frames of reference (the scale of an individual rather than that of a classroom, for instance), mathematics education researchers frequently foreground particular aspects of mathematics identity with a view to deepening our insights relative to that specific aspect. Research in mathematics education and psychology indicate that two prominent aspects of individual student mathematics context that strongly connect to student mathematics performance are prior academic (and mathematics) performance (sometimes referred to as intelligence) (Deary et al., 2007; Frey & Detterman, 2004; Gustafsson & Undheim, 1996; Kuncel et al., 2004) and motivation (Gose et al., 1980; Schicke & Fagan, 1994; Spinath et al., 2006; Steinmayr & Spinath, 2009). These two characteristics are clearly connected in feedback loops wherein strong motivation can fuel higher performance, which can fuel further increases in performance as well as strengthened motivation; the reverse can also hold wherein lower academic performance can dampen motivation, which can contribute to further declines in performance (Guay et al., 2003; Marsh & Yeung, 1997). In the remainder of this section, we foreground the notion of motivation before focusing on the influences of prior performance; then we revisit the connections between these aspects of student mathematics identity in a culminating discussion of the construct of mathematical disposition.

Mathematics education and psychological researchers rely on a variety of interacting constructs to formulate different theories of motivation such as Bandura's (1986) social cognitive theory (Schunk & DiBenedetto, 2020), expectancy-value theories (Eccles & Wigfield, 2002), and self-determination theory (Ryan & Deci, 2002); see Schukajlow et al. (2017) for a more comprehensive review of motivational theories in mathematics education. Psychological constructs that contribute to student motivation include expectancy-value (Eccles et al., 1983), task interest (Cleary & Chen, 2009; Cleary & Kitsantas, 2017), and math anxiety (Pajares & Graham, 1999) among others. However, research indicates the possibility that self-efficacy may correlate with mathematics performance more strongly than other motivational constructs (Cleary & Kitsantas, 2017; Pajares & Graham, 1999). Because of this potentially stronger correlation of student self-efficacy with student performance, the notion of self-efficacy merits further elaboration.

The set of a student's mathematical self-perceptions, particularly self-efficacy and/or self-concept (Steinmayr & Spinath, 2009) are central to student internal context and to their mathematical identities. Though these two constructs sound and seem very similar (and, indeed share several commonalities (Bong & Skaalvik, 2003)), many educational psychologists have constructed both theoretical and empirical distinctions between self-concept and self-efficacy (Zimmerman, 2000; Bong & Skaalvik, 2003; Parker et al., 2013; Chmieleqski et al., 2013). Self-concept and self-efficacy are similar in that they are taken to (at least partially) explain an individual's thought, emotion and action in a given context where the individual's perceived skills and abilities come into play. Further, both self-concept and self-efficacy are domain specific (Bong & Skaalvik, 2003), so an individual can have a strong self-concept or self-efficacy in one area but not in another. Bong and Skaalvik (2003) turn to Bandura (1986) to draw distinctions between the two constructs:

> While self-concept represents one's general perceptions of the self in given domains of functioning, self-efficacy represents individuals' expectations and convictions of what they can accomplish in given situations. For example, the expectation that one can high-jump 6 ft is an efficacy judgment (Bandura, 1986). It is not a judgment of whether one is competent in high jumping in general but a judgment of how strongly a person believes that [they] can successfully jump that particular height under the given circumstances. Self-efficacy researchers thus tend to emphasize the role played by specific contexts in efficacy appraisals. (p. 5)

In the context of mathematics classrooms, then, mathematics self-concept discloses individuals' perceptions about themselves in the area of mathematics (or perhaps in more specific domains like algebra or geometry) while mathematics self-efficacy, following Bong and Skaalvik's (2003) articulation, is a more bounded, context-dependent estimation of the extent to which students believe they can succeed at given specific mathematical tasks to certain levels. Both mathematics self-efficacy and mathematics self-concept can influence students' mathematical identities, mathematical dispositions, mathematical classroom experiences and the mathematics they construct in practically meaningful ways, not the least of which is by fueling oneself with the motivation to persevere.

