# Three-Dimensional Stress State of Earth Dams Under Static Loads

**Doniyor Juraev** ⓘ **, Paxridin Matkarimov** ⓘ **, and Mirziyod Mirsaidov** ⓘ

**Abstract** The article is devoted to the study of the stress–strain state and strength of various earth dams under the action of static loads. The stress–strain state of the Gissarak, Sokh and Pachkamar earth dams built in Central Asia is studied in a three-dimensional formulation under the action of body forces and hydrostatic water pressure. It was established that under the action of body forces and hydrostatic water pressure, the plane deformed state for some dams is realized in a very narrow range along the length of the dam, and for other dams, this state is realized in a wide range. The equivalent stress for the three-dimensional stress state of the dams under consideration was estimated using the fourth maximum-strain-energy theory, and the results obtained were compared with the allowable stress for the soil material of the dams, which showed sufficient strength of these dams under the action of static loads.

**Keywords** Earth dam · Three-dimensional problem · Static load · Stress–strain state · Equivalent stress · Maximum-strain-energy theory

## 1 Introduction

When designing earth dams in seismic regions, there arises the problem of choosing a calculation model for studying their stress–strain state (SSS) and dynamic behavior in general. In most cases, simplified plane design schemes are used in the study of such types of structures.

D. Juraev · M. Mirsaidov (✉)
National Research University -Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Tashkent, Uzbekistan
e-mail: theormir@mail.ru; diyorbekmuhammadamir@mail.ru

P. Matkarimov
Namangan Institute of Engineering and Technology, Namangan, Uzbekistan

M. Mirsaidov
Institute of Mechanics and Seismic Stability of Structures of the Academy of Sciences of the Republic of Uzbekistan, Tashkent, Uzbekistan

Simplified design schemes are unable to describe many effects of the spatial work of real structures. In this regard, the material of the structures is often used non-rationally. This causes an overrun of the material and the impossibility of providing the required margin of safety and reliability of the structure.

The solution to the above problem, considering the factors listed, can be most fully obtained using numerical methods, for example, the finite element method (FEM) or the finite difference method (FDM) [1–5, 22].

To date, there are a number of scientific publications devoted to the study of the stress–strain state of earth structures, both in a plane or spatial formulation.

The stress–strain state of various earth dams in a plane or spatial formulation is considered in [6–15]; this formulation takes into account the design features of structures, the moisture properties of soil, the interaction of structures with the water environment of the reservoir, and other features of structures.

The study in [16] presents the results of the assessment of the stress–strain state of high earth dams depending on time (consolidation analysis) in a plane formulation. Some results of calculations to determine the effect of water pore pressure on the stress–strain state and subsidence of the dam are analyzed.

In [17], the stress state of earth dams under static and dynamic effects was studied by the finite element method, taking into account the elastic–plastic strain of the soil of the dam. The numerical results obtained were compared with the results of field measurements during the Wenchuan earthquake.

The study in [18] analyzes in detail the use of non-traditional materials (rock and soil mixes) to ensure the stability of the slopes of earth dams.

In [19], scientific achievements and main conclusions are described; i.e. accumulated experience in the construction of high rock-and-earth dams is systematically summarized, and major technical issues are discussed, including control of strains, seepage, slope stability, safety assessments and other issues related to earth dams.

In [20], a review and analysis of the results of studies of the stress–strain state of rockfill dams with a reinforced concrete screen, performed by different authors, were presented. The results of analytical, experimental and numerical studies are considered. The models used to reproduce the non-linear nature of the screen deformation and rockfill in the numerical simulation of the stress–strain state of dams are described.

As the review shows, the stress–strain state and strength of earth dams, taking into account design features and real work in a three-dimensional formulation, were not studied sufficiently, therefore, research in this direction is of great scientific interest. The prediction of the behavior of earth dams should be based on the most complete consideration of all factors affecting their SSS and strength under various types of loads.

