



# Green Design of Single Frequency Networks by Multiband Robustness and a Hybrid Metaheuristic

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**Abstract.** The passage to a second generation of broadcasting Single Frequency Networks has generated the need for reconfiguring and redesigning existing networks. In this work, we present a robust optimization model for the green design of such networks based on the digital television DVB-T2 standard. Our robust model pursues protection against uncertainty of signal propagation in a complex real-world environment. As reference model, we adopt Multiband Robustness and we propose to solve the resulting model by a hybrid metaheuristic that combines mathematically strong formulations of the optimization model, with an exact large neighborhood search. We report computational tests based on realistic instances, showing that the multiband model grants highly protected solutions without reducing service coverage and without leading to a high price of robustness.

**Keywords:** Green Wireless Networks · Single Frequency Networks · Integer Programming · Robust Optimization · Hybrid Metaheuristics

## 1 Introduction

All around the world, in the last century, cities have experienced a huge expansion, becoming critical socio-economic hubs, and intergovernmental organizations like the United Nations estimates that, by 2050, two out of every three people in the world will live in cities [44]. In this evolution process of cities, Information and Communication Technologies (ICT) have played a crucial role, supporting a better digital interaction of the inhabitants with the urban environment, government and services, leading to the introduction of the concept of “Smart City” [31,42]. Among the fundamental elements of a smart city, telecommunication infrastructures and services have gained a major role and provide a

fundamental basis for the six pillars of a smart city (see e.g., [24,32]). The introduction of the 5th Generation of wireless networks (5G) has in particular been recognized as an accelerator of the realization of actual smart cities, allowing to support the launch of new services with unprecedentedly high performance [12,26,41,43,45]. However, besides such last generation services, elderly broadcasting wireless services like radio and television continue to be regarded as fundamental mass media that must be guaranteed to people for an immediate and plain access to news. A milestone in the evolution of broadcasting radio and television communications was constituted by the passage from analogue to digital transmissions, which has enabled an enhanced exploitation of the frequency spectrum and allowed to furnish services of higher quality [3,23,28]). Within this new digital context, it has been possible to introduce the concept of Single Frequency Networks (SFNs), namely a network that uses only one single frequency channel for broadcasting and in which all the broadcasting stations transmit the same bits on this single channel (see e.g., [25,34,36,39]). The vast majority of SFNs providing digital television broadcasting services are based on the standard DVB-T (Digital Video Broadcasting - Terrestrial): first published in 1997, this standard has quite recently been improved through its second generation (DVB-T2), in order to increase its data carrying capacity, support major operation flexibility and signal reception robustness and allow a major number of broadcasters to co-exist in the same band. Due to the change of the standard, there was the possibility for new broadcasting enterprises to enter the market and old companies have had to update the configuration of their networks. Such reset of the networks has led to revive the attention towards approaches that could automatically design SFN networks on the basis of Mathematical Programming techniques.

In this work, we address the question of developing a Robust Optimization model for designing SFN networks based on the DVB-T2 standard, while taking into account the uncertainty that naturally affects wireless signal propagation. Specifically, our original contributions are:

1. We present a binary linear programming model for representing the robust counterpart based of an SFN design problem including signal-to-interference constraints. The objective of the problem is to pursue a green optimization of the network, finding the minimum total power emission that allows to serve a target fraction of the population of a region. The counterpart is defined according to the principles of Multiband Robust Optimization [8], a refined version of the classical  $\Gamma$ -Robust Optimization model [6].
2. Since the resulting robust optimization model may result very challenging even for a state-of-the art optimization software like IBM ILOG CPLEX, we define a hybrid metaheuristic for its solution, proposing to combine a probabilistic variable fixing procedure with an exact large variable neighborhood search. The probabilistic fixing exploits the precious information that can be derived from a tight linear relaxation of the model adopted to represent the SFN design problem, whereas the exact search consists of exploring a solu-

tion neighborhood formulating the search as an optimization problem that is solved at the optimum.

3. We highlight the performance of our new modelling and algorithmic approach by means of tests conducted on realistic SFN/DVB-T instances, showing the superior performance of the multiband approach with respect to both a benchmark robust and deterministic model.