Bandura (2010) noted that an individual's self-efficacy relates to their belief in their ability to exert an influence over events pertaining to their lives, a belief which connects right to the foundation of human motivation and emotional well-being as well as (academic) performance. Bandura (2010) points out that, unless individuals believe that they can produce "desired effects by their actions, they have little incentive to undertake activities or to persevere in the face of difficulties." As one may expect, mathematics education researchers have devoted much energy to intensive studies of student mathematics self-efficacy, from investigating potential sources of student mathematics self-efficacy (Lent et al., 1991; Lopez & Lent, 1992; Usher & Pajares, 2009) to exploring connections between self-efficacy and mathematics performance (Hackett & Betz, 1989; Pajares & Graham, 1999; Pajares & Miller, 1995) to the potential roles of mathematics self-efficacy in achievement, problem solving, and even career choice (Betz & Hackett, 1983; Lopez & Lent, 1992; Pajares & Miller, 1994; Randhawa et al., 1993). Researchers also point to the importance of mathematics self-concept to persistence (Parker et al., 2013) and to mathematics performance. Seaton et al. (2014) suggest that addressing students' mathematics self-concept may be as influential to their mathematics performance as building their mathematical fluency.

These decades of research indicate that the way students view themselves as learners of mathematics is centrally important to the mathematics those students might construct. In the next section, we discuss how students' views of themselves interfaces with their views of mathematics as they engage in processes of learning mathematics.

Students' views on mathematics

Understanding the ways students construct mathematical knowledge by actively seeking to make sense of their realities through adaptation to new experiences connects to the previous discussion of students' mathematical identities, because, as we saw, the particular characteristics students construct about themselves as mathematics learners can influence their construction of mathematical insights. Another aspect of mathematical learning environments also connects, however, to ways students might be primed to undertake mathematical knowledge construction, and that is students' views of the field of mathematics (Kilpatrick et al., 2001). As McLeod (1992) notes, students in school mathematics settings regularly experience a range of emotions; the frequency, ferocity, and severity of these emotions relates to a student's affective attitude towards the discipline of mathematics itself. McLeod (1992) asserts that, "the improvement of mathematics education will require changes in affective responses of both children and adults" (p. 575).

In school mathematics settings, students' views about the nature of mathematics (what mathematics is, what mathematics is like, what mathematics is not—from psychosocial perspectives) is informed by their experiences as mathematics students. These mathematical experiences are categorized by Medley (1987) as Type C, Interactive Mathematics Teacher Activities and Type B, Student Mathematics Learning Activities. Unsurprisingly, students who have experienced success in school mathematics performance tend to continue to experience success in school mathematics performance (Archambault et al., 2012; Reynolds, 1991). Concerningly, however, students who have less successful school mathematics prior achievement tend to struggle with mathematics performance (Archambault et al., 2012; Reynolds, 1991). These trends make sense both from cognitive and affective perspectives, because, if a child has somehow succeeded in learning mathematics content, it is reasonable to surmise that they may be successful in learning more, and perhaps related, mathematics content. Further, it is reasonable that a student experiencing success in school mathematics likely is boosted in their mathematics identity (self-concept, self-efficacy), which can increase the chances that that student persists and perseveres as well as enjoys mathematics while connecting school mathematics to other aspects of their lives. We have realized for decades, from the work of Eccles and others, that students' impressions about and experiences with mathematics in school inform and relate to both their mathematics identities and their construction of mathematical knowledge (Eccles, 1983; Meece et al., 1990; Simpkins et al., 2006).

However, decades after the National Council of Teachers of Mathematics (NCTM, 1989, 2000) embarked on its monumental reform movement, students are still frequently taught mathematics from a traditional perspective where mathematics is seen as a static discipline, a "sets of preexisting facts and procedures that is passed along from teacher to student in an authoritarian manner" (Wilkins & Ma, 2003). In such classrooms, students' activities are dominated by silent, individual seatwork and rote note taking, so many students suspect that mathematics is about memorization of content (Wilkins, 2000). In such classroom environments, students often reasonably surmise that mathematics is dry, boring, and potentially a waste of time. Responding to this concerning student view of mathematics, mathematics educators have, in the past few decades, begun to focus increasingly on supporting productive student mathematical dispositions, which Kilpatrick et al. (2001) characterized in their report *Adding it Up* as an intertwining of students' views of mathematics with their views of themselves as mathematics learners:

