External Context-Related Research: Digital Resources as Transformers of the Mathematics Teachers' Context

Ghislaine Gueudet and Birgit Pepin

1 Introduction

The contribution of this chapter addresses current issues associated with the evolution of research in mathematics education related to the external context for mathematics teachers' professional activity. The external context combines many elements, such as for example materials, facilities, community support (Manizade et al., 2019). We contend that, in this digital era, digital resources play an essential role in this external context. We primarily focus on research concerning the external context and related with digital resources: this includes research about the digital resources themselves, as well as research about for example community support for the integration of digital resources by teachers, or educational policy linked with digital resources.

In other words, we have concentrated on Medley's (1987) Type I (external context) variable in the context of digital resources. By doing that, we focus on the "materials" variable (Manizade et al., 2019), with a specific focus on digital materials, and on other materials when they are combined with digital materials. For the sake of the size of the chapter, we do not review research concerning school administration (e.g., Hunter, 2019), supervision (e.g., Yang et al., 2021), community support (e.g., Nicol, 2018), and parental support systems (e.g., Wadham et al., 2020), when they are not linked with digital resources.

Returning to our focus, digital technologies have led to tremendous changes in these external context variables: not only changes in the access to available digital

G. Gueudet (✉)
UR Etudes sur les Sciences et les Techniques, University Paris-Saclay, 407 rue du Doyen George Poitou, 91400 Orsay, France
e-mail: ghislaine.gueudet@universite-paris-saclay.fr

B. Pepin
Eindhoven School of Education, Eindhoven University of Technology (TU/E), 23, Cascade, P.O. Box 513, 5600 MB Eindhoven, The Netherlands
e-mail: b.e.u.pepin@tue.nl

A. Manizade et al. (eds.), *The Evolution of Research on Teaching Mathematics*,
Mathematics Education in the Digital Era 22,
https://doi.org/10.1007/978-3-031-31193-2_10

materials, but also changes in what can be called "community support" through digitalization (e.g., on platforms), or more generally for supporting the integration of digital resources. Research in mathematics education about these external context variables has undergone very significant changes. The factors 'causing' these changes in the focus of research studies were related to the technologies themselves, but also to other factors, such as events in the society impacting the educational system (e.g., the COVID-19 pandemic), have played a role. Moreover, these studies not only concentrate on the changes in the external context (Type I) variables; we evidence in this chapter that they also address the influence of digital resources as elements of the external on several online variables and their interactions, in particular D (teachers' pre-post-out-of-class activities) and E (teachers' competence, knowledge and skills), and also the process variables B (students' mathematics learning activities) and C (teacher-student interactions in class).

Given the large number of research publications concerning the teaching of mathematics in the digital era and related to the selected external context variables, we restricted our search to the identification of important trends. In this chapter, we address the following research question:

> Which are the evolutions of research in mathematics education about digital resources as context for mathematics teachers' professional activity?

We considered recent research literature, research published between 2016 and 2020. This was done, because during that period a large body of research emerged that addressed the changes due to digital resources. We also included selected seminal pieces cited in the literature, covering the last 20 years. This included conference proceedings of the following conferences: Congress of the European Society for Research in Mathematics Education (CERME10,[1] 2017), CERME11, 2019); Mathematics Education in the Digital Age (MEDA,[2]2018; MEDA,[3]); International Conference on Technology in Mathematics Teaching (ICTMT13,[4] 2017), ICTMT14,[5] 2019); International Conference on Mathematics Textbooks Research and Development (ICMT3,[6] 2019). Further, we included journal articles: we searched the 2016–2020 issues of Educational Studies in Mathematics (ESM), Journal for Research in Mathematics Education (JRME), ZDM Mathematics Education, Digital Experiences in Mathematics Education (DEME). Moreover, we searched the following books: Hoyles and Lagrange (2010, ICMI Study 17 about technology), Clark-Wilson et al., (2014, 2021), Drijvers et al. (2016), Monaghan et al. (2016), Trouche et al. (2019). We systematically searched for papers or chapters about digital technologies and digital resources as contexts in mathematics teaching: we used the keywords "technology", "digital technology", "digital resources", "digital platforms", "digital

[1] All the CERME proceedings are available at http://erme.site/cerme-conferences/.

[2] https://www.math.ku.dk/english/research/conferences/2018/meda/proceedings/.

[3] https://www.jku.at/linz-school-of-education/steam/meda-conference-2020/.

[4] https://hal.archives-ouvertes.fr/hal-01632970.

[5] https://duepublico2.uni-due.de/receive/duepublico_mods_00048820.

[6] https://tagung.math.uni-paderborn.de/event/1/.

tools", crossed with "teaching", "teacher", "teacher professional development". We excluded papers whose central focus concerned topics addressed in other chapters of this book (e.g., mathematics teacher affect studies; studies on mathematics teacher professional development; mathematics teacher knowledge). Nevertheless, we did not restrict ourselves to the research about digital resources because studying the influence of a given digital resource on the teaching of mathematics often includes studying its use by teachers, or indeed the knowledge development through the use of such resources. At the end of this process, we retained 160 papers and chapters. We noted for each of these papers the questions addressed, and the main results obtained.

We have chosen the following organization for this chapter: After this Introductory Sect. 1, we present theoretical elements guiding our review of the literature (Sect. 2). In Sect. 3, we discuss evolution of research about educational policies and about teachers' professional activity, including assessment. Section 4 focusses on research about the quality of digital curriculum resources, while Sect. 5 concerns selected current evolutions. In Sect. 6 we present our conclusions.

2 Theoretical Frames guiding our Review

In this section we introduce the concepts that guided our review of the literature. We present in particular what we mean by (1) educational technology as compared to digital curriculum resources; and (2) mathematics teachers' professional activity for the purpose of this chapter.

2.1 Digital (Curriculum) Resources and Educational Technology

The literature reviewed in this chapter concerns what we call *digital resources*. Digital resources can be defined as materials that have been conceived and created digitally or by converting analogue materials to a digital format. Examples of digital resources are simulations, models, graphics, e-books, and e-notes intended to make learning more engaging, accessible and contextualized. Over the past decade there have been numerous research studies investigating the use of digital resources for mathematics teaching (e.g., Clark-Wilson et al., 2014; Drijvers et al., 2016; Hoyles & Lagrange, 2010).

Within the general category of digital resources, we distinguish between *digital curriculum resources* (DCRs) and *educational technologies* (ETs), following Pepin et al. (2017a) who defined DCRs as follows:

> It is the attention to sequencing—of grade-, or age-level learning topics, or of content associated with a particular course of study (e.g., algebra)—so as to cover (all or part of) a curriculum specification, which differentiates DCRs from other types of digital instructional tools or educational software programmes. (p. 647).

ETs can be defined as the digital tools that are used in and for education by students or teachers (e.g., platforms). Once these tools are used for teaching (and learning) a particular curriculum content, and built into for example a lesson plan, they would have become DCRs.

Pepin et al. (2017a) observed that research about DCRs pays particular attention to:

1. The aims and content of teaching and learning mathematics;
2. The teacher's role in the instructional design process (i.e., how teachers select, revise, and appropriate curriculum materials);
3. Students' interactions with DCRs in terms of how they navigate learning experiences within a digital environment;
4. The impact of DCRs in terms of how the scope and sequence of mathematical topics are navigated by teachers and students;
5. The educative potential of DCRs in terms of how teachers develop capacity to design pedagogic activities.

For our review, it makes sense that we bring these two together (DCRs and ETs), as teachers are working in environments that are influenced by both. Nevertheless, as we will see in what follows, the distinction between them can contribute to refining our understanding of the research literature: for example, conceptualizing *quality* (see Sect. 4) is a need that emerged from the studies about DCRs.

2.2 Teachers' Professional Activity

Reviewing the literature about digital resources as external context for teachers' professional activity depends on the perspective chosen on this professional activity. Indeed, the external context and this professional activity are intertwined.

Borba and Villarreal (2005) started with the premise that technologies have changed humankind, and emphasized that:

> [...] humans-with-media, human-media or humans-with-technologies are metaphors that can lead to insights regarding how the production of knowledge itself takes place [...] this metaphor synthesizes a view of cognition and of the history of technology that makes it possible to analyze the participation of new information technology 'actors' in these thinking collectives in a way that we do not judge whether there is 'improvement' or not, but rather identify transformations in practice. (p. 23)

The "Humans-with-media" perspective challenges the borders between what is external and what is internal for the teachers interacting with a context comprising digital resources. In terms of the framework of research on teaching mathematics (Manizade et al., Chap. 1), it invites to consider that some of the digital media do not only belong to the offline Type I variables, but can also be considered as belonging to the online Type E variables, e.g., because the teacher-with-media can be seen as a hybrid entity.

Using a historical lens, when digital resources became available for teachers and mathematics classrooms, and teachers were increasingly encouraged to use those 'tools', the research literature also reflected this turn. Questions such as the following were asked: What is the teacher-tool relationship (e.g., Brown, 2009)? In which ways does the 'tool' influence teachers' practices, or indeed their knowledge development concerning the use of the 'tool'? Particular theoretical lenses were developed to face the challenges associated with answering such questions. The instrumental approach to didactics (e.g., Guin et al., 2005) and the documentational approach to didactics (e.g., Trouche et al., 2020a) provide us with theoretical tools which have been useful to face this challenge in our review. The instrumental approach (Guin et al., 2005) was developed to study, and theorize, the integration of computer tools into mathematics education. It distinguishes between an *artifact*, a product of the human activity, designed for a goal-directed activity, and an *instrument* developed by the user along their activity for a given goal. The subject (e.g., the student) develops an instrument, incorporating the artifact (external) and knowledge (internal). Two different subjects even with the same goal do not develop the same instrument. The development of an instrument is called instrumental genesis. This genesis comprises two inseparable processes: instrumentation that describes how the features of the artifact influence the subject's activity; and instrumentalization which describes how the subject modifies the artifact, according to their pre-existing knowledge.

The instrumental approach has been used in mathematics education research to analyze how students learned with educational technologies (the calculator, in particular). The concept of orchestration was introduced by Trouche (2004) to address the question: "How do teachers use technology in class, and why do they use it this way?". With the perspective of the instrumental approach, this question was formulated as: "How do teachers orchestrate the students' instrumental geneses with a given educational technology?". The instrumental orchestration was defined as the systematic organization, arrangement and didactical use of artifacts in the classroom, and therefore, concerns both Type C (interactive mathematics teacher activities) and Type D (pre- and post-active mathematics teacher activities) of Medley's variables.