We remark that, while the deterministic (i.e., not considering data uncertainty) optimal design of wireless networks based on signal-to-interference ratios has received wide attention, the use of optimization under uncertainty techniques, such as Robust Optimization and Stochastic Programming, has received less attention and has especially considered the effects of traffic uncertainty. This is also true for the case of DVB-T, in which optimization approaches have tended to not address data uncertainty by means of optimization under uncertainty techniques (e.g., [2, 17, 30, 33, 34, 37, 38]). To the best of our knowledge, just the work [18] has tried to adopt an optimization under data uncertainty method, in the form of a heuristic min-max regret approach, for tackling signal propagation uncertainty and this paper is the first work that discusses the adaption of a robust optimization approach to DVB-T and presents a hybrid metaheuristic for the solution of the resulting complex problem.

## 2 Optimal SFN Design

To derive an optimization model, we refer to an SFN network based on the DVB-T standard: in such network, all broadcasting stations synchronously transmit identical data on the same frequency channel according to the OFDM modulation scheme [23]. Broadcasting services are spread over a target territory to reach the receiving devices of a population. Following the recommendations of telecommunications regulatory bodies (e.g., [1, 27]), we discretize the target territory into a raster of pixels: each pixel represents a fragment of territory so small that the signal strength measured at its center (testpoint) can be considered representative for the strength of signals in any other point of the pixel. If we denote by  $S$  the set of broadcasting stations and by  $T$  the set of testpoints, the network design problem can be essentially described as that of i) setting the power emission of every station and b) selecting the serving station of each pixel/testpoint, with the objective of minimizing the total emitted power (green perspective) under the condition of granting service to a given fraction of the people located in the target territory.

The previous problem belongs to the family of *Wireless Network Design Problems* (e.g., [14, 37]) and, in particular, it constitutes a variant of the Scheduling and Power Assignment Problem, known to be NP-Hard [11, 37]. We can model the two decisions taken in the considered problem by means of two sets of binary variables, namely:

- to represent the power emission of a station  $s \in S$ , we introduce a set of equally spaced discrete power values  $P = \{p_1, p_2, \dots, p_n\}$  on which each station may emit. We also introduce the set  $L = \{1, 2, \dots, n\}$  to represent the index

of the discrete power values, which we call power levels. Given the set  $P$ , we introduce a set of binary variables  $y_{s\ell} \in \{0, 1\}$  to represent whether a station  $s \in S$  emits on power value  $p_\ell \in P$ , such that  $y_{s\ell} = 1$  when  $s$  emits on  $p_\ell$  and  $y_{s\ell} = 0$  otherwise (in what follows, we also alternatively write that a station  $s \in S$  emits on power level  $\ell \in L$ ).

- to represent whether a testpoint  $t \in T$  is served by a station  $s \in S$ , we introduce a binary variable  $x_{ts} \in \{0, 1\}$  such that  $x_{ts} = 1$  when  $t$  is served by  $s$  and  $x_{ts} = 0$  otherwise.

In order to evaluate whether a testpoint  $t \in T$  is covered with service with reference signal  $s \in S$ , the signal-to-interference ratio must be above a threshold  $\delta > 0$  ([40]):

$$SIR_{ts} = \frac{\sum_{\sigma \in U(t,s)} a_{\sigma t} \cdot \left( \sum_{\ell=1, \dots, n} p_\ell \cdot y_{\sigma\ell} \right)}{N + \sum_{\tau \in I(t,s)} a_{\tau t} \cdot \left( \sum_{\ell=1, \dots, n} p_\ell \cdot y_{\tau\ell} \right)} \geq \delta$$

in which i)  $N > 0$  is the noise of the system, ii)  $\delta > 0$  is the minimum SIR value requested for considering the testpoint served, iii) the power emitted by a station  $\sigma \in S$  is expressed by the combination of the power values  $p_\ell$  by the corresponding binary power activation variable  $y_{\sigma\ell}$  (i.e.,  $\sum_{\ell=1, \dots, n} p_\ell \cdot y_{\sigma\ell}$ ). The summation distinguish between useful and interfering signal according to a DVB-T time-window detection explained in detail in [23, 35, 37].