> Productive disposition […] includes the student's habitual inclination to see mathematics as a sensible, useful, and worthwhile subject to be learned, coupled with a belief in the value of diligent work and in one's own efficacy as a doer of mathematics. (Kilpatrick, 2001, p. 107)

Gresalfi (2009) emphasizes that, just as the content students learn is inseparable from the ways in which they learn it, students' dispositions—their social, affective, and motivational factors such as persistence, collaboration, and engaging with novel problems—are both central to and inseparable from learning processes. Dispositions involve students' ideas about, perspectives towards, and their interactions with content; dispositions "capture not only…what one knows but how [they] know it…not only the skills one has acquired, but how those skills are leveraged" (Gresalfi, 2009, p. 329). Students' mathematical dispositions are clearly pertinent to mathematics teachers, who may seek to better understand their students' mathematical dispositions in relation to classroom mathematics practice. Clark et al. (2014) emphasize four aspects of student identity that can provide information for mathematics teachers relative to their students' mathematical dispositions, which includes students'

- perceptions of their mathematics ability and the ways these perceptions influence their mathematics performance
- perceptions of the importance of mathematics inside and beyond their current experiences in the mathematics classroom
- perceptions of the engagement in and exposure to particular forms of mathematical activity and the ways these engagements influence students seeing themselves as mathematics learners, and

- motivations to perform at a high level and attributions of their success or failure in mathematical contexts (p. 251).

4 Connections Among Individual Student Internal Contexts and Broader Social Scales

Contexts of social systems (students, teachers, schools, families, communities, or disciplines of study) are filled with relationship and connection. Although, in all of the previous sections of this chapter, we have repeatedly attempted to plant this theme, we use this section to specifically foreground these influences and connections seemingly external to the scale of an individual student. These influences include relationships between and among students and their classroom peers, their teachers, their school community, their home contexts, their local social community, their individual and familial histories and material contexts. Because of the limitations both of language itself and of our (lack of) prowess with language, we are constrained to list (rather incompletely) these relationships as if they are connected only to the student and are separate from their own interconnections, but this is inaccurate. All of these relationships are in dynamic and nondeterministic (although self-organizing) conversation with all of the other relationships (and more), pinging and twinging in transformative interdependence such that one apparently isolated experience that may, for instance, directly connect only the student and their teacher activates this entire biosystem of relationships (though perhaps not all to the same extent), buzzing them alive with energy that can (un)make and/or (trans)form them. Thus, when we have the privilege to interact with a student, we must also be interacting with their entire relationship biosystem.

For us, this illuminates the impossibility of cleaving a mathematics student from their ambient realities and contexts. As Aydeniz and Hodge (2011) explain,

> students' identities in relation to science or mathematics cannot be fully understood without considering the multiple communities in which students participate including home community, school community, and the online social communities that now define most students' daily social lives in western societies. (p. 513)

For us, this means that, in a classroom relationship biosystem, it is not possible, for instance, to affirm a student's mathematical contributions while delegitimizing a seemingly distinct aspect of that student, such as their non-English primary language, because that student's construction of mathematics is inextricably, meaningfully connected to that core, identity-influencing experience of being, for instance, bilingual. Additionally, we maintain that it is also not possible to generatively value, say, a student's mathematical persistence while delegitimizing, however indirectly, a seemingly different but core aspect of that student's reality, such as their gender identities. When we (as people in general or especially as teachers) are dismissive, even offhandedly, of the efforts of persons (like our students, for instance) to have their humanity embraced and legitimized, we risk hindering for ourselves, and perhaps for some time, the potential to affirm students in their mathematical processes. Because students' identities are complex, dynamic, and interconnected, teachers cannot assume that they can separate a student's mathematical identity from the student's holistic identity.

Further, a stunting dismissal of our fellow humans' identities and lived realities tends to atrophy classroom relationship biosystems, as one might imagine, at (various) scale(s) and can profoundly influence students' mathematical (just to name one) identities, even for students apparently adjacent to the target of the dismissal. That is, a delegitimization (very often) resulting in a degeneration of relationship can tinge not just the holistic biosystems of the persons directly involved (the doer of the delegitimization and the direct recipient(s)), but also those of all of the persons cognizant of the delegitimization; vicarious experiences can powerfully impact our efficacies (Zimmerman, 2000). Teachers, in practice, interact not just with a student but with their entire relationship biosystem, and these interpersonal interactions contribute to classroom-level interactions, which also interact with the holistic relationship biosystems of all aware persons in those interactions as well as impinging upon the classroom-level network of relationship.