Introduced by Trouche (2004) and refined by Drijvers (2012), the concept of instrumental orchestration has been a first step in the development of studies referring to the instrumental approach and investigating the teacher's role. This direction of research has rapidly developed, with authors considering teachers' instrumental geneses. In particular, Haspekian (2014) introduced the concept of teachers' double instrumental geneses: the teachers had to learn the technical functionalities of the artifact, and at the same time had to learn how to use the artifact for their teaching goals. While these studies from the instrumental approach still focused on educational technologies, the available DCRs were rapidly growing, and opening new avenues for a mathematics teacher's (curriculum/tool/task) design activities (e.g., Pepin et al., 2017b), individually and collaboratively.

The proliferation of available DCRs and the need to understand its consequences for teachers' professional activity led to the introduction of the *documentational approach to didactics* (DAD, Gueudet & Trouche, 2009; Gueudet et al., 2012; Trouche et al., 2019, 2020a). This approach considered the interactions between teachers and (digital) *resources* mobilized for their teaching. Referring to Adler (2000), the term resource was used with a very general meaning, namely anything that can re-source the teacher's practice is a resource. All the elements of teachers' professional external context: (digital) curriculum resources, students' productions, discussions with colleagues can constitute resources. Even elements of their personal context can become resources for their teaching: discussions with a member of a family, a journal where the teacher notices interesting statistics etc. Teachers' documentational work (searching for resources, selecting them, modifying them and using them in class) is central to their professional activity.

The documentational approach drew on the instrumental approach and introduced a distinction between a given set of resources, and a *document*, developed by the teacher about their use of these resources for the goals of their activity. The document connects the recombined resources and the teacher's professional knowledge. The development of a document was called *documentational* genesis. Like the instrumental genesis, it encompasses two associated processes of instrumentation and instrumentalization.

The DAD viewed a teachers' professional activity as continuous design work and considered teachers as (co-)designers. The availability of a wealth of resources, digital resources in particular, opened new possibilities for teachers but also created new complexity, requiring the development of teachers' (co-)design capacity (Pepin et al., 2017b). According to the documentational approach, Medley's variables Type C (interactive mathematics teacher activities) and Type D (preactive mathematics teacher activities) are strongly linked, and the DAD can be considered as a conceptualization of the links between variables of Type I, C, D and E.

At the end of this review of theory, we retain certain points that seem particularly important to us, and we make certain choices for the rest of the chapter:

- In what follows we use "digital resources" as the most general term. We acknowledge that it is complex to distinguish between Digital Curriculum Resources (DCRs) and Educational Technologies (ET), and that both are often combined in teachers' practice. Nevertheless, this distinction can be useful for some aspects of the literature.
- There are still many terms relevant for our study that are not always precisely defined (e.g., "digital platform" can be used for very different digital resources, depending on the cultural context in particular).
- The instrumental approach introduced the distinction between an artifact (external) and an instrument (both external and internal) developed by teacher interacting with this artifact. The documentational approach introduced a similar distinction between resource (external) and document (both external and internal). While we do not consider here studies focusing on teacher knowledge, we included in our review studies focusing on the interactions between teachers and digital resources.).

3 Evolution of Research about External Context Variables linked with Digital Resources and about their Influence on Teacher Work and Teacher Knowledge

In this section we analyze the evolution of research considering external context variables (Type I) and their influence on the online variables Type C, D and E, keeping our focus on teachers working with DCRs. While Medley (1987) considered that external context variables influenced the Type E-D relation (between teachers' competencies, knowledge and skills and teachers' pre-post-out-of-class activities), we align here with the new model proposed by Manizade et al. (Chap. 1 in this book) by also considering their influence on the Type D-C relation (between teachers' pre-post-out-of-class activities and teachers' interactions with students in class).

We claim that one way that research about external context variables has evolved concerns investigations about educational policies (including official curricula and reforms) addressing the provision and use of DCRs. We present this research and its evolution in Sect. 3.1. The research about teacher integration (or non-integration) of DCRs has also evolved during the last 20 years. Since this integration is strongly influenced by educational policies, we consider that research about teacher integration addresses the influence of Type I variables on Types C-D-E, and will discuss this in Sect. 3.2. One of the levers used by educational policies to influence teacher integration of DCRs is assessment; we focus on this issue in Sect. 3.3.

3.1 Educational Policies as Context for Teachers' Work with DCRs

Educational policies, including curriculum reforms, were not listed by Medley (1987) amongst the examples of Type I variables. Nevertheless, educational policies of their respective countries and institutions are an important element of the teachers' external context. In the "Challenges in basic mathematics education" brochure, Artigue (2011) stresses that "Quality education for all today cannot be achieved without taking technological factors into account" (ibid p. 35). Within mathematics education research, work on educational policies, and how they contribute to shaping the teachers' use of DCRs (how this Type I variable influences Types C and D) or how educational authorities use DCRs in their attempts to shape teachers' practices has developed during the last 20 years.

Educational Policies and Access to Technology

Between 2000 and 2010, many studies investigated how educational policies and projects at a national scale tried to promote through different means the use of technologies in the mathematics classroom (UNESCO, 2005). These studies, often comparing different national situations, examined in particular the issue of access to technology. Specifically, how the policies try to develop this access, and does the actual provision of computers permit the design by the teacher of classroom orchestrations where students exploit the potential of relevant software in their mathematical activity?

Julie et al. (2010) described the situations in four countries (Russia, Hong Kong, Vietnam, South Africa) and one region (Latin-America). They noted similarities in the educational policies of these countries and particularly the acceptance at the political and bureaucratic level of the use of digital technologies for mathematics teaching and learning. The translation of policy into practice took very different forms (in terms of equipment in computers, Internet access, provision of digital resources, and teacher education), according to the different economic situations of these countries. Nevertheless, in all countries they observed that unequal access to technologies remained, and that the actual use of digital technologies in schools was rare.

Sinclair et al. (2010) compared five projects concerning the use of technologies in the teaching and learning of mathematics that had been undertaken at a national scale in different parts of the world. These projects were: Mexico's Enciclomedia, Italy's M@t.abel; the US's Sketchpad for Young Learners, Lithuania's Mathematics 9 and 10 with The Geometer's Sketchpad, and Iran's E-content initiative. The authors introduced three axes, for their comparison of the projects: (1) The curriculum axis (Technology activities support existing curriculum vs. Technology activities encourage new content); (2) The teacher practices axis (Technology activities reify existing teacher practices vs. Technology activities endorse new practices); and (3) Activity design ("Open" activity design for students vs. "Closed" student activity design). Their analysis led them to observe shifts in the projects, such as increasing participation of the teachers as co-designers and epistemic value (supporting the learning of mathematics) of the technologies being progressively foregrounded relative to its pragmatic value (e.g., obtaining a numerical result). Nevertheless, at least in some of the countries, difficulties of access to computers were an obstacle for the implementation of these projects.

Analyzing Evolution of the Policies and their Implications

The work by Trouche et al. (2013) can be considered as a transition between the 'early' (2000–2012) works about educational policies and technologies, where the issue of access was central, to more recent works (2013–2021) where DCRs are used by educational authorities to support teacher design, and at the same time to try to influence teacher classroom practices.

Trouche et al. (2013) analyzed the issues connected to policy implications on two continua/ dimensions, as shown in Fig. 1.

1. bottom-up to top-down policy approaches (e.g., "A top- down policy could be a national directive of imposing access to graphing calculators during national examinations; whereas support for teachers who start to design their own online resources can be seen as a bottom-up policy" (p. 2).); and
2. access—support approaches (e.g., "In the United States, the National Council of Teachers of Mathematics (NCTM), in its 2008 Position Statement, claims

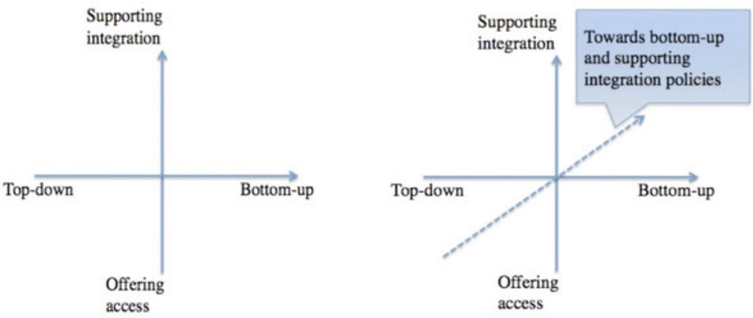

Fig. 1 The two policy dimensions (left) with potential orientation towards bottom-up and supporting policies (right), extracted from Trouche et al. (2013)

> that "all schools must ensure that all their students have access to technology" but also that "Programs in teacher education and professional development must continually update practitioners' knowledge of technology and its classroom applications" (NCTM, 2008, p. 13).)

This evolution of the policies as envisaged by Trouche et al. (2013) is also linked with evolution in research foci. Researchers in mathematics education have increasingly investigated how the educational policies support teacher integration of technologies, and teacher design. The researchers themselves sometimes participate in this effort, by developing curricula in particular. This evolution of the policies (and of associated research) is linked with another kind of evolution: the educational authorities increasingly use DCRs to provide resources for teacher design (supporting this design, acting on Type D variables), with further aim of influencing the classroom practices (acting on Type C variables).

Use of DCRs for Supporting Teacher Design and Shaping Teacher Practices

The Cornerstone Maths project in England is an illustrative example (e.g., Clark-Wilson & Hoyles, 2019) of systematically scaling-up of innovations involving DCRs. It began in 2011 by designing curriculum units that embedded digital technology for learning mathematics (called dynamic mathematical technology, DMT). Such technologies were said to offer the potential for teachers and pupils to (re-)express their mathematical understandings. The national curriculum for mathematics (introduced in England in 2012) specifies the content of the school mathematics curriculum (5–16 years) but offers little pedagogical guidance with regard to the use of technology, implying that teachers should use their judgement about when ICT tools should be used, and how. Consequently, there were no government-funded initiatives to support either secondary school mathematics teachers to develop ways of integrating DMTs into their classroom practices or for mathematics departments to embed such approaches within their school-designed schemes of work. Some use of technology across the secondary curriculum was expected and lightly monitored within the school inspection regime. There was the need to support within-school upscaling. The team judged it important to design and test an in-school professional development toolkit as that could help instructional leaders in schools to support other colleagues and to develop as leaders. The present step (reported in the article) concerned the issue of supporting a large-scale and sustained use of this curriculum units, and this was done via a web-based toolkit. However, there were also difficulties in the context: due to a shortage of mathematics teachers, which acted as a barrier for schools to sustain innovations and innovative practices. The study is also interesting in terms of the importance of the schools as sites for supporting professional development of teachers with respect to their work with digital curriculum resources.

While researchers sometimes participate in the design and dissemination of DCRs to contribute to teacher professional development, the national educational authorities more generally have offered resources to teachers. Their aim is to influence teachers' practices in and out-of-class, and to contribute to their professional development (in particular in the context of reforms). In many countries, digital platforms propose DCRs to teachers.