Based on the above, this work is devoted to the development of methods for assessing the stress–strain state and strength of the Gissarak, Sokh, and Pachkamar earth dams built in Central Asia in a three-dimensional formulation, taking into account the design features, physical and mechanical properties of building materials. In this article, the finite element method (FEM) is used as a computing tool.

## 2 Methods

A three-dimensional non-homogeneous deformable system (Fig. 1) is considered here, i.e. a model of an earth dam that occupies volume $V = V_1 + V_2 + V_3$. The area of the base of the system along the bottom is $\Sigma_4''$ and the areas of two coastal faces $\Sigma_4'$, $\Sigma_4''$ are rigidly fixed; the surface of the lower slope and crest are stress-free. It is assumed that body forces $\vec{f}$ and hydrostatic water pressure $\vec{p}$ act on surface $S_p$ of the earth dam.

Here: $V_1$, $V_3$ are the volumes of the upper and lower prisms, $V_2$ is the volume of the core.

It is necessary to determine the components of the displacement vectors and the components of the stress tensor in a three-dimensional body (Fig. 1) under the action of force $\vec{f}$, and water pressure $\vec{p}$.

For the mathematical formulation of the problem, the principle of virtual displacements, the generalized Hooke's law, the Cauchy relation for a three-dimensional body [8], and kinematic boundary conditions are used.

Boundary conditions are:

$$\vec{x} \in \Sigma_4' + \Sigma_4'' + \Sigma_4''' : \quad \vec{u} = 0 \tag{1}$$

Here $\vec{u} = \{u_1, u_2, u_3\} = \{u, v, w\}$ are the components of the displacement vector of the point of the body; $\{x\} = \{x_1, x_2, x_3\} = \{x, y, z\}$ are the coordinates of the point of the body; $i, j, k = 1, 2, 3$.

We determine the functions of displacements $\vec{u}(\vec{x})$, strains $\varepsilon_{ij}(\vec{x})$ and stresses $\sigma_{ij}(\vec{x})$ arising under static forces $(\vec{f})$ and pressure $(\vec{p})$ in the body of the system (Fig. 1).
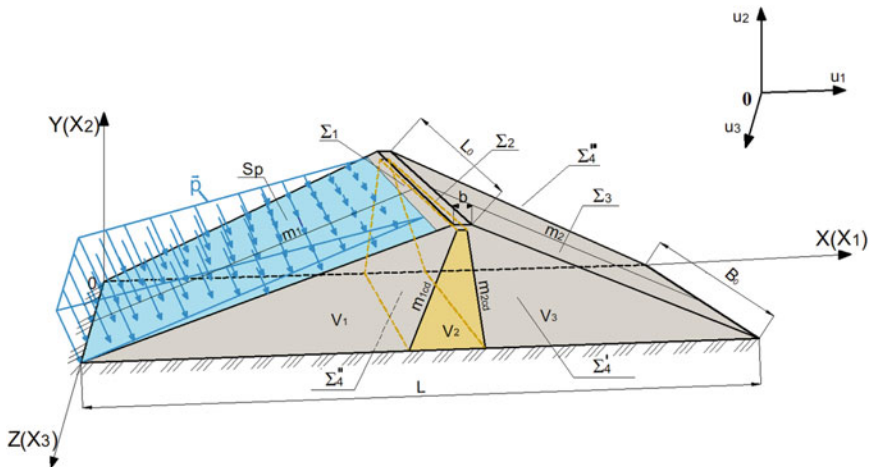


**Fig. 1** Calculation scheme of a non-homogeneous three-dimensional system

The finite element method (FEM) is the most appropriate in order to solve this problem for a domain of non-canonical shape (Fig. 1); it allows taking into account both the geometry features and the properties of the material of the system.

Here, the domain occupied by the body is divided into sub-domains with different physical and mechanical characteristics, then the sub-domains are automatically partitioned into finite elements. As a result, a discrete model of the system is created.

When solving this three-dimensional problem, first-order volumetric elements were used in the form of tetrahedra with 12 degrees of freedom. A linear function is used to approximate the displacement field inside a volumetric tetrahedron.