To dismiss or belittle aspects of students' selves has potential restrictive ramifications for how a teacher can meaningfully interact with students in their mathematics classroom. Conversely, there is a potential for a more expansive student–teacher connection when teachers' interpersonal behaviors consistently communicate how highly each student and their intellectual contributions are valued. Student perception of teachers' behaviors indicating tolerance, care for student wellbeing, and relative lack of authoritarianism is an important feature of student–teacher connections (Van Petegem et al., 2008), so implications for teachers are profound on the scale of students and, we emphasize, on the level of the classroom community relationship biosystem as well. Teachers, because of their position in the classroom, have considerable opportunity and responsibility to mindfully facilitate equitable status relationships with all students; otherwise, students' academic progress and classroom participation (among other potential damages) are at risk (Alexander et al., 1987; Cohen & Lotan, 1995; Fuller & Clarke, 1994).

The interconnected nature of student mathematics cognition, identity, and disposition to wider contexts like mathematics classrooms, schools, and communities uncovers several issues pertaining to student access to opportunities to develop and nurture productive identities and dispositions within dynamic and rigorous classroom environments. Access to important mathematics content is far from equitably attainable (Reddy, 2005), as academic curriculum tracks (Oakes, 2005), along with intergenerational and geographically-dependent disparities in school funding structures (Kozol, 2012) pose non-trivial barriers and opportunity gaps (Horn, 2012) to the fulfillment and facilitation of every students' potential to be their most full mathematical selves.

Much as student internal context cannot be bifurcated into disconnected pieces like cognitive vs. emotional processes, individual students cannot be separated from their multi-leveled social and historical contexts. The discussion in this section illustrates the futility, as understood within mathematics education scholars, of attempting to isolate mathematical cognitive processes in students and interact (or operate with) solely those processes. Perhaps even more critically, this section emphasizes the responsibilities and obligations of teachers to advocate for student growth, development, and health in holistic and connected ways.

5 Implications for Teachers

Though the reasons for inequitable access to high quality, rigorous, engaging mathematics learning opportunities are many, systemic, and extending beyond bounded educational structures, we focus in this section on several aspects of mathematics learning environments that teachers and students can more directly influence, and which are closely connected to our previous discussions of student mathematics learning processes, student mathematical identity constructions, and student attitudes and views towards mathematics. These aspects include the existence and roles of status in mathematics classrooms and socially just and affirming pedagogies in mathematics.

Socially just and affirming mathematics pedagogies not only provide opportunities for mathematics teachers to provide students with enactive experiences, which strongly inform students' efficacy beliefs (Zimmerman, 2000) and which can foster productive mathematics dispositions, but they can also facilitate students' cognitive growth. For instance, culturally responsive teaching is one such pedagogically affirming approach, which Hammond (2014) characterizes as

> An educator's ability to recognize students' cultural displays of learning and meaning making and respond positively and constructively with teaching moves that use cultural knowledge as a scaffold to connect what the student knows to new concepts and content in order to promote effective **information processing**. All the while, the educator understands the importance of being in a relationship and having a social-emotional connection to the student in order to create a safe space for learning. (p. 15, emphasis in original)

Culturally responsive teaching in mathematics asks teachers to be open and expansive to a variety of 'real worlds' that their students navigate and negotiate daily (Gay, 2002). Further, socially just and affirming pedagogy expects teachers to validate students lived realities in part by creating space for those realities in mathematics learning processes. Simultaneously, culturally responsive mathematics teaching expects that high quality, rigorous, important mathematics teaching and learning occurs in classrooms. Our understandings of such justice-oriented pedagogies indicates that powerful and affirming mathematics teaching draws on and leverages broader contexts that students inhabit beyond their individual selves and expands students' and teachers' awareness of the ways mathematics holds power to critically analyze and interpret our worlds, potentially opening up spaces for justice-oriented agency and action (Aguirre & Zavala, 2013).