The SIR lies at the core of every wireless network design problem (see e.g., [11, 14, 16, 22, 29, 37, 40]), and for the specific case that we consider, the model, which we denote by DVB-MILP, is:

$$\min \sum_{s \in S} \left( \sum_{\ell \in L} p_\ell \cdot y_{s\ell} \right) \tag{1}$$

$$\sum_{\sigma \in U(t,s)} a_{\sigma t} \cdot \left( \sum_{\ell \in L} p_\ell \cdot y_{\sigma\ell} \right) - \delta \sum_{\tau \in I(t,s)} a_{\tau t} \cdot \left( \sum_{\ell \in L} p_\ell \cdot y_{\tau\ell} \right) + M(1 - x_{ts}) \geq \delta \cdot N \quad t \in T, s \in S \tag{2}$$

$$\sum_{s \in S} x_{ts} \leq 1 \quad t \in T \tag{3}$$

$$\sum_{t \in T} \sum_{s \in S} \pi_t \cdot x_{ts} \geq \alpha \cdot \sum_{t \in T} \pi_t \tag{4}$$

$$\sum_{\ell \in L} y_{s\ell} = 1 \quad s \in S \tag{5}$$

$$y_{s\ell} \in \{0, 1\} \quad s \in S, \ell \in L \tag{6}$$

$$x_{ts} \in \{0, 1\} \quad t \in T, s \in S \tag{7}$$

in which, a) the objective function (1) pursues the minimization of the total power emission; b) the quality-of-service conditions are expressed by the SIR formulas (easily reorganized by simple algebra operations) and including the big-M term  $M(1 - x_{ts})$  to activate or deactivate the constraint depending on whether testpoint  $t$  is served by station  $s$  (see [14, 17, 19, 29] for details); c) constraints (5)

imposes that each station emits by exactly one power value whereas (3) imposes that each testpoint is served by at most one station. The constraints (4) impose that at least a fraction  $\alpha \in [0, 1]$  of the total population of the region must be covered with service ( $\pi_r$  is the population of one pixel). Finally, (6) and (7) are the decision variables previously defined.

### 3 Protecting Against Propagation Uncertainty

The fading coefficients  $a_{ts}$  that are part of the SIR constraints (2) are naturally subject to uncertainty because of the wide range of factors that influence signal propagation in a real environment (e.g., landscape, obstacles, weather, etc.) and that are hard to precisely assess [40]. These coefficients are commonly computed by (empirical) propagation models that, using extensive field propagation measurements, provide a formula for computing the coefficient values on the basis of factors like the distance between the communicating points, the portion of the spectrum adopted for transmissions, and the characteristics of the propagation environment (e.g., with many obstacles like tall buildings or in line of sight). As well-known by telecommunication professionals, the actual propagation values may be sensibly different from the values returned by the propagation models and it is thus very important to protect design solutions from possible fluctuations in these values.

Since the fading coefficients constitute *uncertain data*, i.e. data whose value is not exactly known when the problem is solved, we protect against fluctuations in their value that could cause infeasibility or sub-optimality of produced solutions by Robust Optimization (RO). RO is possibly the most successful optimization under uncertainty methodology and is essentially based on defining a robust counterpart of the original problem that identifies the best solution under worst data deviations, allowing to control the price of robustness (i.e., the deterioration in the value of optimal solutions due to excluding non-robust solutions from the feasible set) typically by setting one parameter as in the  $\Gamma$ -RO model (see [5, 6]).