The procedure of the finite element method allows us to reduce the three-dimensional problem under consideration to a system of non-homogeneous high-order algebraic equations, i.e.:

$$[k]\{u\} = \{P\} \tag{2}$$

Here: $[K]$ is the stiffness matrix for the system under consideration (Fig. 1); $\{u\}$ are the sought-for components of the displacement vectors, in the nodes of the finite element; $\{P\}$ are the components of external forces (body and surface ones) acting on the nodes of the finite element.

In the study of specific three-dimensional problems, the partition of given domain $V$ into finite elements is performed considering the design features and physical and mechanical properties of the material of different parts of the dam. When solving this problem, the computer programs developed by the authors, and the standard ABAQUS programs were used. When solving specific problems, the number of unknowns in these equations reached 68,493.

## 3    Results and Discussion

The stress–strain state (SSS) and the strength of the Gissarak, Sokh, and Pachkamar earth dams are investigated in the article under the action of body forces and hydrostatic water pressure, in a three-dimensional formulation.

Body forces $\vec{f}$ acting on the structure and the hydrostatic pressure of water in the upstream face acting on the upper slope of the dam were taken as external impacts, determined by the following formula

$$\vec{p} = \rho_0 g(h - x_2) \tag{3}$$

here, $\rho_0$ is the density of water; $h$ is the height of reservoir filling.

The study was performed for the following earth dams built on the territory of the seismic region of Central Asia. The main geometric parameters of these structures, as well as the physical and mechanical characteristics of soil in various sections of the dam, were taken from the design data:

1) **Gissarak dam** (Fig. 2), H $= 138.5$ m high, built on the Aksu river in the Kashkadarya region of Uzbekistan, with slope coefficients $m_1 = 2.2$, $m_2 = 1.9$. Retaining prisms 1 are laid out of rock mass with the following physical and mechanical parameters E $= 3600$ MPa, soil specific gravity $\gamma = 1.9$ tf/m$^3$, Poisson's ratio $\nu = 0.3$ and cohesion coefficient C $= 2$ kPa. Core 4 is laid from loam with physical and mechanical parameters - E $= 2400$ MPa, soil specific gravity $\gamma = 1.7$ tf/m$^3$, Poisson's ratio $\nu = 0.35$ and cohesion coefficient C $= 20$ kPa. The transition zone is of sandy-gravelly soil. The crest of the dam is b $= 16$ m wide and L $= 660$ m long.

2) **Sokh dam,** H $= 87.3$ m high, was built on the Sokh River in the Fergana region, with slope coefficients $m_1 = 2.5$, $m_2 = 2.2$. Retaining prisms are laid from pebbles with physical and mechanical parameters - E $= 3550$ MPa, specific gravity of soil $\gamma = 2.1$ tf/m$^3$, Poisson's ratio $\nu = 0.35$ and cohesion coefficient C $= 10.9$ kPa. The core is laid from loam with physical and mechanical parameters E $= 2400$ MPa, soil specific gravity $\gamma = 1.75$ tf/m$^3$, Poisson's ratio $\nu = 0.35$ and cohesion coefficient C $= 30 - 50$ kPa. The crest of the dam is b $= 10$ m wide and L $= 487.3$ long.

3) **Pachkamar dam,** H $= 70$ m high, erected in Kashkadarya region, with slope coefficients $m_1 = 2.25$, $m_2 = 2.25$. Retaining prisms are from sand-pebbles with physical and mechanical parameters E $= 3600$ MPa, soil specific gravity $\gamma = 2.25$ tf/m$^3$, Poisson's ratio $\nu = 0.3$ and coefficient of cohesion C $= 11$ kPa. The core is laid from loam with physical and mechanical parameters E $= 2400$ MPa, soil specific gravity $\gamma = 1.78$ tf/m$^3$, Poisson's ratio $\nu = 0.35$ and cohesion coefficient C $= 30$ kPa. The crest of the dam is b $= 8$ m wide and L $= 589$ m long.
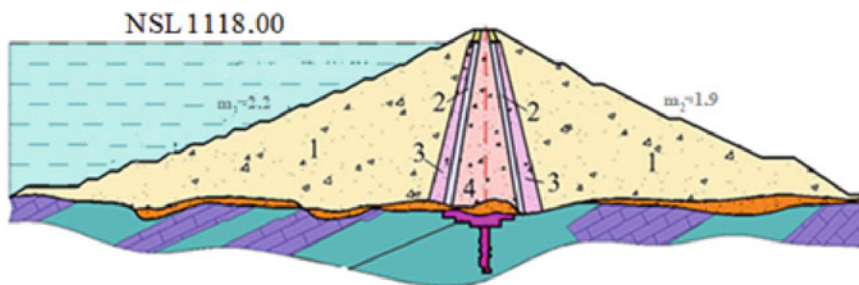