Concerning platforms and their use by teachers, there are issues involving expectation management. For example, in relation to 'design'- developments, whilst in some countries (and schools) mathematics teachers are to some extent expected to (co-) design their curriculum, in others teachers are expected merely to follow the approved textbook (Trouche et al., 2019). However, the free availability of an enormous number of DCRs, leaves the teacher at a loss in regard to assessing the quality of the available DCRs (see Sect. 4 below), and how to design or amend DCRs? The availability of free resources is also of economic importance, as it raises the issue of competition with commercial resources (e.g., textbooks). In some countries, government institutions provide access to or design DCRs themselves. Others offer opportunities for teachers to engage in the creation of resources. The DCRs' design issues cannot be seen on two dimensions; they are more complex, involving a variety of 'systems' and agents with commercial and economic considerations.

In their investigation of digital platforms for mathematics teacher design (Gueudet et al., 2021), the international team members analyzed the affordances and constraints of commonly used digital education platforms available for mathematics teachers (often provided by governments). They used the documentational approach and the concept of 'connectivity' (Pepin, 2021), introduced by Gueudet et al. (2016) in their study of e-textbooks. These authors distinguish between: (1) macro-level connectivity (e.g., connections made between the book and other websites, or between the resource systems of users); (2) micro-level connectivity (e.g., internal mathematical connections made by the authors between different representations; between the mathematical content and real-life contexts). Transferring this concept to digital platforms, Gueudet et al. (2021) compared three contrasting cases of platforms in three European countries (France, Netherlands, and Denmark), in terms of potential instrumentation and instrumentalization processes for users, and of micro- and macro-level connectivity. They found important differences between the platforms that were strongly linked with national educational policies and national perspectives on teachers' work. For example, in Denmark the use of the platforms was compulsory, and their features were chosen to compel teachers to design objective-driven lessons, according to new national standards. In France, the digital platform was designed to support the implementation of the new curriculum. In the Netherlands, the platform was linked with a policy supporting the use of open educational resources.

As claimed by these authors, digital platforms or other digital resources offered by the institution can be seen as interfaces between educational policies and teacher's practices (in class and out-of-class). Hence studies about educational policies are strongly linked with the studies about teachers' integration of digital resources that we discuss in the next section.

3.2 Teachers' Integration of Digital Resources

In this section we analyze the evolution of research about teachers' integration of digital resources. We argue that the studies about integration firstly tried to identify the factors supporting or hindering the use of technologies by teachers in class. The influence of Type I variables (e.g., educational policies) on the uses in class (Type C variable) was identified by these studies; then they noted the importance of teacher knowledge (Type E) as a factor of integration. As the environment for teachers in terms of available resources has become more complex, specific theoretical frameworks have emerged that allow for the consideration of interactions between variables of Types C, D, E and I.

Early research mostly focused on questions concerning the factors explaining the integration or non-integration of educational technologies. The factors identified were firstly external variables, and integration was considered in terms of in class use. We consider these works to address the influence of Type I on Type C variables. Some of these variables were linked with the national or regional educational policies, in terms of equipment, and technical support offered to the teachers in their schools (Thomas, 2006). These policies also led to the presence, or not, of the technologies in the mathematics curricula, and in the official examinations (Trouche, 2016), and this had a strong influence on the extent of integration. Other external factors concerned the level of the school environment, the school culture and the interactions among colleagues in the school (Forgasz, 2006). If the use of technologies was promoted by the school, with support staff, or by way of a collective project drawing on some kind of technology, the integration was favored. On the other hand, if a group of teachers in a school felt that the use of technology was an additional constraint imposed by the superiors, this constituted a strong obstacle to integration in that school.

Progressively, the studies considered Type I variables as factors explaining integration and Type E variables like teachers' experience and teachers' knowledge (e.g., Attard et al., 2020; Geiger et al., 2016; Goos, 2014). This draws a more complex landscape, involving the interaction of four types of variables. The model (framework of research on teaching mathematics) introduced by Manizade et al. (Chap. 1, this book) considers the influence of Type I variables on the E-D and on the D-C relations. But in this model the E-D-C relation is presented as linear: teacher knowledge influences teacher preparation which in turn influences teacher activity in-class. In the studies we consider here, E-D-C can be viewed as a triangle, with two-way interactions along each side of the triangle. Indeed, the researchers investigated how Type E variables influenced the use of technology in-class (Type C) and out-of-class (Type D), still taking-into-account the external context (Type I). The theoretical perspective of the instrumental approach (Guin et al., 2005) strongly associates the technology (Type I), the teacher's knowledge (Type E), and her practice both in class (Type C) and out-of-class (Type D) and has been used by some authors to study the interactions between teacher knowledge and their use of technology.

Assude (2007) introduced the concept of 'instrumental integration stages'. She proposed four stages of increasing technology use in the classroom. In the *instrumental initiation* stage, the teacher wants the students to learn how to use the software; in the *instrumental exploration* stage, the students explore the software through mathematical tasks; the *instrumental reinforcement* means that the software in used to reinforce mathematical knowledge, and finally in *instrumental symbiosis* stage the software and the mathematics are combined in the students' mathematical activity. Assude (2007) explained that these stages do not correspond to stages of professional development. Rather, even in the same lesson, the teacher could propose software use corresponding to instrumental reinforcement at some point, and to instrumental exploration at another moment. Assude (2005) also foregrounded the importance of time as factor hindering or favoring the integration of technology by teachers. The consideration of time economy as an essential variable was realized in particular by Ruthven (2009), who proposed a theoretical framework combining variables of different natures and including this notion of time economy. This framework called "the Structuring Features of the Classroom Practices" (SFCP) associates five features (considered here as variables) that explain how a teacher integrates a new digital technology:

- the *working environment*: classroom equipment, support in the school (Type I);
- the *activity format*: the teacher and their students have a usual activity format (Type C);
- the *curriculum script*: professional knowledge (Type E);
- the *time economy* (Type I);
- the *resource system*: mathematical tools and curriculum materials in use in the classroom (Type I, with interactions with Type C).

The SFCP framework foregrounded the importance of considering different features for understanding the integration or non-integration of a given educational technology. It played an important role in the evolution of research from studies focused on a single educational technology to studies considering sets of resources, including digital resources of different kinds. Considering these five features led to studies evidencing the interactions between Type I, Type E and Type C variables.

The SFCP framework (Ruthven, 2009) was together with the instrumental approach one of the sources of the documentational approach. One of the main developments brought about by the studies referring to the documentational approach has been the strong association of the teacher's work in the classroom and outside the

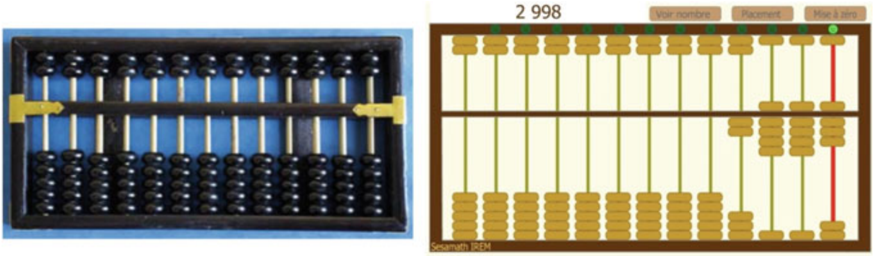

Fig. 2 The Chinese abacus, material (on the left), virtual (on the right) (Gueudet & Poisard, 2018)

classroom, because teacher design as an essential and continuous process takes place in class and out-of-class leading to associations among variables of Types C and D.

Studies referring to the documentational approach (Trouche et al., 2019) considered complex sets of resources (e.g., Gueudet & Poisard, 2018; Wang et al., 2018). The integration by a teacher of an available resource meant that the teacher, using this resource, developed one or several documents. For example, Gueudet and Poisard (2018) studied the integration by a primary school teacher of a set of resources[7] designed by a research team for the teaching of number, using the Chinese abacus, both material and digital, as seen in Fig. 2.

The mathematics teacher planned for her students to use manipulatives. She integrated both the material and the digital abacus in her lesson. This was observed by the researchers through the analysis of the documents developed by the teacher for different aims of her activity. In this case the virtual abacus was never considered as isolated, but as associated with the material abacus, lesson plans, examples of students' productions, and other resources designed by the researchers.

To summarize, the integration of digital resources by teachers is viewed with the perspective of the documentational approach as their integration in teachers' resource systems (Trouche et al., 2020a). Studying this integration process requires to consider the work of the teacher in class (Type C) and out-of-class (Type D), and teacher knowledge (Type E) previously developed that will influence the use of digital resources or developed along the use of these resources. The digital resources offered (e.g., by the educational authorities, but also simply available on the web) are Type I variables; but along their use in class and out-of-class the teachers develop documents, which are mixed entities: research about documents associates Types I-E-D and C variables.

One of the factors shaping links between education policies, digital resources and teachers' work concerns assessment. We consider research about assessment in the next subsection.

[7] http://seminaire-education.espe-bretagne.fr/?page_id=611.

3.3 Assessment, Digital Technologies and Digital Curriculum Resources

The amount of research about assessment has significantly increased since 2000, and this also concerns research about assessment involving digital technologies or digital curriculum resources. Assessment is both a Type I variable, as an aspect of the curriculum, and an outcome of interactions between Type C and D variables, since the teacher designs assessment for their students and implement them in class. Hence, the research about assessment and digital resources concerns Types I, C and D variables. Since most of this research developed during the last ten years, we do not analyze in detail its historical evolution. The most important historical evolution that we want to stress in this subsection is the emergence of research about assessment, and especially about digital resources and assessment.

Stacey and William (2013) introduced a useful distinction, further developed by Drijvers et al. (2016), to categorize this research. Assessment *with* technology concerns the use of technology during an assessment, such as when the students are allowed to use CAS in a written exam. Assessment *through* technology concerns digital assessment, for example with online exercises. According to the distinctions we use here, assessment with technology is linked with DCRs.

Digital technology has been introduced internationally in mathematics curricula. This introduction has been followed by the integration of technology in both formative and summative assessment (Stacey & William, 2013). Teachers designing assessments with technology had new possibilities in their choice of tasks: they could propose rich problems that the students would not be able to solve without technologies (e.g., Leung & Bolite-Frank, 2015). These new possibilities were associated with a new complexity. In particular when designing summative assessments, teachers need to find a delicate balance between proposing tasks that are too complex and creating the possibility of a black-box use of technology. How teachers design assessments with technology also strongly depends on national educational policies, including whether and how technology can be used in mathematics exams (Drijvers et al., 2015). Moreover, technology can be used in very different ways in mathematics exams. Jankvist et al. (2021), presented contrasting examples of the use of CAS in different countries. In Denmark, for example, CAS has been used in the final exams in upper secondary school since 2005, and in lower secondary school since 2013. Jankvist et al. (2021) offered the example of a task presented at the final exam to Grade 9 students, showed that the students could solve this task without any mathematical reasoning, using the CAS as a black-box. They contrasted this example with a task given in Germany for the upper-secondary final exam in Bavaria in 2014. That involved complex mathematical modelling and would have been very difficult to solve without CAS. These differences in exams have strong impacts on the teachers' practices with educational technologies in class.