As Robust Optimization model, we propose here to adopt *Multiband Robust Optimization* (MB) introduced in [8–10] to generalize and refine classical  $\Gamma$ -Robustness [6]: MB uses multiple deviation bands for better modeling arbitrary discrete distributions, under the form of histograms, which are commonly considered by professionals to analyze deviations in the input data in real-world optimization problems (as also illustrated in [4]). Also, MB allows to take into account “good” deviations, typically neglected in canonical RO approaches. As basis, we assume that the actual value of a generic uncertain fading coefficient  $a_{ts}$  belongs to the symmetric interval  $[\bar{a}_{ts} - d_{ts}, \bar{a}_{ts} + d_{ts}]$  (here,  $\bar{a}_{ts}$  is the nominal value of the uncertain coefficient, while  $d_{ts}$  is its maximum allowed deviation). Practically,  $\bar{a}_{ts}$  could be the value provided by a propagation model, while  $d_{ts}$  could be set as the maximum deviation that the network planner wants to consider according to its risk aversion. Following the principles of MB, for the uncertain fading coefficient we define the following MB Uncertainty Set:

1. we partition the overall deviation range  $[-d_{ts}, d_{ts}]$  into  $K$  bands, defined on the basis of  $K$  deviation values:  
 $-d_{ts} = d_{ts}^{K^-} < \dots < d_{ts}^{-1} < d_{ts}^0 = 0 < d_{ts}^1 < \dots < d_{ts}^{K^+} = d_{ts}$ ;
2. through these deviation values,  $K$  deviation bands are defined, namely: a set of positive deviation bands  $k \in \{1, \dots, K^+\}$  and a set of negative deviation bands  $k \in \{K^- + 1, \dots, -1, 0\}$ , such that a band  $k \in \{K^- + 1, \dots, K^+\}$  corresponds to the range  $(d_t^{k-1}, d_t^k]$ , and band  $k = K^-$  corresponds to the single value  $d_t^{K^-}$ . Note that  $K = K^+ \cup K^-$ ;
3. we define a lower and upper bound on the number of values that may experience a deviation of value in each band: for each band  $k \in K$ , two bounds  $l_k, u_k \in \mathbb{Z}_+$ :  $0 \leq l_k \leq u_k \leq |T| \cdot |S|$  are introduced.

The linear robust counterpart of an uncertain SIR constraint defined for a couple  $(s, t)$  is obtained according to the theoretical results of Multiband Robust Optimization substituting each SIR constraint (2) of  $(s, t)$  with constraints:

$$\sum_{\sigma \in U(s,t)} a_{t\sigma} \cdot \left( \sum_{\ell=1,\dots,n} p_\ell \cdot y_{s\ell} \right) - \delta \sum_{\sigma \in I(s,t)} a_{t\sigma} \cdot \left( \sum_{\ell=1,\dots,n} p_\ell \cdot y_{s\ell} \right) - \left( \sum_{k \in K} \theta_{ts}^k \cdot w_{ts}^k + \sum_{s \in S} z_{ts} \right) + M(1 - x_{ts}) \geq \delta \cdot N \tag{8}$$

$$w_{ts}^k + z_{ts} \cdot \left( \sum_{\ell=1,\dots,n} p_\ell \cdot y_{s\ell} \right) \geq d_{ts}^k \left( \sum_{\ell=1,\dots,n} p_\ell \cdot y_{s\ell} \right) \quad k \in K \tag{9}$$

$$w_{ts}^k \geq 0 \quad k \in K \tag{10}$$

$$z_{ts} \geq 0 \tag{11}$$

which includes the additional dual constraints (9) and variables (10), (11) for linearly reformulating the original (non-linear) robust multiband SIR constraints.

The robust optimization problem that we consider and that we denote by Robu-DVB-MILP is obtained by DVB-MILP substituting each SIR constraints with (8) and the auxiliary dual constraints and variables (9), (10), (11).

## 4 A Hybrid Solution Algorithm

The previous robust problem results challenging to be solved even for a state-of-the-art optimization solver like CPLEX [13], especially because of the presence of the complicating (robust) SIR constraints. To solve it, we thus propose a hybrid metaheuristic that combines heuristic exploration of the feasible set with the adoption of exact optimization methods (i.e., guaranteeing convergence to an optimal solution) for suitable subproblems of the complete problem. Specifically, we propose a metaheuristic that follows the algorithmic principles presented in [15,20], to which we refer the reader for more details. It is mainly based on a *probabilistic variable fixing* procedure integrated with an *exact large neighborhood search*. The probabilistic fixing procedure combines an a-priori and an

a-posteriori fixing measures. In our case, the a-priori measure is provided by a linear relaxation of the robust model (model (1)–(7) with SIR constraints (2) replaced by (8)–(11)), denoted by Robu-DVB-MILP, while the a-posteriori measure is given by a (tighter) linear relaxation of the model (1)–(7), denoted by DVB-MILP (where a subset of variables has been fixed in value). At the end of each cycle of variable fixing, the a-priori fixing measure is updated, evaluating how good were the applied fixing. Once reached a time limit, the fixing cycle stops and an exact search runs for trying to improve the best solution found.