**Fig. 2** Cross section of the Gissarak dam 1 - retaining prisms, 2 - I layer of transition zones, 3 - II layer of transition zones, 4 – core

## 3.1   Evaluation of the Stress–Strain State of Earth Dams

The SSS of earth dams is studied in a three-dimensional formulation. The calculation results are the components of displacement vectors $u_1$, $u_2$, $u_3$, strains $\varepsilon_{11}$, $\varepsilon_{22}$, $\varepsilon_{33}$, $\varepsilon$ and stresses $\sigma_{11}$, $\sigma_{22}$, $\sigma_{33}$, $\sigma$ for all points of the structure (Fig. 1).

To analyze the results in the characteristic transverse and longitudinal sections of the dam, isolines of equal values of the components of displacements, strains, and stresses were constructed.

Figure 3 shows the distribution field of equal values of vertical $u_2$ - (along the $x_2$ axis) and longitudinal $u_3$- (along the $x_3$ axis) displacements in the middle of the longitudinal section of the Gissarak (a, d), Sokh (b, e), and Pachkamar (c, f) dams obtained in a three-dimensional formulation under the own weight of the structures.

An analysis of the results presented (Fig. 3), shows that in the longitudinal section of the Gissarak dam, there is a sharp change in the distribution pattern of vertical $u_2$ and longitudinal $u_3$ displacements of points from the middle part of the dam to the coastal slopes (Fig. 3a). This indicates a significant influence of the left and right banks on the value of vertical and longitudinal displacements of the middle part of the dam.

For this dam (Fig. 3a), a plane deformed stress state is realized in a very narrow range (a section marked in blue) since the geometric dimensions of this dam in three directions are of the same order.

For the Sokh and Pachkamar dams, the area where the plane-deformed stress state is observed is quite large (Fig. 3b, c) since for these structures the influence of coastal slopes on the stress state of the structure is insignificant (a section marked in blue).

The values of longitudinal displacements $u_3$ in the left and right slopes of all dams increase sharply. This phenomenon can lead to the formation of transverse cracks in the upper part of the dams adjacent to the banks.

Along with this, the value of displacements (i.e., absolute deformation) of the profile points of the dams under consideration depends significantly on considering the level of water filling in the reservoir. In this case, the distribution pattern of displacements in the body of the dam completely changes, the symmetry is broken, and the absolute deformation of the dam significantly depends on the value of water pressure in the upstream face, which is especially evident when calculating the structure for the case of a completely filled reservoir.

Figure 4 shows lines of equal levels of horizontal $\sigma_{11}$ (a), vertical $\sigma_{22}$ (b) and shear $\sigma_{12}$ (c) stresses of the Gissarak dam under its own weight and hydrostatic water pressure when the reservoir is completely filled. Such results were also obtained for the Pachkamar and Sokh earth dams in three-dimensional and spatial formulations at different levels of reservoir filling.