In the literature assessment *through* technology is called Computer-Aided-Assessment (CAA) or sometimes Computer Assessment System is also used, but we use here CAA in order to avoid a possible confusion with Computer Algebra

Systems. Sangwin et al. (2010) identified three possible outcomes generated by a CAA: a numerical mark; written feedback or statistics concerning a cohort's achievement. A numerical mark and automated feedback on technical errors offer to teachers the possibility of concentrating their own feedback on understanding (Olsher et al., 2016). Statistical overviews of cohort achievement are also a new element in teacher's external context, which can lead to adaptations of the content of the course. These can include adaptions, for example, when using clickers at university level (e.g., Lockard & Metclaf, 2015).

Other outcomes of CAA have been identified. For example, the FASMED project (Formative Assessment in Science and Mathematics Education), Aldon et al. (2017b) also foregrounded the possibilities opened by CAA in terms of automatically generated feedback or statistical overviews. They added possibilities for tracking students' learning paths, through access to statistics, and also to rich data about the students' mathematical activity.

Finally, technological advances have opened the way for a new kind of association between assessment *with* technology and assessment *through* technology resulting from automated scoring. Drijvers (2018) studied the automated scoring of students through digital means, using Intelligent Tutoring Systems. While online assessment sometimes means multiple-choice quizzes focused on technical skills, these new automated scoring tools have the potential to assess complex reasoning, and students' productions with educational technologies, particularly Dynamic Geometry Systems. These new tools offer possibilities for the design of digital assessment, associated with a subtle automatic scoring. As designers, teachers can propose dynamic and interactive tasks; as graders, they save a lot of time and have access to analyses of their students' work (Drijvers, 2018).

To summarize Sect. 3, it can be said that the research in mathematics education concerning the influence of digital resources and other associated Type I variables (e.g., educational policies) has evolved during the past 20 years towards more complexity and more refined analyses of the interactions between different kinds of variables. This refinement has been associated with an increasing complexity of the teachers' working environment in terms of digital resources, which has contributed to the development of specific theories. These theories have evidenced the need for considering different dimensions and the ways they are linked. These dimensions corresponded to Type I (e.g., the digital resources themselves, but also, for example, the time economy), Type C (teachers' practices in class), Type D (teachers' practices out-of-class) and Type E (teacher knowledge).

4 On the Quality of Digital Curriculum Resources

In this section, we review the literature concerning the quality of digital resources in teachers' contextual environment, as we assume that a conceptualization of quality influences teachers' choice of resources. Considering that over the past decade a considerable amount of research in mathematics education has investigated teachers' lesson planning and enactment of designed lessons involving the use of digital resources (e.g., Aldon & Trgalová, 2017; Clark-Wilson et al., 2014, 2021; Trouche et al., 2019), it is surprising that a similar amount has not attended to the resources' quality within teachers' contexts of teaching. However, there are a number of studies attending to this issue in teacher design of their curriculum. In conceptualizing the quality of DCRs, we have to distinguish between (1) different aspects of quality criteria, and (2) quality of which kinds of digital resources.

4.1 Quality of Dynamic Mathematics Materials

Whilst there is no consensus of what 'platform', actually is, many dynamic mathematics materials are 'deposited' on platforms of some kind. The literature presents research on numerous online platforms providing a large number of Open Educational Resources (OER) for teaching mathematics: e.g., GeoGebra Materials, 2016; LearningApps, 2016; I2Geo, 2016. Teachers find it difficult to choose amongst the enormous quantity of resources and note inconsistency in their quality (Trgalová et al., 2011). Quality variability is particularly likely if the platform is not supported by 'gatekeepers', that is a dedicated 'editorial team' that checks on the quality of 'self-made' resources that are often freely available or shared by different types of users (Camilleri et al., 2014).

There are several platforms that provide mechanisms for assessing the quality of their resources, in order to be able to rank the materials according to their quality. In the context of the Intergeo project, for example, Trgalová et al., (2011, p. 1163) identified nine "relevant indicators" of the quality of dynamic geometry resources on their platform I2Geo: "metadata, technical aspect, mathematical dimension of the content, instrumental dimension of the content, potential of the DG, didactical implementation, pedagogical implementation, integration of the resource into a teaching sequence, [and] usage reports." In that project a questionnaire was developed based on these nine quality indicators. The assessment of the quality of a particular resource on the I2Geo platform required users to respond to nine broad statements, which can be extended optionally to 59 questions (ibid).

In her search for quality aspects of 'dynamic materials', Kimeswenger (2017) interviewed experts in electronic resource development, who described their views on educationally valuable use of dynamic materials. The analysis of the expert interviews revealed eight core "quality dimensions" as crucial factors: (1) author, (2) mathematical content, (3) resource type, (4) supporting the learning of mathematics, (5) integration into teaching, (6) advantages of dynamic material, (7) design and presentation, and (8) technical aspects. She also provided examples of these quality criteria. For example, for the criterion "Supporting the learning of mathematics", the following question is asked: "Does the dynamic material support the learning of mathematics?" and in of the following ways:

- Allows students to explore with the dynamic construction;
- Allows students to discover mathematics;
- Encourages students to make their own assumptions;
- Encourages students to formulate insights.

She also emphasized that the majority of experts stated that there is/was/should be a strong correlation between the 'quality' of the author and the created material, and the authors' views on learning.

In another study, Ladel et al. (2018) developed the ACAT framework for the evaluation of apps, in order to provide information on quality of apps and also on the various possibilities for teachers to evaluate apps in an efficient and reliable way. Artifact-Centric Activity Theory (ACAT) is a model developed to capture complex situations that arise when digital technology is introduced in classroom situations. They proposed five steps and questions for the evaluation: (S1) What is the mathematical *object* of the app? (S2) How do students *interact* with the mathematical object, mediated by the app? (S3) How does the *interaction* develop? (S4) Is the app suitable for teaching and learning the mathematical object? (S5) How can the app be used in classroom instruction?

Leaning on selected theories in mathematics education (e.g., cognitive load), Donevska-Todorova and Weigand (2018) developed three design principles for 'resources and tasks for technology-enhanced teaching and learning mathematics'. These were; (P1) Reduction of the total cognitive load by decreasing extraneous cognitive load; (P2) Reduction of the total cognitive load by decrease of the intrinsic cognitive load; (P3) Connection of active engagement and focus on mathematical content. Donevska-Todorova (2019) also developed a framework for evaluating the quality of tablet apps in primary mathematics education and their integration in student-centered learning environments. Focusing on the didactical potentials of tablet-apps, she identified six overarching categories: (1) mathematical content and relation to curriculum, (2) communication, collaboration and cooperation, (3) differentiation, (4) feedback and assessment, (5) connections and networking and (6) logistics. She claimed that the proposed model may become "meaningful for teachers' decision making when selecting and implementing touchpad-apps in their instructional practices but also for developmental surveying of existing apps, their re-designs and further novel designs involving identified potentials" (p. 121). Based on this framework, Donevska-Todorova and Eilerts (2019) also developed review criteria related to a particular content area: *space and shape*.

4.2 Quality of E-Textbooks

The e-textbook can be seen a system of digital curriculum resources. In their efforts to identify aspects of the quality of e-textbooks, Pepin et al. (2015) distinguished between three models of currently available e-textbooks – dynamic, evolving or "living", and interactive. In the "dynamic" model, a static textbook (traditional or digital) is linked to other learning objects. In the "living" model, textbooks are dynamically and cumulatively authored by a community, often a community of teachers (e.g., Gueudet et al., 2013). The third model of e-textbooks – interactive – is based on a toolkit model, and is anchored in a set of learning objects, where tasks and interactive materials can be linked and combined in different ways. These distinctions also relate to the quality aspect of 'coherence'. Drawing on Gueudet et al. (2013) and Yerushalmy and Chazan (2008), distinguished between two types of coherence in textbooks. First, coherence of the design of a textbook encompasses aspects such as mathematical correctness, epistemological stance toward mathematical topics, sequencing that avoids gaps in the mathematical progression, consistent handling of mathematical objects, and consistency with national curricula. These aspects of coherence are constituted in the textbook's expositions, its tasks, and ways in which technology is made available to students. The second type of coherence-in-use is the coherence of what teachers actually propose to their students, drawing on the textbook, or on other curricular material. The e-textbook is changing the boundary between coherence of design and coherence in use. Issues pertaining to sequencing and availability of technology, which have been considered aspects of design of a linear textbook, are becoming aspect of coherence in use, as teachers re-design the textbook (e.g., Gueudet, et al., 2018). In order to help teachers to use digital resources in ways that provide a coherent learning trajectory for students, Confrey and her team (e.g., Confrey et al., 2017) have designed tools and materials to help teachers develop learning trajectories through a "bag of resources" in alignment with particular standards (in this case US Common Core State Standards).

4.3 Quality of Dynamic Mathematical Tasks

Concerning the quality in dynamic mathematics tasks, one of these quality aspects related to authentic tasks, which require realistic objects and questions (e.g., Jablonski et al., 2018). An example is MathCityMap which takes up the idea of outdoor mathematics through the creation of math trails by using an app and a web portal in which every registered user is allowed to create and publish their own tasks. Through a constantly growing community and the provision of a particular quality of the published material, the system is based on a multistep review process and several criteria for published tasks. Criteria for tasks in a MathCityMap math trail include the following: (1) Uniqueness (every task should provide a picture that helps identify the object of the task and what the task is about); (2) attendance (authenticity- the task can only be solved at the object location); (3) activity (embodied mathematics, i.e., mathematics can only be fully comprehended through an active experience); (4) multiple solution (solvable in different way); (5) reality (meaningful relevance); (6) hints (every task should provide at least one hint in terms of solving the task); (7) school math and tags (the task should feature a connection to school math); (8) solution formats (e.g., each task should be based on a meaningful answer format, such as intervals for measurement tasks); (9) tools (the task should be solved without special and extraordinary tools). A math trail idea is a combination of different tasks that should harmonize as a trail. Therefore, the whole trail comes into the review process after every task of a trail has been through it.