In the probabilistic fixing procedure, a number of solutions are built iteratively: at every iteration, a partial solution (i.e., a solution where only a subset of variables has its value fixed) is available and we can fix the value of an additional variable. Once the value of all the variables has been fixed, we obtain a complete solution whose quality is evaluated by means of its objective value. The fixing procedure is based on the observation that once the power emission variables have been fixed in value, it is possible to easily check which testpoints are covered with service by some station and compute the value of the objective function. We thus base the fixing procedure on deciding the values assumed by the power variables. At a generic iteration of the construction cycle of a feasible solution, we have at disposal a partial solution to the problem (obtained by having chosen the power emissions of a subset of stations  $S^{\text{FIX}} \subseteq S$  by fixing their variables  $y_{sl}$  while respecting (5)). We probabilistically choose the next station whose power is fixed by the formula: whose power emission is fixed by means of the following formula, defined  $\forall s \in S \setminus S^{\text{FIX}}, l \in L$ :

$$p_{sl} = \frac{\alpha \tau_{sl} + (1 - \alpha) \eta_{sl}}{\sum_{s \in S \setminus S^{\text{FIX}}} \sum_{\lambda \in L} [\alpha \tau_{s\lambda} + (1 - \alpha) \eta_{s\lambda}]}, \quad (12)$$

which expresses the probability of fixing the power emission of station  $s \in S \setminus S^{\text{FIX}}$  to power level  $P_l$  by considering all the couples  $\sigma \in S \setminus S^{\text{FIX}}, \lambda \in L$  of stations whose emission is not yet fixed. In the formula,  $\tau_{sl}$  is the a-priori attractiveness measure obtained from the optimal value of Robu-DVB-MILP including power-indexed variables, while  $\eta_{sl}$  is given by the value of a tight linear relaxation of DVB-MILP including fixing of variables done in previous iterations. The two measures are combined by a coefficient  $\alpha \in [0, 1]$ . If we set  $y_{sl} = 1$  for some  $(s, l)$ , due to (5) we can set  $y_{s\lambda} = 0$  for all  $\lambda \in L : \lambda \neq l$ .

After having defined the power emissions of all stations (assume this is denoted by a binary power vector  $\vec{y}$ ), all the SIR ratios can be easily computed. On the basis of the value of these ratios, we can also easily check which testpoints are covered with service and thus derive a valorization of the server assignment variables  $\vec{x}$ . The resulting solution  $(\vec{y}, \vec{x})$  which is feasible for DVB-MILP is accepted as robust when it maintains its feasibility also when the fading coefficients are deviating to their worst value. Once a round of construction of feasible solutions has been operated, the a-priori measures are updated through formula:

$$\begin{aligned} \tau_{sl}(h) &= \tau_{sl}(h-1) + \sum_{\text{SOL}=1}^{\gamma} \Delta\tau_{sl}^{\text{SOL}} \\ \Delta\tau_{sl}^{\text{SOL}} &= \tau_{sl}(0) \cdot \left( \frac{OG(v^{\text{AVG}}, u) - OG(v^{\text{SOL}}, u)}{OG(v^{\text{AVG}}, u)} \right) \end{aligned} \quad (13)$$

where  $\tau_{sl}(h)$  is the a-priori measure of fixing station  $s$  at power level  $P_l$  at the  $h$ -th execution of the cycle and  $\Delta\tau_{sl}^{\text{SOL}}$  is the modification to the value of the a-priori measures, computed over a summation that considers the last  $\gamma$  solutions that have been constructed. Moreover,  $u$  is an upper bound on the optimal value of the problem,  $v^{\text{SOL}}$  is the value of the SOL-th feasible solution built in the last construction cycle,  $v^{\text{AVG}}$  is the average of the values of the last  $\gamma$  solutions that have been constructed. The optimality gap  $OG(v, u)$  measures how far is the value  $v$  of a solution from the upper bound  $u$  and is defined as  $OG(v, u) = (u - v)/v$ . The role of formula (13) is to update the a-priori measure rewarding (penalizing) those fixing that have lead to a solution with lower (higher) optimality gap in comparison to the moving average value  $v^{\text{AVG}}$ .