Just as our field has pointed to connectivity and generativity over the past several decades, it has also emphasized the capability of (all) students to deeply think and reason, so an affirming classroom relationship system is not devoid of student questioning and debating. When responding to a student contribution, for instance, an affirming expectation can be that the student will articulate, explain, and justify their contribution and, if necessary, attempt to persuade peers in the classroom to agree with their mathematical justification. An affirming mathematics classroom relationship system does not let just any suggestion prevail; it acts—with often analytical purpose—to ascertain which suggestions are viable for the mathematics community and to justify why that viability exists. At the same time, however, we are increasingly (if belatedly) aware of important commitments to social justice that mathematics teachers entail as political agents in an unjust social system. We can only weakly attempt to maintain the neutrality of mathematics and the teaching of mathematics; this neutrality is exposed as a fiction, which, for us, implies that mathematics teachers' socio-political self (and community) awareness is more and more paramount. Beyond implementing mathematical instructional practices that deeply and actively engage students in contextual and meaningful mathematics, Aguirre and Zavala (2013) argue that culturally responsive mathematics teachers must

- develop a socio-cultural political consciousness
- understand and embrace social constructivist and socio-cultural theories of learning
- get to know and leverage the mathematical resources of students, their families, and their communities.

Socially justice-oriented mathematics classrooms can provide a potential-filled connective space for nurturing and facilitating the aspects of student internal and individual contexts we have discussed in this chapter.

6 Concluding Comments

The adapted framework for this volume articulates a number of factors, characteristics, and contexts relevant for teaching and learning in mathematics classrooms. Each of these is important on their own, and perhaps even more so in concert with other contexts; for the authors of this chapter, we agree with Dewey (1906) and many other educational scholars that students and their contexts are crucially important to mathematics learning. In this chapter, we have provided an overview of several influential aspects of individual student internal context (Type H), including student mathematical identities, self-efficacy, and disposition. We have also emphasized the significance of the connections between all of these aspects and other educational and social considerations and contexts; students exist, as we all do, within a cosmos of relationship rather than in an isolated vacuum.

Given this overview, several avenues for future research emerge in our field, as we hope they do for the reader. Though each person may see different research potentials arising from the work scholars in our field have already done, the potentials we see are for an increased presence of postmodern research perspectives on student internal context and for greater consideration of critical theoretical approaches in our consideration of student and educational contexts. Postmodern perspectives disrupt static boundaries and binaries as well as linear, predictable pathways while emphasizing the importance of context and acknowledging the existence of indeterminacy (Stinson & Bullock, 2012). These perspectives offer promise to our work in student mathematical internal context because of the dynamic complexity of identity and its non-linear enmeshment in broader conditions; we have the potential to (re)envision our notions of student identity and student mathematics performance in ways that are open to irregularity and spontaneity while maintaining rigor in research. Simultaneously, we see ourselves as undertaking an important responsibility to refrain from treating students as isolated subjects; in socially aware and critical educational research, it is incumbent upon us to deepen our understandings of the systemic nature of concealed, asymmetric relationships of power (Stinson & Bullock, 2012) and the ways those of social contexts of inequity reveal themselves in children's educational lived experiences and identities.

One social context that has emerged in the past two decades as a potentially powerful research focus is that of online communities and the potential to inhabit yet another identity as a virtual being in virtual worlds. Though mathematics researchers have been studying the connections between technology and student mathematics motivation, achievement, and attitude (Higgins et al., 2019) since the mid-1980's, the possibility to understand how online spaces and realities impact and are impacted by students' online mathematical identities is more recently being realized (Rosa & Lerman, 2011), especially in the context of gamification (Lo & Hew, 2020). Technology has progressed from desktop computers placed in classrooms to hand-held devices providing not only unprecedented access to information but also potential for identity transformation and (re)construction. The responsive, adaptive, and dynamic aspects of critical postmodern research perspectives seem well poised to contribute to our understandings of students' mathematical identities and internal social contexts in a variety of technological mathematical learning environments, including gaming environments, online mathematics classrooms, and social media environments while also pushing us to better understand patterns and asymmetries in student access to important online mathematics learning communities.

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