4.4 Quality of Curriculum Programs

Choppin et al. (2014) created a typology for analyzing the quality of digital curricula in mathematics education. They documented two distinct curriculum types, individualized learning programs and digitized versions of traditional textbooks. In order to help educators better understand the characteristics of these materials, they developed and applied a framework to analyze a representative sample of digital curriculum programs. The framework has three distinct themes:

Theme 1 relates to students' interactions with the programs, and was subdivided into three categories that describe students' interactions with the programs:

1. Student learning experiences (what students see and do in the program);
2. differentiation/individualization (features that enable teachers to select content according to their perceptions of students' abilities); and
3. social/collective features (features of the programs aimed at virtually connecting groups of students or other stakeholders).

Theme 2 concerns curriculum use and adaptations, that address the flexibility of each program in terms of providing tools and resources to sequence and design lessons for teachers. Choppin et al. (2014) analyzed programs according to four categories that provide teachers the ability to:

1. Map and sequence lessons;
2. Design content of lessons;
3. Locate and use multi-media presentation materials; and
4. Make and store notes for future planning.

Theme 3 encompasses the analysis of assessment systems. As assessment systems offer the potential for online assessments and the ability to automatically analyze and report assessments, they proposed criteria for the analysis of the assessment systems, built into the programs, and focused on the following four categories of functionality:

1. Create assessments;
2. Record and store results of assessments;
3. Generate dashboard or other summaries of data; and
4. Generate and transmit reports/results to multiple audiences, including teachers, parents, and administrators.

Choppin et al. (2014) claimed that while the programs offered some of the features identified as transformative, particularly with respect to assessment systems that rapidly and visually report student performance, there were many features that did not take full advantage of the digital medium.

To summarize this section, it appears that there is a huge variety of DCRs. As different DCRs (and types of DCRs) have different affordances and constraints (also as compared to analogue materials), perceptions of what is 'quality' also vary: from interactive, over add-on, to dynamic materials, to name but a few of the quality notions. Moreover, notions of didactic quality seem to change their 'appearance' in teachers' work with digital resources (e.g., what does consistency or coherence means in e-textbooks).

5 Recent Developments and Future Directions for Research

We have seen in the previous sections that the progress of research on teaching taking-into-account digital resources is manifested through development of theories, which propose different dimensions to understand the interactions of teachers with available digital resources, and the consequences of these interactions. We foresee that this progress will continue, since teachers' working environment (and hence their professional activity in class and out-of-class) continue to evolve with new elements, predictable or not at this stage.

In this section, we consider three themes that correspond to recent evolutions in the external contexts of mathematics teachers' professional activity and that are giving rise to a growing body of research. We have chosen themes illustrating different elements of the external context for teachers: (1) official curricula and the integration of programming in these curricula; (2) the collective work of teachers in different kinds of teams or networks related to digital resources; and (3) the COVID-19 pandemic, which foregrounded the importance of digital tools, in particular for distance teaching (and learning).

5.1 Introduction of Programming in the Mathematics Curricula Internationally and Consequences for Teachers

Research in mathematics education about programming is not new (e.g., Papert, 1993), albeit the interest for programming in mathematics education declined at the end of the 90s and beginning of 2000s.

Then a major change happened in the official curricula internationally: between 2010 and 2020 that saw programming introduced in primary and secondary school curricula of many countries (see e.g., Haspekian, 2017; Misfeldt et al., 2020; Modeste, 2015). In some countries programming was introduced as a specific discipline and has been taught by computer science teachers. In others it has been inserted in mathematics curricula and has been taught by mathematics teachers. These changes in curricula have led to a renewal of research about programming and computational thinking in mathematics education. While the early works in the 70s and 80s were mostly focused on students' learning and their development of computational thinking, researchers now acknowledge the importance of the teacher (Benton et al., 2017; Pérez, 2018), and the need to investigate how teachers integrate programming in their mathematics courses.

In the Scratchmath project in UK, Benton et al. (2017) designed a curriculum for primary school teachers, and the team studied teachers' implementation of this curriculum. The authors observed that some primary school teachers were not familiar with programming, and that the concept of an algorithm was difficult for them. Nevertheless, the framework designed by the researchers supported teachers in their implementation of strategies with their students. The choice of strategies depended in particular on their confidence with Scratch. The teachers also made different choices in terms of emphasizing programming, or mathematics. This research was similar to other studies evoked in Sect. 3.2 concerning teacher integration of digital resources, and how they were integrated.

The issue of the links between programming and mathematics that these developments draw attention to has been investigated in several studies. Pérez (2018) proposed a framework evidencing different dimensions of computational thinking; this framework has been actually developed has a tool for secondary school mathematics teachers engaging for the first time with the teaching of programming and facing the need to combine mathematical thinking and computational thinking. Misfeldt et al. (2020) examined the official curricula in Denmark, Sweden and England, and examined the enacted curriculum through selected cases. They identified four possible types of relations between mathematics and programming: "(1) specific relations to mathematical concepts or processes [.]; (2) explicit relations to mathematics [.]; (3) implicit relations to mathematics, [.]; and (4) no or weak relations to mathematics." (ibid. p. 259). How teachers can and do combine mathematics and programming in this new context is a promising and important direction for research.

The role of the teacher in courses combining programming and mathematics has already been the subject of research at university level, where programming has been present in some courses since the early 2000s. One example is the MICA courses (Mathematics Integrated with Computers and Applications) at Brock University in Canada. Buteau and Muller (2014) evidenced that teachers in these courses also intervened as policy makers, and that this role was essential for implementing and sustaining the intervention at the departmental level. In their recent research, Buteau et al. (2020) used the instrumental approach and the theory of orchestration, to study how a teacher in MICA courses supported students' instrumental geneses with a programming language, for mathematical investigations. They showed that the lab setting was a key element in the teacher's orchestration, where the work of the students on their projects were supported. Lockwood & Mørken (2021) called for more research exploring the relationships between computing and mathematics at university level, and this is also certainly a promising direction for research.

Studies about teaching programming and mathematics have much in common with the recent studies evoked in Sect. 3.2, about how teacher integrate digital resources. They have also investigated how a Type I variable (the introduction of programming in the official curriculum) affects Type C, D and E variables. We note, nevertheless, that the nature of Type E variables in this case is specific, since it questions the links between mathematics and programming.

5.2 Teacher Collaborative Activities

Research on mathematics teachers' collective work with DCRs has developed significantly over the last 20 years, and particularly in the most recent of these years. This includes research on teacher's work in established communities as well as in spontaneously set-up communities with a common purpose, and it also includes the collective work online, in schools, at home or in institutions that offer collective work as professional development.

Regarding collective work in organized teacher collectives, Gueudet et al. (2016) provided a window into the collective design of an e-textbook, which was made possible by new "digital" opportunities: e.g., platforms, discussion lists. The context of the collective work was provided by the French Sesamath teacher association and their design of a Grade 10 e-textbook in terms of the "functions" chapter. This study concerned the influence of Type I variables on the collective teacher design (Type D). Here the Type I variables include the digital platform, but also from the point of view of an individual teacher, member of the group, the other members of the group (in this case mathematics teacher and computer science specialists). At the individual level, these variables also influence a member of the group in terms of professional knowledge (Type E), and the Type D-E variable interaction, as described in Medley's framework.

In terms of teachers working 'spontaneously' with colleagues, Trouche et al. (2020b) reported on the collective work of an experienced mathematics teacher at secondary level, who has also worked as a teacher educator in a university department. They investigated her work and professional development with colleagues (e.g., lesson planning), with a particular interest in the digital resources, including both digital curriculum resources (e.g., e-textbooks, online resources) and digital technologies (e.g., for communicating, sharing). Results show that her transition to DCRs was a critical process in her professional learning trajectory. Of importance were the notions of *resource system* for studying the teacher's activity as a whole, and of *documentational trajectory* for studying the teacher's activity over the time. In other words, they point to a teacher's resource system (an organized system of digital and analogue resources) and his/her collective work (over time) as major ingredients for professional learning and development. The authors claimed to contribute to a better understanding of the impact of digital resources on mathematics teachers' work and professional learning over time, and of the ways the context of collaboration shapes their professional work and learning. Hence, they consider the interactions between variables of Type I (digital resources, colleagues) and variables of Type C, D and E (since the practice and the knowledge of the teacher evolved).

Other recent research concerns teacher collective work with DCRs in a context of preservice or in-service teacher education, hence interactions between Type I and Type J offline variables (with consequences for Types C, D and E variables). Although the collective work in the context of teacher education has been researched for more than 20 years, the use of various digital means opened new possibilities, in particular in terms of blended or distant learning, that have been recently investigated. For example, in a study by Borba et al. (2018) online pre-service teacher distance education is the context. The purpose of this study was to analyze the role of digital technologies in two specific contexts: how teachers, tutors, and students play a role in producing interactive DCRs, and how digital technologies themselves can play a role in teaching distance learning courses. However, for these roles to emerge, the authors pointed to the need for participants in online courses to interact collaboratively. Their results showed that the roles are related and that digital technologies transform both teacher and student roles and participation in the virtual classroom, with the result that an 'agency of media' (meaning here the possibility to combine different media, to change media when relevant) emerges in online mathematics education.

Lesson Study (LS, Takahashi, 2014) provides context in which teachers collaborate to design lessons (through cycles of plan—teach—reflect). LS has been investigated for more than 20 years by mathematics education researchers; but they are renewed by digital resources, allowing in particular the organization of blended training. Joubert et al. (2020) reported on a Lesson Study in a blended approach to support isolated mathematics teachers (who could not meet face-to-face), to use and integrate mobile technology in their teaching. They identified eleven aspects playing an important role in the processes: technology; collective/group; learning management system; online facilitation; technological pedagogical content knowledge; (mobile) learning strategies; a lesson planning form; backward design; time; photos, videos and reports; and reflection questions. The eleven aspects that emerged led to the development of a framework consisting of three dimensions of LS, namely Collaboration, Instructional Development, and the Iterative Improvement Process, supported by the identified aspects.

Massive Open Online Courses (MOOCs) are another kind of digital curriculum resource that now contribute in mathematics teachers' in-service education. Hollebrands and Lee (2020) reported on the design of three MOOCs for mathematics teachers' professional learning. The designs were based on principles of effective online professional development that included: self-directed learning, learning from multiple voices, job-connected learning, and peer-supported learning. The team examined how these design principles were enacted in the development of the MOOC-Eds and how they influenced the engagement of 5767 participants. Evidence showed that the three MOOC-Eds were successful in "allowing two experienced mathematics teacher educators to design engaging experiences for teachers that have shown to have positive impacts on their beliefs, perspectives and practices in teaching mathematics and statistics" (p. 872). The authors claimed that scaling-up professional development for teachers requires much more than simply transforming typical in-person experiences into online videos and readings. As they grounded their design in an interconnected model of professional growth (Clarke & Hollingsworth, 2002) and used best practices from mathematics teacher education and design principles for online teacher engagement, they claimed that they could establish a large-scale professional development program that engaged and impacted teachers from around the world.