At the end of the construction cycle, with the aim of improving the best robust solution found, an exact neighborhood search is conducted, i.e. we explore a (very large) neighborhood of the best solution, formulating the search as an optimization problem which is optimally solved by a state-of-the-art solver (see e.g., [7, 21]). The adoption of exact searches is motivated by the fact that, while it can be difficult and long for a solver to solve the complete problem, it is instead possible to efficiently solve to optimality some subproblems. The large neighborhood that we define is built from a robust solution  $(\bar{y}, \bar{x})$  allowing to change the power emission of all stations by either 1) turning off a station  $s$  (i.e., setting  $y_{s0} = 1$  or 2) allowing a modification of the power emission to the adjacent power level set by  $\bar{y}$  (i.e., if  $y_{sl} = 1$  then it is allowed to set  $y_{sl-1} = 1$  or  $y_{sl+1} = 1$ ). The exact search is then conducted by expressing the previous conditions as linear constraints that are added to Robu-DVB-MILP and the resulting problem is solved by an exact solver.

The pseudocode of the matheuristic for solving Robu-DVB-MILP is presented in Algorithm 1. The first step consists of solving the linear relaxation of Robu-DVB-MILP including the power fixing of each couple  $(s, l)$  with  $s \in S$  and  $l \in L$ . The obtained optimal values are employed to initialize the a-priori measures  $\tau_{sl}(0)$ . Then a solution construction cycle is executed until reaching a time limit. In each execution of the cycle, a number of feasible solutions are built first by fixing the power emission binary variables through formula (12), then deriving the corresponding valorization of variables  $x$  and finally checking their robustness. At the end of each execution of the cycle, the a-priori measures  $\tau$  are updated on the basis formula (13). As last step, once the construction time limit is reached, the exact large neighborhood search is conducted, using as basis the best robust feasible solution defined during the construction cycle.



**Algorithm 1**


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1: compute the linear relaxation of the power-indexed version of Robu-DVB-MILP for all  $y_{sl} = 1$ 
   and initialize the values  $\tau_{sl}(0)$  with the corresponding optimal values
2: let  $(x^*, y^*)$  be the best robust feasible solution found
3: while a global time limit is not reached do
4:   for  $SOL := 1$  to  $\gamma$  do
5:     construct a feasible power vector  $\bar{y}$  using the probabilistic fixing formula (12)
6:     check which SIR inequalities are satisfied and derive the corresponding  $\bar{x}$  vector
7:     check the robustness of the feasible solution  $(\bar{x}, \bar{y})$ 
8:     if the coverage granted by  $(\bar{x}, \bar{y})$  is better than that of  $(x^*, y^*)$  then
9:       update  $(x^*, y^*)$  with  $(\bar{x}, \bar{y})$ 
10:    end if
11:   end for
12:   update  $\tau$  according to (13)
13: end while
14: execute the exact large neighborhood search using  $(x^*, y^*)$  and the modified power-indexed
   version of Robu-DVB-MILP as basis
15: return  $(x^*, y^*)$ 

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## 5 Preliminary Computational Results