The results obtained also showed that in the central part of the Sokh and Pachkamar dams, the conditions of plane deformation of the theory of elasticity are observed, which are not observed in the Gissarak dam.

a) $u_2$, m

b) $u_2$, m

c) $u_2$, m

d) $u_3$, m

e) $u_3$, m

f) $u_3$, m
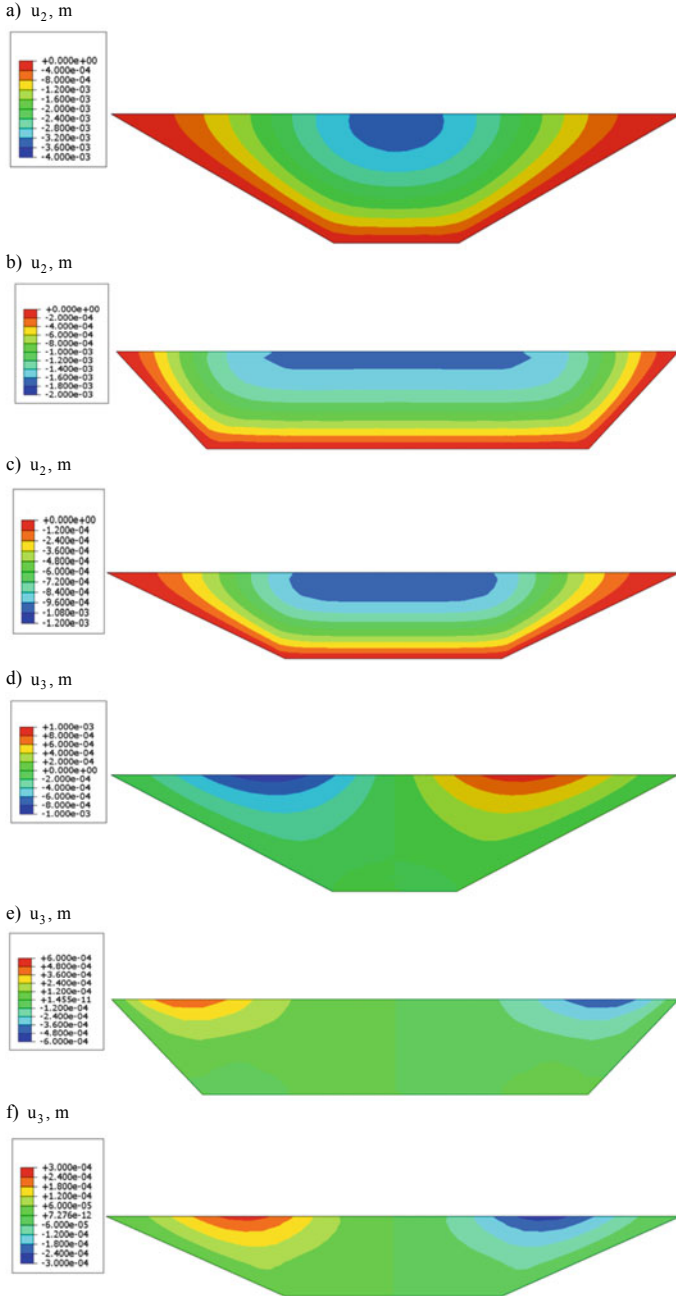
**Fig. 3** Distribution field of equal values of vertical $u_2$ (along the $x_2$ axis) and longitudinal $u_3$ (along the $x_3$ axis) displacements in the middle of the longitudinal section of the Gissarak (a, d), Sokh (b, e) and Pachkamar (c, f) dams in a three-dimensional formulation under the action of their own weight
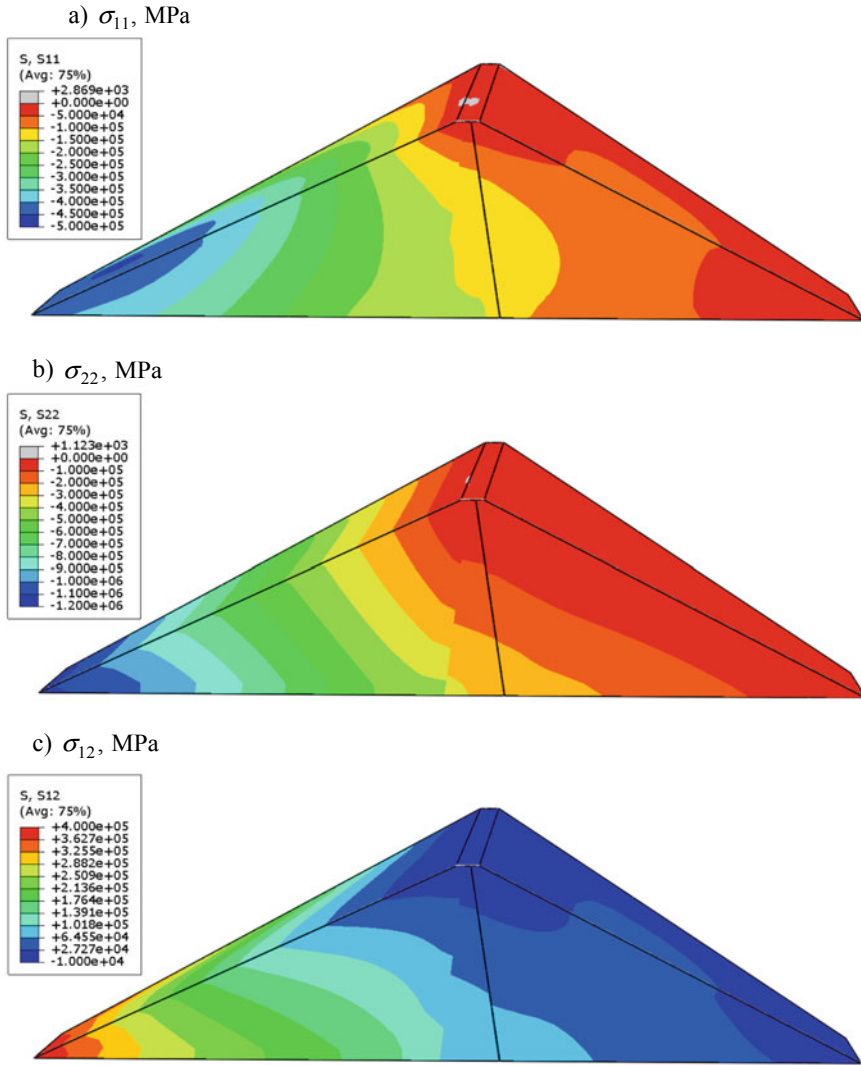
a) $\sigma_{11}$, MPa