An assessment of design principles used to guide the development of MOOCs for teachers was conducted by Aldon et al. (2017a). They examined how instructors' practices influenced collaboration and participation in MOOCs implemented in France (eFAN Maths MOOC) and Italy (UniTo: Geometria MOOC and Numeri MOOC). The MOOCs from these countries supplemented discussion forums with the use of other collaborative tools (e.g., Padlet, social networks, collaborative project spaces). There were differences noted in how the instructors facilitated collaboration. With those in the French MOOCs focused on fostering local collaboration while the Italian MOOCs encouraged collaboration among all participants within the MOOC. The study pointed to the importance of examining not just the design of a MOOC for teachers, but also how such MOOCs are enacted and experienced by participants.

Many possibilities for combining digital resources and mathematics teachers' collective work exist, and can have different consequences for teacher knowledge and teacher practice (within a teacher training program or more informally). Cai et al. (2020) suggest that digital technologies can contribute to the design of shared knowledge base for mathematics teachers and for researchers in mathematics education. The effective realization of these new possibilities constitutes a challenge for the mathematics education research communities and a promising direction for future research.

5.3 Digital Resources in Mathematics Education, Equity, and COVID-19

The socio-economic environment in which students live is also a critical component of the professional context for teachers. Research in mathematics education is increasingly taking this context into account, and there is interest in how teaching can contribute to equity (Forgasz & Rivera, 2012). Questions have been raised in particular about the use of technologies because students have different accesses and relationships to technology, depending on their socio-cultural background, how can teaching be equitable when teachers use technology in their mathematical courses? Can they use technology to create opportunities for students from different socio-cultural backgrounds? Forgasz et al. (2010) present a synthesis or research investigating such issues. They showed that obstacles to the use of technologies linked to issues of access seem to have decreased in rich countries, whereas they remain prevalent in developing countries. They also presented teaching interventions (in rich countries) where technology was used to create mathematical learning opportunities for all students.

While resources in the form of computers, software, and Internet access have tremendously increased since these early studies, important disparities in terms of access to digital technologies remain at an international level (e.g., Bethell, 2016). The external context for mathematics teachers is thus very different according to the country in which teachers work, and we acknowledge that research synthesized in this chapter mostly addresses the context of teachers in rich countries. Nevertheless, even in these rich countries, socio-economic differences exist between different schools. In the U.S. Kitchen and Berk (2016) argue that the use by teachers of computer assisted instruction in schools that predominantly serve low-income students may favour work on technical tasks, instead of problems fostering a rich mathematical activity. This reduces the opportunities of learning for these students.

Research in mathematics education has increasingly considered equity issues, and how digital resources can contribute to equitable teaching. Referring to the framework guiding this book, we consider that this research investigates how a Type I variable (digital resources) can be used to counterbalance negative effects of another Type I variable (the socio-economic background) on the relations between processes (Types C and B) and product (Type A, learning outcomes). For example, in a study conducted in a primary school in 'unfavorable' (in socio-economic terms) contexts in Mexico, Sandoval and Trigueros (2021) observed that when primary school teachers create a classroom culture grounded on mutual respect, listening to each other, and combined this with the use of software supporting students' problem-solving activity, all the students can grasp the important mathematical ideas.

Finally, major changes in the mathematics teachers' external context in recent years have been due to the COVID-19 pandemic. From Kindergarten to University, teachers all over the world were forced to teach online of at least some of the time over several months. This dramatic context is also a new theme (or a large set of new themes) for research.

From the first lockdown, researchers in mathematics education launched questionnaires to investigate the consequences of this situation for teachers' practices, including naturally their use of technologies.

Drijvers (2020) and his colleagues, for example, conducted a study entitled "Math@Distance study" in Flanders, Germany and the Netherlands. They asked 1719 secondary school mathematics teachers about their teaching practices during lockdown. The use of digital resources was an important aspect in their study. They observed that the use of video conferencing software drastically increased. More surprisingly, the use of online exercises and online learning environments decreased. During the synchronous video lessons, the teacher presents, the students answer questions; but the collective work of students was scarce. Hodgen et al. (2020) reached similar conclusions, analyzing questionnaires and interviews with 49 heads of mathematics departments in secondary schools in England. Moreover, disadvantaged pupils were less engaged in the teaching due to problems of access, low parental support, and new personal and familial difficulties. Solomon (2021) stressed that equity is one of the most difficult challenges in the COVID-19 context; at the same time this context presented new opportunities for teachers to access student thinking using some of the technologies utilized during distant teaching.

Technological equipment and online teaching practices have changed since these "early" in the pandemic chronology studies. We assume that "Which digital resources can support teachers, and students, in secondary school mathematics for distant or hybrid teaching in a context of pandemic?" will remain an important research question for some years to come. This is because the pandemic unfolds over several years and the research will need several years of setbacks to understand these phenomena.

6 Conclusions

The question leading this chapter was:

> How has the evolution of research in mathematics education about digital resources impacted the context of mathematics teachers' professional activity?

Reviewing the relevant literature, we have observed a very large number of changes in the research studies. We have selected and presented particular directions in these that seemed to be the most pertinent. Our focus was not only on digital resources themselves (e.g., e-textbooks, mathematical software, digital platforms, online assessment systems, tools for distant collaboration, videos, and other kinds of digital media), but on various aspects of teachers' external context linked with digital resources: community support, or time economy for example. Moreover, we have shown that the research studies on these topics strongly associate Type I, Type E, Type D and Type C variables.

The changes we observed and insights we gained can be summarized as follows:

- *Evolution in the research about educational policies*: early studies considered the policies in terms of material equipment of the schools, and then the place of the educational technologies in national curricula. They evidenced some discrepancies between the intended curriculum and the enacted curriculum, linked with a lack of equipment, and teachers' professional development concerning the use of technologies. The role of technology in national assessments (often very limited) was an important factor explaining the discrepancies. Recent research has been more focused on DCRs (e.g., digital platforms), proposed by the educational authorities to support teachers' design, in particular in a context of reforms.
- *Evolution in research about teachers' integration of digital resources*: the research questions evolved from the integration of a single educational technology by a teacher to questions about complex sets of resources available in a digital environment. This evolution in the questions being asked was linked with the development of theoretical frameworks and new conceptualizations of digital resource integration by teachers. New questions arose about the role of the teacher. In a digitalized context, students develop as self-directed learners together with support from their peers, and teachers become the scaffolders of knowledge development. The research also highlighted new requirements for the teachers, including a need to change their perspective on the mathematics (e.g., seeing programming as an integral part of mathematics). Finally, an increasing number of studies considered the potential and actual collective dimensions of teachers' work and how these have been impacted by digital resources.

- *Evolution in the research about the digital resources*: Concurrent with development and use of new digital resources, new issues have emerged and have been developed. These included the quality of digital resources. Research has produced different kinds of tools for assessing this quality, and revealed the need to re-conceptualize quality, to consider new possibilities for connectivity, and new perspectives on the teachers as designers of their own curriculum (Type D and Type C variables, since the design takes place out-of-class and in-class). It has become evident that new technologies and digital resources necessitate and drive new pedagogical approaches. In other words, questions are not only concerned with how the teacher may be able to suitably integrate resources, but with the digital resources themselves (e.g., digital learning environments) require and force teachers to take a different stance and build their 'teaching' (or coaching) around the new digital environment.

Different causes were combined to produce these changes. Each time that a new digital resource is introduced in school, it is a new element in the external context for teachers and opens the way for research on the potential of this digital resource, on its actual use, on its impact on teaching and on teacher knowledge (Types C, D and E). The general evolution of research on mathematics education has also influenced research about DCRs as part of teachers' context (this can involve any of the variables). New research issues (e.g., assessment; teachers' and students' collective work) encompass studies about digital resources and mathematics teaching. The socio-political turn, and the value of research addressing equity issues is also an important trend in recent research present in the literature we reviewed.

We foresee further evolution in research in all the directions mentioned above that stress the need for more research on:

- Educational policies pertaining to the offering of digital curriculum (e.g., digital platforms) and the tensions between supporting teacher creativity (with these resources) and efforts of the national agencies offering the resources to help teachers align with education reforms;
- Provision and quality of particular DCRs (e.g. for particular mathematical topic areas, including programming);
- Digital assessment procedures, developing from simple tests to complex digital environments where students can work collaboratively on tasks;
- Distant and hybrid teaching at all school levels, and its links with equity issues.

Moreover, in an external context requiring teachers to become the designers of their own curriculum, more research is needed on educative digital resources for teacher professional development (Type J variable). We contend that digital resources as elements of the external context for mathematics teachers' professional activity are often underestimated, and their affordances, constraints, and potential to drive under-researched. For us, this review was an eye-opener, and we believe that there are many avenues for mathematics education research in this field.

References

Adler, J. (2000). Conceptualising resources as a theme for teacher education. *Journal of Mathematics Teacher Education, 3*, 205–224.

Aldon, G., Arzarello, F., Panero, M., Robutti, O., Taranto, E., & Trgalová, J. (2017a). MOOC for mathematics teacher training: design principles and assessment. In G. Aldon & J. Trgalová (Eds.), *Proceedings of the 13th International Conference on Technology in Mathematics Teaching* (pp. 200–209). Lyon, France.

Aldon, G., Cusi, A., Morselli, F., Panero, M., & Sabena, C. (2017b). Formative assessment and technology: Reflections developed through the collaboration between teachers and researchers. In G. Aldon, L. Bazzini, & U. Gellert (Eds.), *Mathematics and technology* (pp. 551–578). Springer.

Aldon, G., & Trgalová, J. (2017). In *Proceedings of the 13th International Conference on Technology in Mathematics Teaching*. Lyon, France: Institut Français de l'Education.

Artigue, M. (2011). *Challenges in basic mathematics education.* UNESCO. https://unesdoc.unesco.org/ark:/48223/pf0000191776_eng

Assude, T. (2007). Teachers' practices and degree of ICT integration. In D. Pitta-Pantazi & G.N. Philippou (Eds.), *Proceedings of the Fifth Congress of the European Society for Research in Mathematics Education* (pp. 1339–1348). Larnaka: Department of Education, University of Cyprus.

Assude, T. (2005). Time management in the work economy of a class. *Educational Studies in Mathematics, 59*(1), 183–203.