The robust optimization approach was tested on 15 instances including realistic data defined from regional DVB-T networks deployed in Italy, including up to about 300 stations and 4000 testpoints. The revenue associated with covering a testpoint is represented by the population of the testpoint, so, in what follows, the value of the best solution found by an algorithm is expressed as the percentage of the population covered with service. As optimization software, we used IBM ILOG CPLEX [13] and the algorithms were tested on a Windows machine with 2.70 GHz Intel i7 and 8 GB of RAM. The hybrid metaheuristic of Algorithm 1 ran with a time limit of 1 h (50 min are devoted to the solution construction and 10 min are reserved to the execution of the exact neighborhood search). The parameters  $\alpha$  and  $\gamma$  are set equal to 0.5 and 5, respectively. The robust model takes into account a deviation range that allows deviation up to 20% of the value of the fading coefficients and that is partitioned into 5 deviation bands. In order to evaluate the performance of the multiband robustness model, we considered the coverage of the population that is able to guarantee and the corresponding Price of Robustness (PoR), which we recall to be the reduction in solution optimality that we must pay in order to guarantee protection against uncertain coefficients. We also generated 1000 scenarios of realizations of the uncertain fading coefficients for evaluating the protection that the best found robust solution is able to guarantee. The preliminary results of the computational tests are presented in Table 1, where: i) ID identifies the instance; ii) *COV* is the percentage coverage of the population associated with the best solution found within the time limit and is reported for three models of the design problem, namely *Det*, which is the model not considering the presence of uncertain fading coefficient, *Full*, which considers the model including all the fading coefficients set to their worst value, and *Multi*, which is the Multiband Robust Optimization model; ; iii) *PROT* is the percentage of scenarios in which the best solution found results feasible (specified for the three considered models); iv) *PoR%* is the price of

robustness, expressed as percentage increase in the total power value emitted by all stations.

**Table 1.** Computational results

ID	COV%			PROT%			PoR%		
	Det	Full	Multi	Det	Full	Multi	Det	Full	Multi
1	96.5	81.6	95.7	83.4	100	100	0	32.5	15.5
2	96.7	83.0	96.2	86.0	100	100	0	24.8	13.7
3	95.6	84.2	95.3	88.1	100	100	0	21.3	12.8
4	95.4	83.4	95.9	86.7	100	100	0	30.2	14.3
5	96.3	85.5	96.4	87.5	100	100	0	24.8	12.9
6	95.8	82.9	95.4	87.6	100	100	0	25.7	13.2
7	96.9	83.3	96.3	86.7	100	100	0	25.2	12.4
8	95.6	84.7	95.7	88.8	100	100	0	28.7	14.1
9	95.7	81.5	95.4	87.4	100	100	0	29.3	16.0
10	96.3	85.6	96.8	86.3	100	100	0	27.1	13.7
11	96.2	83.3	96.2	92.4	100	100	0	25.5	14.6
12	95.8	81.0	96.1	90.3	100	100	0	24.2	12.7
13	96.4	84.8	95.7	87.9	100	100	0	27.9	11.6
14	95.5	82.9	95.6	89.6	100	100	0	28.8	13.6
15	95.8	84.6	96.3	88.9	100	100	0	25.7	16.8

Looking at the table, a first observation that can be made is that the percentage coverage granted by the solution associated with full robustness is much lower than those by the deterministic and multiband models (on average only an unsatisfying 83% of the population is covered). This is not so surprising, since imposing full robustness forces the model to take into account all worst data deviations occurring simultaneously and this leads to a substantial shrinkage of the feasible set and to the identification of robust solutions that are unnecessarily conservative (it is indeed highly unlikely that all data jointly deviate to their worst value). In contrast, multiband robustness allows to guarantee a percentage coverage of the population that is very close to that of the deterministic model (on average 96.0% granted by the deterministic model versus 95.9% of the multiband model). This (superior) performance of the multiband model must be observed also taking into account the protection that is offered: the multiband model is able to offer the same full 100% protection of the full robustness model, which is much higher than that associated with the deterministic model, whose solutions turn out to be infeasible for about 12% of the cases. Finally, if we look at the price of robustness, the multiband model is able to entail a percentage increase in total power which is about halved on average with respect to full robustness (naturally, the deterministic model is associated with null price

of robustness since it does not provide any protection). Looking jointly at the three performance indicators, multiband robustness is thus able to guarantee a full protection against deviations in propagation while maintaining the same level of coverage of the deterministic model and granting a substantial reduction in the price of robustness with respect with the full robustness model.

As future work, we intend to widen the computational experience to a larger set of instances, also conducting a study about the impact of parameter tuning. Moreover, we intend to also better study the impact of different characterization of the uncertainty set on the robustness of solutions.

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