b) $\sigma_{22}$, MPa

c) $\sigma_{12}$, MPa

**Fig. 4** Isolines of equal values of horizontal $\sigma_{11}$ **a** vertical $\sigma_{22}$ **b** and shear $\sigma_{12}$ **c** stresses for the Gissarak dam in 3D formulation when the reservoir is completely filled

## 3.2 Strength Assessment of Earth Dams

Using the results obtained in paragraph 3.1 for stresses, the strength of the three dams under consideration was estimated in a three-dimensional formulation using the maximum-strain-energy theory. It is assumed that the potential energy of deformation of the material in the elastic region is the sum of the potential energy of a change in its volume and the potential energy of a change in its shape.
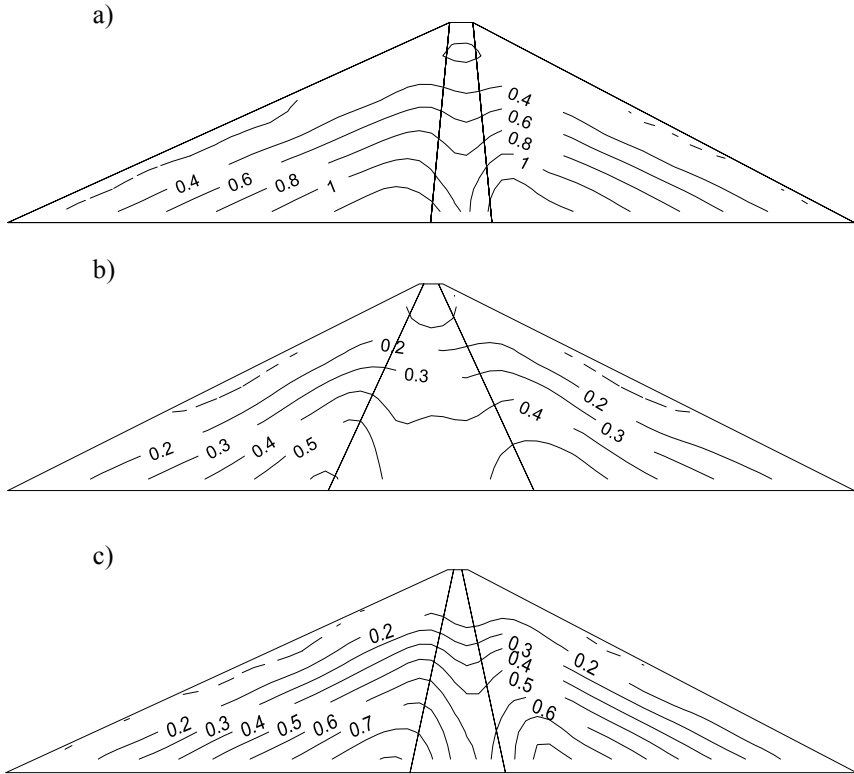
**Fig. 5** Isolines of equal values of equivalent stress $\sigma_{equiv}$ in the body of the Gissarak **a** Sokh **b** and Pachkamar **c** earth dams