Attard, C., Calder, N., Holmes, K., Larkin, K., & Trenholm, S. (2020). Teaching and Learning Mathematics with Digital Technologies. In J. Way, C. Attard, J. Anderson, J. Bobis, H. McMaster, & K. Cartwright (Eds.), *Research in mathematics education in Australasia 2016–2019* (pp. 319–347). Springer. https://doi.org/10.1007/978-981-15-4269-5_13

Benton, L., Hoyles, C., Kalas, I., & Noss, R. (2017). Bridging Primary programming and mathematics: some findings of design research in England. *Digital Experiences in Mathematics Education, 3*(2), 115–138. https://doi.org/10.1007/s40751-017-0028-x

Bethell, G. (2016). *Mathematics education in Sub-Saharan Africa: Status, challenges, and opportunities.* World Bank.

Borba, M. C., Chiari, A. S., de S., & de Almeida, H. R. F. L. (2018). Interactions in virtual learning environments: New roles for digital technology. *Educational Studies in Mathematics, 98*(3), 269–286. https://doi.org/10.1007/s10649-018-9812-9

Borba, M., & Villarreal, M. (2005). *Humans-with-media and the reorganization of mathematical thinking: Information and communication technologies, modeling, experimentation and visualization.* Springer (Mathematics Education Library).

Brown, M. W. (2009). The teacher-tool relationship: Theorizing the design and use of curriculum materials. In J. T. Remillard, B. A. Herbel-Eisenmann, & G. M. Lloyd (Eds.), *Mathematics teachers at work: Connecting curriculum materials and classroom instruction* (pp. 17–36). Routledge.

Buteau, C., Muller, E., Santacruz Rodriguez, M., Gueudet, G., Mgonbelo, J, & Sacristán, A.-I. (2020). Instrumental orchestration of using programming in mathematical investigations. In A. Donevska-Todorova, E. Faggiano, J. Trgalová, Z. Lavicza, R. Weinhandl, A. Clark-Wilson & H.-G. Weigand (Eds.), *Proceedings of the MEDA2020 Conference* (pp. 443–450). Linz University.

Buteau, C., & Muller, E. (2014). Teaching roles in a technology intensive core undergraduate mathematics course. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital Era* (pp. 163–188). Springer.

Cai, J., Morris, A., Hohensee, C., Hwang, S., Robison, V., Cirillo, M., Kramer, S. L., & Hiebert, J. (2020). Improving the impact of research on practice : Capitalizing on technological advances for research. *Journal for Research in Mathematics Education, 51*(5), 518–529. https://doi.org/10.5951/jresematheduc-2020-0165

Camilleri, A. F., Ehlers, U. D., & Pawlowski, J. (2014). *State of the art review of quality issues related to open educational resources (OER)*. Publications Office of the European Union.
Choppin, J., Carson, C., Borys, Z., Cerosaletti, C., & Gillis, R. (2014). A typology for analyzing digital curricula in mathematics education. *International Journal of Education in Mathematics, Science, and Technology, 2*(1), 11–25.
Clarke, D., & Hollingsworth, H. (2002). Elaborating a model of teacher professional growth. *Teaching and Teacher Education, 18*, 947–967.
Clark-Wilson, A., Donevska-Todorova, A., Faggiano, E., Trgalová, J., & Weigand, H.-G. (2021). *Mathematics education in the digital age*. Routledge.
Clark-Wilson, A., & Hoyles, C. (2019). A research-informed web-based professional development toolkit to support technology-enhanced mathematics teaching at scale. *Educational Studies in Mathematics, 102*(3), 343–359. https://doi.org/10.1007/s10649-018-9836-1
Clark-Wilson, A., Robutti, O., & Sinclair, N. (2014). *The mathematics teacher in the digital Era*. Springer.
Confrey, J., Gianopulos, G., McGowan, W., et al. (2017). Scaffolding learner-centered curricular coherence using learning maps and diagnostic assessments designed around mathematics learning trajectories. *ZDM Mathematics Education, 49*, 717–734. https://doi.org/10.1007/s11858-017-0869-1
Donevska-Todorova, A. (2019). Towards a theoretical foundation for quality tablet app-enriched learning environments in primary school mathematics education. *International Journal of Technology in Mathematics Education, 26*(3), 121–129.
Donevska-Todorova, A., & Eilerts, K. (2019). Designing and disseminating review criteria for quality of tablet apps in primary school mathematics. In B. Barzel, R. Bebernik, L. Göbel, M. Pohl, H. Ruchniewicz, F. Schacht, D. Thurm (Eds.), *Proceedings of the 14th International Conference on Technology in Mathematics Teaching* (pp. 279–288). Essen, Germany.
Donevska-Todorova, A., & Weigand, H.-G. (2018). Design principles for resources and tasks for technology-enhanced teaching and learning mathematics. In H.-G. Weigand, A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, N. Groenbaek, & J. Trgalová (Eds.), *Proceedings of the MEDA Conference* (pp. 51–58). Copenhagen, Denmark: University of Copenhagen.
Drijvers, P. (2020). Math@distance: Distance mathematics teaching during the COVID-19 lockdown. In *Moving forward in the midst of a pandemic: International lessons for math teachers*. Hosted by the U.S. National Commission of Mathematics Instruction National Academies of Sciences, Engineering, and Medicine.
Drijvers, P. (2018). Digital assessment of mathematics: Opportunities, issues and criteria. *Mesure Et Évaluation En Éducation, 41*(1), 41–66.
Drijvers, P. (2012). Teachers transforming resources into orchestrations. In G. Gueudet, B. Pepin, & L. Trouche (Eds.), *From text to 'lived' resources: Mathematics curriculum materials and teacher development* (pp. 265–281). Springer.
Drijvers, P., Ball, L., Barzel, B., Heid, M. K., Cao, Y., & Maschietto, M. (2016). *Uses of technology in lower secondary mathematics education: A concise topical survey*. Springer.
Drijvers, P., Monaghan, J., Thomas, M., & Trouche, L. (2015). *Use of technology in secondary mathematics: Final report for the International Baccalaureate*. International Baccalaureate Organization. Retrieved from www.ibo.org/globalassets/publications/
Forgasz, H. (2006). Factors that encourage or inhibit computer use for secondary mathematics teaching. *Journal of Computers in Mathematics and Science Teaching, 25*(1), 77–93.
Forgasz, H., Vale, C., & Ursini, S. (2010). Technology for mathematics education: Equity, access and agency. In C. Hoyles & J.-B. Lagrange (Eds.), *Mathematics education and technology : Rethinking the terrain: The 17th ICMI study* (pp. 385–404). Springer.
Forgasz, H., & Rivera, F. (2012). *Towards equity in mathematics education gender, culture, and diversity*. Springer. https://doi.org/10.1007/978-3-642-27702-3
Goos, M. (2014). Technology integration in secondary school mathematics: The development of teachers' professional identities. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital Era* (pp. 139–162). Springer.

Geiger, V., Calder, N., Tan, H., Loong, E., Miller, J., & Larkin, K. (2016). Transformations of teaching and learning through digital technologies. In K. Makar, S. Dole, J. Visnovska, M. Goos, A. Bennison, & K. Fry (Eds.), *Research in mathematics education in Australasia 2012–2015* (pp. 255 280). Springer. https://doi.org/10.1007/978-981-10-1419-2_13

Gueudet, G., Pepin, B., Courtney, S., Kock, Z.-J., Misfeldt, M., & Tamborg, A. (2021). Digital platforms for mathematics teacher curriculum design: Affordances and constraints. In A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, J. Trgalová, & H.-G. Weigand (Eds.), *Mathematics education in the digital age: Learning, practice and theory* (pp. 84–98). Routledge.

Gueudet, G., Pepin, B., Sabra, H., Restrepo, A., & Trouche, L. (2018). E-textbooks and connectivity: Proposing an analytical framework. *International Journal for Science and Mathematics Education, 16*(3), 539–558.

Gueudet, G., Pepin, B., Sabra, H., & Trouche, L. (2016). Collective design of an e-textbook: Teachers' collective documentation. *Journal of Mathematics Teacher Education, 19*(2–3), 187–203.

Gueudet, G., Pepin, B., & Trouche, L. (2013). Collective work with resources: An essential dimension for teacher documentation. *ZDM—Mathematics Education, 45*(7), 1003–1016.

Gueudet, G., Pepin, B., & Trouche, L. (Eds.). (2012). *From Textbooks to 'lived' resources: Mathematics curriculum materials and teacher documentation.* Springer.

Gueudet, G., & Poisard, C. (2018). Design and use of curriculum resources for teachers and teacher educators: Example of the Chinese abacus at primary school. *International Journal of Educational Research, 93*, 68–78.

Gueudet, G., & Trouche, L. (2009). Towards new documentation systems for teachers? *Educational Studies in Mathematics, 71*(3), 199–218.

Guin, D., Ruthven, K., & Trouche, L. (Eds.). (2005). *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument.* Springer.

Haspekian, M. (2014). Teachers' instrumental geneses when integrating spreadsheet software. In A. Clark-Wilson, O. Robutti, & N. Sinclair (Eds.), *The mathematics teacher in the digital Era* (pp. 241–275). Springer. https://doi.org/10.1007/978-94-007-4638-1_11

Haspekian, M. (2017). Computer science in mathematics new curricula at primary school: new tools, new teaching practices? In G. Aldon, & J. Trgalová (Eds.). *Proceedings of the 13th International Conference on Technology in Mathematics Teaching* (pp. 23–32). France.

Hodgen, J., Taylor, B., Jacques, L., Tereshchenko, A., Kwok, R., & Cockerill, M. (2020). *Remote mathematics teaching during COVID-19: Intentions, practices and equity*. UCL Institute of Education.

Hollebrands, K. F., & Lee, H. S. (2020). Effective design of massive open online courses for mathematics teachers to support their professional learning. *ZDM Mathematics Education, 52*(5), 859–875. https://doi.org/10.1007/s11858-020-01142-0.

Hoyles, C., & Lagrange, J. B. (Eds.). (2010). *Mathematics education and technology—Rethinking the terrain. The 17th ICMI study.* Springer.

Hunter, J. M. (2019). *Support provided by administrators for mathematics teachers: a case study of one North Louisiana high school* (Doctoral dissertation). Lousiana Tech University, USA.

Jablonski, S., Ludwig, M., & Zender, J. (2018) Task quality versus task quantity. A dialog-based review system to ensure a certain quality of tasks the MathCityMap web community. In H.G. Weigand, A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, N. Groenbaek, & J. Trgalová (Eds.), *Proceedings of the MEDA2018 Conference* (pp. 115–124). Copenhagen, University of Copenhagen, Denmark.

Jankvist, U.-T. Dreyøe, J., Geraniou, E., Weigand, H.-G., & Misfeldt, M. (2021). CAS from an Assessment Point of View: Challenges and Potentials. In Clark-Wilson, A., Donevska-Todorova, A., Faggiano, E., Trgalová,J. & Weigand, H.-G. *Mathematics education in the digital age. Learning, practice and theory* (pp. 99–120). Routledge.

Joubert, J., Callaghan, R., & Engelbrecht, J. (2020). Lesson study in a blended approach to support isolated teachers in teaching with technology. *ZDM Mathematics Education, 52*, 907–925. https://doi.org/10.1007/s11858-020-01161-x

Julie, C., Leung, A., Thanh, N., Posadas, L., Sacristán, A. I., & Semenov, A. (2010). Some regional developments in access and implementation of digital technologies and ICT. In C. Hoyles & J.B. Lagrange (Eds.), *Mathematics education and technology—Rethinking the terrain. The 17th ICMI study* (Vol. 13, New ICMI Study Series, pp. 361–383). Springer. https://doi.org/10.1007/978-1-4419-0146-0_19

Kimeswenger, B. (2017). Identifying and assessing quality criteria for dynamic mathematics materials on platforms. In T. Dooley & G. Gueudet (Eds.), *Proceedings of the Tenth Congress of the European Mathematical Society for Research in Mathematics Education* (pp. 2414–2421). CERME 10, Dublin, Ireland: DCU Institute of Education and ERME.

Kitchen, R., & Berk, S. (2016). Research commentary : Educational technology: An equity challenge to the common core. *Journal for Research in Mathematics Education, 47*(1), 3–16. https://doi.org/10.5951/jresematheduc.47.1.0003

Ladel, S., Kortenkamp, U., Larkin, K., & Etzold, H. (2018). Evaluation of apps using the ACAT framework. In H.G. Weigand, A. Clark-Wilson, A. Donevska-Todorova, E. Faggiano, N. Groenbaek, & J. Trgalová (Eds.), *Proceedings of the MEDA 2018 Conference* (pp. 171–178). Copenhagen, Denmark: University of Copenhagen.

Leung, A., & Bolite-Frant, J. (2015). Designing mathematics tasks: The role of tools. In A. Watson & M. Ohtani (Eds.), *Task design in mathematics education : An ICMI study 22* (pp. 191–228). Springer. https://doi.org/10.1007/978-3-319-09629-2

Lockard, S. R., & Metcalf, R. C. (2015). Clickers and Classroom voting in a transition to advanced mathematics course. *Primus, 25*(4), 326–338. https://doi.org/10.1080/10511970.2014.977473

Lockwood, E., & Mørken, K. (2021). A call for research that explores relationships between computing and mathematical thinking and activity in RUME. *International Journal of Research in Undergraduate Mathematics Education*, *7*(3), 404–416. https://doi.org/10.1007/s40753-020-00129-2

Manizade, A., Mellone, M., Makonye, J.-P., Ribeiro, M., & Jakobsen, A. (2019). International perspectives on evolution of research on teaching mathematics. In M. Graven, H. Venkat, A. Essien & P. Vale (Eds.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 179–180). Pretoria, South Africa: PME.

Medley, D. M. (1987). Evolution of research on teaching. In M. Dunkin (Ed.), *The international encyclopedia of teaching and teacher education* (pp. 105–113). Pergamon.

Misfeldt, M., Jankvist, U. T., Gerianou, E., & Bråting, K. (2020). Relations between mathematics and programming in school: juxtaposing three different cases. In A. Donevska-Todorova, E. Faggiano, J. Trgalová, Z. Lavicza, R. Weinhandl, A. Clark-Wilson & H.-G. Weigand (Eds.), *Proceedings of the MEDA2020 Conference* (pp. 255–262), Linz University.

Modeste, S. (2015). Impact of informatics on mathematics and its teaching. On the importance of epistemological analysis to feed didactical research. In F. Gadducci, M. Tavosanis (Eds.), *History and philosophy of computing, series: IFIP advances in information and communication technology* (Vol. 487). Springer.

Monaghan, J., Borwein, J., & Trouche, L. (2016). *Tools and mathematics: Instruments for learning*. Springer.

NCTM (2008, March). *The role of technology in the teaching and learning of mathematics. A position of the National Council of Teachers of Mathematics.* Retrieved http://www.nctm.org/uploadedFiles/About_NCTM/Position_Statements/Technology%20final.pdf

Nicol, C. (2018). Connecting mathematics, community, culture and place: Promise, possibilities, and problems. In G. Kaiser, H. Forgasz, M. Graven, A. Kuzniak, E. Simmt, & B. Xu (Eds.), *Invited Lectures from the 13th International Congress on Mathematical Education*. ICME-13 Monographs. Cham: Springer. https://doi.org/10.1007/978-3-319-72170-5_24

Olsher, S., Yerushalmy, M., & Chazan, D. (2016). How might the use of technology in formative assessment support changes in mathematics teaching? *For the Learning of Mathematics, 36*(3), 11–18.

Papert, S. (1993). *Mindstorms: Children, computers and powerful ideas.* Basic Books.

Pepin, B. (2021). Connectivity in support of student co-design of innovative mathematics curriculum trajectories. *ZDM Mathematics Education, 53*, 1221–1232. https://doi.org/10.1007/s11858-021-01297-4

Pepin, B., Choppin, J., Ruthven, K., & Sinclair, N. (2017a). Digital curriculum resources in mathematics education: Foundations for change. *ZDM—Mathematics Education, 49*(5), 645–661.

Pepin, B., Gueudet, G., & Trouche, L. (2017b). Refining *teacher design capacity*: Mathematics teachers' interactions with digital curriculum resources. *ZDM Mathematics Education, 49*(5), 799–812. https://doi.org/10.1007/s11858-017-0870-8

Pepin, B., Gueudet, G., Yerushalmy, M., Trouche, L., & Chazan, D. (2015). E-textbooks in/for teaching and learning mathematics: A disruptive and potentially transformative educational technology. In L. English & D. Kirshner (Eds.), *Handbook of international research in mathematics education* (pp. 636–661). Taylor & Francis.

Pérez, A. (2018). A framework for computational thinking dispositions in mathematics education. *Journal for Research in Mathematics Education, 49*(4), 424–461. https://doi.org/10.5951/jresematheduc.49.4.0424

Ruthven, K. (2009). Towards a naturalistic conceptualisation of technology integration in classroom practice: The example of school mathematics. *Education & Didactique, 3*(1), 131–149.

Sandoval, I., & Trigueros, M. (2021). Technology, knowledge and learning. In A. Clark-Wilson, O. Robutti & N. Sinclair (Eds.), *The mathematics teacher in the digital Era* (2nd ed.). Springer.

Sangwin, C., Cazes, C., Lee, A., & Wong, K. L. (2010). Micro-level automatic assessment supported by digital technologies. In C. Hoyles & J.-B. Lagrange (Eds.), *Mathematics education and technology: Rethinking the terrain: The 17th ICMI study* (pp. 227–250) Springer.

Sinclair, N., Arzarello, F., Trigueros, Lozano, M.D., Dagiene, V., Behrooz, E., & Jackiw, N. (2010). Implementing digital technologies at a national scale. In C. Hoyles, & J.-B. Lagrange (Eds.), *Mathematics education and technology: Rethinking the terrain: The 17th ICMI study* (pp. 61–80). Springer.

Solomon, Y. (2021). Mathematics education in and beyond the COVID-19 pandemic: challenges and opportunities for mathematics education. Thinking about inclusion, diversity and equity. In *Communication at the pre-CERME12 virtual event*.

Stacey, K., & Wiliam, D. (2013). Technology and assessment in mathematics. In M. A. Clements, A. Bishop, C. Keitel, J. Kilpatrick, & F. Leung (Eds.), *Third international handbook of mathematics education* (pp. 721–751). Springer.

Takahashi, A. (2014). The role of the knowledgeable other in lesson study: Examining the final comments of experienced lesson study practitioners. *Mathematics Teacher Education and Development, 16*(1).

Thomas, M. O. J. (2006). Teachers using computers in the mathematics classroom: A longitudinal study. In J. Novotna, H. Moraova, M. Kratka, & N. Stelikhova (Eds.), *Proceedings of the 30th Annual Conference of the International Group for the Psychology of Mathematics Education* (Vol. 5, pp. 265–272). Prague: PME.

Trgalová, J., Soury-Lavergne, S., & Jahn, A. P. (2011). Quality assessment process for dynamic geometry resources in Intergeo project. *ZDM—Mathematics Education, 43*(3), 337–351.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*(3), 281–307.

Trouche, L. (2016). Integrating tools as an ordinary component of the curriculum in mathematics education. In J. Monaghan, L. Trouche, & J.M. Borwein (Eds.), *Tools and mathematics. Instruments for learning* (pp. 267–304). Springer.

Trouche, L., Drijvers, P., Gueudet, G., & Sacristán, A. I. (2013). Technology-driven developments and policy implications for mathematics education. In A. J. Bishop, M. A. Clements, C. Keitel, J. Kilpatrick, & F. K. S. Leung (Eds.), *Third international handbook of mathematics education* (pp. 753–789). Springer.

Trouche, L., Gueudet, G., & Pepin, B. (2020a). Documentational approach to didactics. In S. Lerman (Ed.), *Encyclopedia of mathematics education* (2nd ed., pp. 239–247). Springer. An updated version and translations in different languages in the context on the DAD-Multilingual project is available at https://hal.archives-ouvertes.fr/DAD-MULTILINGUAL/

Trouche, L., Gueudet, G., & Pepin, B. (2019). *The "Resource" approach to mathematics education*. Springer.

Trouche, L., Rocha, K., Gueudet, G., Pepin, B., & Wang, C. (2020b). Transition to digital resources as a critical process in teachers' trajectories: The case of Anna's documentation work. *ZDM Mathematics Education, 52*, 1243–1257. https://doi.org/10.1007/s11858-020-01164-8

UNESCO. (2005). *Towards knowledge societies* (UNESCO World Report). Retrieved from http://www.unesco.org/en/worldreport

Wadham, B., Darragh, L., & Ell, F. (2020). Mathematics home-school partnerships in diverse contexts. *Mathematics Education Research Journal*. https://doi.org/10.1007/s13394-020-00357-4

Wang, C., Trouche, L., & Pepin, B. (2018). An investigation of Chinese mathematics teachers' resources work and their professional development in collectives. In G. Schubring, L. Fan, & V. Giraldo (Eds.), *Proceeding of the Second International Conference on Mathematics Textbooks, Research and Development*. Rio, May 8–11, 2017.

Yang, W., Huang, R., Li, Y., & Li, H. (2021). Training teacher-researchers through online collective academic supervision: Evidence from a postgraduate teacher education programme. *Journal of Computer Assisted Learning, 37*, 1181–1193.

Yerushalmy, M., & Chazan, D. (2008). Technology and curriculum design: The ordering of discontinuities in school algebra. In L. English (Ed.), *Handbook of international research in mathematics education* (2nd ed., pp. 806–837). Routledge.