In accordance with the requirements of the maximum-strain-energy theory, the following condition must be met at all points of earth dams:

$$\sigma_{equiv}\sqrt{\frac{1}{2}\left[(\sigma_1 - \sigma_2)^2 + (\sigma_1 - \sigma_3)^2 + (\sigma_3 - \sigma_2)^2\right]} \leq [\sigma] \qquad (4)$$

Here $\sigma_1, \sigma_2, \sigma_3$ are the principal stresses, and $[\sigma]$ is the allowable normal stress.

To assess the strength of the Gissarak, Sokh, and Pachkamar earth dams in a three-dimensional formulation under the action of their own weight and hydrostatic water pressure in a completely filled reservoir, the values of equivalent stress $\sigma_{equiv}$ at all points of the dam body were determined using expression (4). The results obtained, i.e. isolines of equal values of $\sigma_{equiv}$ for the average cross-section are shown in Fig. 5.

To compare the results of $\sigma_{equiv}$ with the value of $[\sigma]$, the following estimate can be made: in [21], it was noted that the high strength of individual particles of gravel-pebble soil predetermines the boundary of mass damage at $[\sigma] = 2.0 \pm 0.5$ MPa (the values of $[\sigma]$ were determined experimentally).

An analysis of the distribution pattern of equivalent stress $\sigma_{\text{equiv}}$ in the body of the dams shows that the value of $\sigma_{\text{equiv}}$ in the body of the dams under consideration completely satisfies the condition of the maximum-strain-energy theory of mechanics (4) at all points of these dams. The results obtained show that the required strength of the dams under the action of body forces and hydrostatic water pressure is ensured at completely filled reservoirs.

## 4  Conclusions

1. The Stress–strain State and Strength of Three Earth Dams of Different Heights in a Three-Dimensional Formulation Under the Action of Body Forces and Hydrostatic Water Pressure Were Studied.
2. It was determined that under the action of body forces and hydrostatic water pressure, the plane deformed state for the Gissarak dam is realized in a very narrow range along the length of the dam, and for other dams, this state is realized in a wide range.
3. Equivalent stress for the three-dimensional stress state for these dams was estimated using the maximum-strain-energy theory and the results obtained were compared with the allowable stress for soil; this showed sufficient strength of these dams under the action of static loads.

## References

1. Konstantinov, I.A.: Dynamics of hydro-technical structures. Part **2**, 196, LPI (1976)
2. Mirsaidov, M.M.: Theory and methods for calculating earth structures for strength and seismic resistance. Tashkent: "FAN," 312 (2010)
3. Zaretsky, Y.K., Lombardo, V.N.: Statics and dynamics of earth dams, p. 256, Energoizdat (1983)
4. Krasnikov, N.D.: Seismic resistance of hydro-technical structures made of earth materials, p.240, Energoizdat (1981)
5. Lyakhter, V.M.: I.I.N. Seismic resistance of earth dams, p. 233, Nauka (1986)
6. Sinenko, E.G., Konisheva, O.V.: Kinematics and geometry of misaligned epicyclic mechanisms. J. Irkutsk State Univ. Commun. Modern Technol. **4**(40), 39–43 (2013)
7. Mirsaidov, M.: An account of the foundation in assessment of earth structure dynamics. In: E3S Web of Conferences (2019). https://doi.org/10.1051/e3sconf/20199704015
8. Mirsaidov, M.M., Toshmatov, E.S.: Spatial stress state and dynamic characteristics of earth dams. Mag. Civil Eng. (2019). https://doi.org/10.18720/MCE.89.1
9. Pinyol, N.M., Alonso, E.E.: Earth dam, spatial model, stress-strain state, dynamic characteristic, natural frequency, modes of oscillations. Int. J. Civil Eng. **17**, 501–513 (2019)
10. Mirsaidov, M., Sultanov, T., Yarashov, J., Toshmatov, E.: Assessment of dynamic behaviour of earth dams taking into account large strains. In: E3S Web of Conferences, vol. 97 (2019). https://doi.org/10.1051/e3sconf/20199705019
11. Mirsaidov, M.M., et al.: Mathematical simulation and the methods to assess the strength of earth dams. In: International Conference on Information Science and Communications Technologies:

Applications, Trends and Opportunities, ICISCT 2019, p. 9011818 (2019). https://doi.org/10.1109/ICISCT47635.2019.9011818

12. Urazmukhamedova, Z., Juraev, D., Mirsaidov, M.: Assessment of stress state and dynamic characteristics of plane and spatial structure. J. Phys: Conf. Ser. (2021). https://doi.org/10.1088/1742-6596/2070/1/012156
13. Mirsaidov, M.M., Sultanov, T.Z., Yarashov, J.Y.: Strength of earth dams considering elastic-plastic properties of soil. Maga. Civil Eng. (2021). https://doi.org/10.34910/MCE.108.13.8.pp.10813
14. Fu, Z., Chen, S., Li, G.: Hydrodynamic pressure on concrete face rockfill dams subjected to earthquakes. J. Hydrodyn. **31**, 152–168 (2019)
15. Wang, M., Chen, J., Xiao, W.: Experimental and numerical comparative study on gravity dam-reservoir coupling system. KSCE J. Civil Eng. **22**, 3980–3987 (2018)
16. Germanov, T.: Effect of the pore water pressure on the stress-strain behavior of earth dams. In: GEOTECH YEAR 2000, Developments in Geotechnical Engineering, pp. 429–438 (2000)
17. Kong, X.J., Liu, J.M., Zou, D.G.: Numerical simulation of the separation between concrete face slabs and cushion layer of Zipingpu dam during the Wenchuan earthquake. Sci. China Technol. Sci. (2016). https://doi.org/10.1007/s11431-015-5953-6
18. Alonso, E.E., Cardoso, R.: Behavior of materials for earth and rockfill dams: perspective from unsaturated soil mechanics. Front. Archit. Civil Eng. China (2010). https://doi.org/10.1007/s11709-010-0013-6
19. Ma, H., Chi, F.: Major technologies for safe construction of high earth-rockfill dams. Eng. (2016). https://doi.org/10.1016/J.ENG.2016.04.001
20. Soroka, V.B., Sainov, M.P., Korolev, D.V.: Rockfill dams with reinforced concrete screen: experience of stress-strain state studies. Bull. MGSU Hydraulics. Geotech. Hydraulic Eng. **14**(2), 207–224 (2019)
21. Grishin, M.M.: Hydro-technical structures. Part 1. High School, 615 (1979)
22. Ilichev, V.A., Yuldashev, S.S., Matkarimov, P.J.: Forced vibratins of an inhomogeneous planar system with passive vibratinal insulation. Soil Mech. Found. Eng. **36**, 50–54 (1999). https://doi.org/10.1007/BF02469084